Certain Topological Indices and Polynomials for Semitotal Line Graph and its Line Graph for Dutch Windmill Graph [1]

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Abstract

Dutch windmill graph [1, 2] and denoted by D_m^n. Order and size of Dutch windmill graph are $(n-1)m+1$ and mn respectively. In this paper, we computed certain topological indices and polynomials *i.e.* Zagreb polynomials, hyper Zagreb index, Redefined Zagreb indices, modified first Zagreb index, Reduced second Zagreb index, Reduced Reciprocal Randić index, 1^{st} Gourava index, 2^{nd} Gourava index, 1^{st} hyper Gourava index, 2^{nd} hyper Gourava index, Product connectivity Gourava index, Sum connectivity Gourava index, Forgotten index, Forgotten polynomials, M-polynomials and some topological indices in terms of M-polynomials *i.e.* 1^{st} Zagreb index, 2^{nd} Zagreb index, Modified 2^{nd} Zagreb index, Randić index, Reciprocal Randić index, Symmetric division index, Harmonic index, Inverse Sum indeg index, Augmented Zagreb index for the semitotal line graph and line graph of semitotal line graph for Dutch windmill graph.

Keywords and Phrases: Dutch windmill graph, Operations on graphs, Subdivision of graph, Semi total line graph, Line graph.

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1 Definitions, Notations and Results

All the graphs in this paper are simple, finite and undirected. In a graph G, $V(G)$ and $E(G)$ are the sets of vertices and edges respectively. Let $d_G(u)$ denotes the degree of a vertex u. Topological indices have been found to be useful in establishing relation between the structure and the properties of molecules. Topological indices mainly used in Quantitative Structure Property Relationship (QSPR) and Quantitative Structure Activity Relationships (QSAR) [3]. Some topological indices are degree based and some are distance based.

The *Zagreb indices* were introduced more than thirty years ago by Gutman and Trinajstić [4]. After ten years, Balaban *et.al* named them *Zagreb group index*, presented by M_1 and M_2. Later it was abbreviated to *Zagreb index* [5], where M_1 and M_2 represents *first Zagreb index* and *second Zagreb index* respectively. If d_u and d_v are the degrees of vertices u, v for simple graph G. Then *first Zagreb index* [5, 6] is defined as

$$M_1(G) = \sum_{p \in V(G)} \left(d_G(p)\right)^2$$
$$= \sum_{pq \in E(G)} d_G(p) + d_G(q)$$

Second Zagreb index is defined as

$$M_2(G) = \sum_{pq \in E(G)} d_G(p) d_G(q)$$

Third Zagreb index introduced by Fath-Tabar [7] in 2011. Which is denoted by $ZG_3(H)$ for a simple graph G and defined as

$$ZG_3(G) = \sum_{pq \in E(G)} \mid d_p - d_q \mid$$

In the same year 2011, same author proposed the concept of *First, Second and Third Zagreb polynomials* [7], for a simple graph G is defined as

$$ZG_1(G, x) = \sum_{pq \in E(G)} x^{d_p + d_q} \tag{1}$$

$$ZG_2(G, x) = \sum_{pq \in E(G)} x^{d_p d_q} \tag{2}$$

$$ZG_3(G, x) = \sum_{pq \in E(G)} x^{|d_p - d_q|} \tag{3}$$

Modified Zagreb index [8] is also an important degree based graph invariant. *First modified Zagreb index* for a graph H is denoted by ${}^m M_1(G)$, defined as

$$\begin{aligned} {}^m M_1(G) &= \sum_{q \in V(G)} \frac{1}{(d_q)^2} \\ &= \sum_{pq \in E(G)} \frac{1}{d_p + d_q}. \end{aligned} \tag{4}$$

Similarly *second modified Zagreb indices* is denoted by ${}^{m}M_2(G)$, defined as

$$ {}^{m}M_2(G) = \sum_{pq \in E(G)} \frac{1}{d_p d_q}. $$

Shirdel *et.al* [9] acquainted a new degree based graph invariant named as *hyper-Zagreb index*, which is defined as

$$ HM(G) = \sum_{pq \in E(G)} (d_p + d_q)^2 \tag{5} $$

Forgotten topological index is also a degree based topological index, denoted by $F(G)$ for simple graph G. It was encountered in [10], defined as

$$ \begin{aligned} F(G) &= \sum_{q \in V(G)} (d_q)^3 \\ &= \sum_{pq \in E(G)} [(d_p)^2 + (d_q)^2] \end{aligned} \tag{6} $$

Forgotten polynomial for a graph G is defined as

$$ F(G, x) = \sum_{pq \in E(G)} x^{[(d_p)^2 + (d_q)^2]} \tag{7} $$

Randić connectivity index was put forward by Randić in 1975, defined as

$$ \begin{aligned} R_{-1/2}(G) &= \sum_{pq \in E(G)} (d_p d_q)^{-1/2} \\ &= \sum_{pq \in E(G)} \frac{1}{\sqrt{d_p d_q}} \end{aligned} \tag{8} $$

and extended to *general Randić connectivity index* [11]. If α is a real number, then it is defined as

$$R_\alpha(G) = \sum_{pq \in E(G)} (d_p d_q)^\alpha$$

In 2014, I. Gutman *et.al*. [12] proposed *reciprocal Randić, reduced second Zagreb index* and *reduced reciprocal Randić index*, these are degree based graph invariants, defined as

$$RR(G) = \sum_{pq \in E(G)} \sqrt{d_p d_q}$$

$$RM_2(G) = \sum_{pq \in E(G)} (d_p - 1)(d_q - 1) \tag{9}$$

and

$$RRR(G) = \sum_{pq \in E(G)} \sqrt{(d_p - 1)(d_q - 1)} \tag{10}$$

Harmonic index for a graph G defined as

$$H(G) = \sum_{pq \in E(G)} \frac{2}{d_p + d_q}$$

Many researchers are working on these graph invariants as only one index does not fully describe chemical properties of a chemical structure. Recently in 2017, V. R. Kulli proposed many new graph invariants [13–16], which are named as Gourava indices, product connectivity Gourava index, sum connectivity Gourava index and hyper Gourava indices.

First Gourava index and *Second Gourava index* [13] are defined as

$$G_1O(G) = \sum_{pq \in E(G)} [d_p + d_q + d_p d_q] \tag{11}$$

and

$$G_2O(G) = \sum_{pq \in E(G)} [(d_p + d_q)(d_p d_q)] \tag{12}$$

respectively.

we can rewrite second Gourava index as

$$= \sum_{pq \in E(G)} [d_p^2 d_q + d_q^2 d_p] \tag{13}$$

Product Connectivity Gourava index is denoted by PGO and defined as

$$PGO(G) = \sum_{pq \in E(G)} \frac{1}{\sqrt{(d_p + d_q)(d_p d_q)}}. \tag{14}$$

Sum connectivity Gourava index is given by

$$SGO(G) = \sum_{pq \in E(G)} \frac{1}{\sqrt{(d_p + d_q) + (d_p d_q)}}. \tag{15}$$

Hyper-Gourava indices i.e. first and second hyper Gourava index are defined as

$$HGO_1(G) = \sum_{pq \in E(G)} [(d_p + d_q) + (d_p d_q)]^2 \tag{16}$$

and

$$HGO_2(G) = \sum_{pq \in E(G)} [(d_p + d_q)(d_p d_q)]^2 \tag{17}$$

respectively. There are some new degree based graph invariants, which plays an important role in chemical graph theory. These topological indices are quite useful for determining total surface area and heat formation of some chemical compounds. These graph invariants are as follows *Symmetric division index*

$$SDD(G) = \sum_{pq \in E(G)} \left\{ \frac{min(d_p, d_q)}{max(d_p, d_q)} + \frac{max(d_p, d_q)}{min(d_p, d_q)} \right\}$$

inverse sum index

$$I(G) = \sum_{pq \in E(G)} \frac{d_p d_q}{d_p + d_q}$$

and *augmented Zagreb index* for a graph H

$$A(G) = \sum_{pq \in E(G)} \left\{ \frac{d_p d_q}{d_p + d_q - 2} \right\}^3$$

In 2009, Zhou and Trinajstić proposed *sum-connectivity index* defined as

$$\chi_{-1/2}(G) = \sum_{pq \in E(G)} [(d_p + d_q)]^{-1/2} \tag{18}$$

In 2013, Ranjini [17] introduced *redefined Zagreb indices i.e. redefined first, second* and *third Zagreb indices* of a graph G. For a graph G, these indices were computed by following formulas *i.e. redefined first Zagreb index*,

$$ReZG_1(G) = \sum_{pq \in E(G)} \frac{d_p + d_q}{d_p d_q} \tag{19}$$

redefined second Zagreb index

$$ReZG_2(G) = \sum_{pq \in E(G)} \frac{d_p d_q}{d_p + d_q} \tag{20}$$

and *redefined third Zagreb index*

$$ReZG_3(G) = \sum_{pq \in E(G)} (d_p d_q)(d_p + d_q) \tag{21}$$

As *algebraic polynomials* are quite significant in determining the bioactivity of chemical compounds. *M-polynomial* [18] is also one of these useful algebraic polynomials, defined as

$$M(G, x, y) = \sum_{\delta \le i \le j \le \Delta} m_{ij}(H) x^i y^j \tag{22}$$

Where $\delta = min\{d_G(p)\}$, $\Delta = max\{d_G(p)\}$ and $m_{ij} = |E(G)|$ for $p, q \in V(G)$ of graph G. Some of the topological indices are directly determined by $M - polynomial$ for $x = y = 1$. These topological indices are stated as (refer to [18])

First Zagreb index

$$M_1(G) = (D_x + D_y)(M(G; x, y))_{x=y=1} \tag{23}$$

Second Zagreb index

$$M_2(G) = (D_x D_y)(M(G; x, y))_{x=y=1} \tag{24}$$

Modified second Zagreb index

$${}^m M_2(G) = (S_x S_y)(M(G; x, y))_{x=y=1} \tag{25}$$

Randić index

$$R_\alpha(G) = (S_x^\alpha S_y^\alpha)(M(G;x,y))_{x=y=1} \tag{26}$$

Inverse Randić index

$$RR(G) = (D_x^\alpha D_y^\alpha)(M(G;x,y))_{x=y=1} \tag{27}$$

Symmetric division index

$$SDD(G) = (D_x S_y + D_y S_x)(M(G;x,y))_{x=y=1} \tag{28}$$

Harmonic index

$$H(G) = 2S_x J(M(G;x,y))_{x=y=1} \tag{29}$$

Inverse sum index

$$I(G) = S_x J D_x D_y(M(G;x,y))_{x=y=1} \tag{30}$$

and *Augmented Zagreb index*

$$A(G) = S_x^3 Q_{-2} D_x^3 D_y^3(M(G;x,y))_{x=y=1} \tag{31}$$

Where

$$D_x M(G(x,y)) = x\frac{\partial(M(G(x,y)))}{\partial x}$$

$$D_y M(G(x,y)) = y\frac{\partial(M(G(x,y)))}{\partial y}$$

$$S_x M(G(x,y)) = \int_0^x \frac{M(G(t,y))}{t}dt$$

$$S_y M(G(x,y)) = \int_0^x \frac{M(G(x,t))}{t} dt$$
$$JM(G(x,y)) = M(G(x,x))$$

and

$$Q_\alpha M(G(x,y)) = x^\alpha M(G(x,y)).$$

Since last thirty years, many scholars and researchers have been working on *composite graphs*. There are various graph operations which are applied directly on simple graphs to study their properties under these operations. Many authors computed several topological indices of some graph operations and line graph of these graph operations for certain families [19–25], *e.g.* composition, disjunction, Cartesian product, corona product, indubala product and wreath product of two graphs.

Subdivision S(G) [21–25] of a graph is acquired by embedding a vertex referred as the *white vertex* into each edge of G. Two black vertices are *related* in $S(G)$ if they are adjacent in G. So $R(G)$ is obtained from $S(G)$ by joining each pair of *related* black vertices. Two white vertices are *related* in $S(G)$ if their corresponding edges are adjacent in G. Semitotal-line graph $Q(G)$ is obtained by joining each pair of *related* white vertices. Similarly two white vertices are *related* in $S(G)$ if their corresponding edges are adjacent in G. $Q(G)$ is obtained by joining each pair of *related* white vertices. Total graph $T(G)$ is simply the union of $R(G)$ and $Q(G)$. *Line graph* $L(G)$ is another graph operation. Utilization of the line graph might have been initiated from the precise starting of the structural chemistry. In 1981, Bertz [26] introduced first topological index based on line graph. Many authors computed the line graph of certain families of graphs in [19, 20, 24, 27, 28]. Many authors computed several topological indices for these four graph operations. M. Faisal *et.al*. [19] computed ABC_4 and GA_5 indices of the line graph of tadpol, wheel and ladder graphs using the notion of subdivision. In [20] Y.Gao *et.al*. provided the formulas for some topological indices of line graphs of the subdivision graphs of Nanotube and Nanotorus of TUC_4C_8. M. Reza *et.al*. [21] considered Wiener index and Hosoya polynomial of the line graph of the wheel graphs using the concept of subdivision and explored new results. He [22] also executed the exact values of Schultz and modified Schultz polynomial of the subdivision graph and line graph subdivision graph for wheel. M. Ajmal *et.al*. [24] worked on forgotten polynomial and forgotten index of line graphs of Banana tree graphs, Firecracker graph and subdivision graphs.

2 Certain Topological Indices and polynomials of $Q(D_n^m)$

In this section, we compute Zagreb polynomials, hyper Zagreb index, Redefined Zagreb indices, modified first Zagreb index, Reduced second Zagreb index, Reduced Reciprocal Randić index, 1^{st} Gourava index, 2^{nd} Gourava index, 1^{st} hyper Gourava index, 2^{nd} hyper Gourava index, Product connectivity Gourava index, Sum connectivity Gourava index, Forgotten index, Forgotten polynomials, M-polynomials and some topological indices in terms of M-polynomial *i.e.* 1^{st} Zagreb index , 2^{nd} Zagreb index, Modified 2^{nd} Zagreb index, Randić index, Reciprocal Randić index, Symmetric division index, Harmonic index, Inverse Sum indeg index, Augmented Zagreb index for the semitotal line graph of Dutch windmill graph. The semitotal line graph of Dutch windmill graph *i.e.* $Q(D_n^m)$ is shown in figure 1.

Theorem 2.1. *Let G be the Semitotal-line graph of a Dutch Windmill graph i.e $Q(D_n^m)$. Then forgotten polynomial and Zagreb polynomials are given by*

$$\begin{aligned}
F(G,x) &= 2mx^{8m^2+8m+4} + mx^{8m^2+16m+8}(2m-1) + 2mx^{4m^2+8m+8}(1+x^{12}) \\
&\quad - mx^{20}(4+3x^{12}) + mnx^{20}(x^{12}+2) \\
ZG_1(G,x) &= 2mx^{4m+2} + mx^4[2mx^{4m} - x^{4m} + 2mx^{2m}] + 2mx^6[x^{2m}+n-2] + mx^8(n-3) \\
ZG_2(G,x) &= mx^{4m}[2x^{4m^2} + 2mx^{4m^2+4m+4} - x^{4m^2+4m+4} + 2x^4 + 2x^{4m+8}] + mnx^8[x^8+2] \\
&\quad - mx^8[3x^8-4] \\
ZG_3(G,x) &= 2m[x^2+m+x^{2m}+x^{2m-2}-2-2x^2] + mn[1+2x^2] - m
\end{aligned}$$

Proof. Consider the semitotal-line graph of Dutch windmill graph,

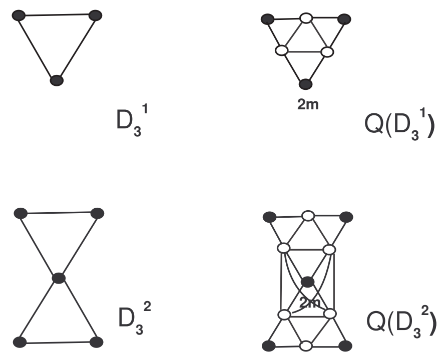

Figure 1: Semitotal-line graph of Dutch Windmill graph *i.e.* $Q(D_n^m)$

we denotes this graph by $Q(D_n^m)$. Order and size of $Q(D_n^m)$ is $2mn-(m-1)$ and $2m^2-2m+3mn$ respectively. $Q(D_n^m)$ contains the vertices with degree $2m$, $(2m+2)$, 4 and 2. We partition the edges of $Q(D_n^m)$ by degrees of end points of edges *i.e.* $E_{(d_u,d_v)}$ denotes the class of those edges uv of $Q(D_n^m)$, where end points of edge uv have degrees d_u, d_v. In $Q(D_n^m)$, we get the edge partition $E_{(2m,2m+2)}$, $E_{(2m+2,2m+2)}$, $E_{(2m+2,2)}$, $E_{(2m+2,4)}$, $E_{(4,4)}$ and $E_{(4,2)}$, as given in Table 1

$$\begin{aligned}
\mathbf{F(G,x)} &= \sum_{\mathbf{pq\in E(G)}} \mathbf{x}^{\mathbf{(d_p)^2+(d_q)^2}} \\
&= 2mx^{(2m)^2+(2m+2)^2} + 2m^2x^{(2m+2)^2+(2m+2)^2} - mx^{(2m+2)^2+(2m+2)^2} + 2mx^{(2m+2)^2+(2)^2} \\
&+ 2mx^{(2m+2)^2+(4)^2} + mnx^{(4)^2+(4)^2} - 3mx^{(4)^2+(4)^2} + 2mnx^{(4)^2+(2)^2} - 4mx^{(4)^2+(2)^2} \\
&= 2mx^{8m^2+8m+4} + mx^{8m^2+16m+8}(2m-1) + 2mx^{4m^2+8m+8}(1+x^{12}) - mx^{20}(4+3x^{12}) \\
&+ mnx^{20}(x^{12}+2)
\end{aligned}$$

$$\begin{aligned}
\mathbf{ZG_1(G,x)} &= \sum_{\mathbf{pq\in E(G)}} \mathbf{x}^{\mathbf{d_p+d_q}} \\
&= \sum x^{2m+2m+2} + \sum x^{2m+2+2m+2} + \sum x^{2m+2+2} + \sum x^{2m+2+4} + \sum x^{4+4} \\
&+ \sum x^{4+2} \\
&= 2mx^{4m+2} + mx^4[2mx^{4m} - x^{4m} + 2mx^{2m}] + 2mx^6[x^{2m} + n - 2] + mx^8(n-3)
\end{aligned}$$

Edges of type	Number of edges
$E_{(2m,2m+2)}$	$2m$
$E_{(2m+2,2m+2)}$	$m(2m-1)$
$E_{(2m+2,2)}$	$2m$
$E_{(2m+2,4)}$	$2m$
$E_{(4,4)}$	$(n-3)m$
$E_{(4,2)}$	$(2n-4)m$

Table 1: Edge partition of $Q(D_n^m)$

$$\begin{aligned}
\mathbf{ZG_2(G,x)} &= \sum_{\mathbf{pq\in E(G)}} \mathbf{x^{d_p d_q}} \\
&= \sum x^{2m(2m+2)} + \sum x^{2m+2(2m+2)} + \sum x^{(2m+2)2} + \sum x^{(2m+2)4} + \sum x^{4(4)} \\
&+ \sum x^{4(2)} \\
&= (2m)x^{2m(2m+2)} + (2m^2)x^{2m+2(2m+2)} - (m)x^{2m+2(2m+2)} + (2m)x^{(2m+2)2} \\
&+ (2m)x^{(2m+2)4} + (mn)x^{4(4)} - (3m)x^{4(4)} + (2mn)x^{4(2)} - (4m)x^{4(2)} \\
&= mx^{4m}[2x^{4m^2} + 2mx^{4m^2+4m+4} - x^{4m^2+4m+4} + 2x^4 + 2x^{4m+8}] + mnx^8[x^8+2] \\
&- mx^8[3x^8-4]
\end{aligned}$$

$$\begin{aligned}
\mathbf{ZG_3(G,x)} &= \sum_{\mathbf{pq\in E(G)}} \mathbf{x^{|d_p-d_q|}} \\
&= \sum x^{2m-2m-2} + \sum x^{2m+2-2m-2} + \sum x^{2m+2-2} + \sum x^{2m+2-2} + \sum x^{4-4} \\
&+ \sum x^{4-2} \\
&= 2mx^2 + 2m^2 - m + 2mx^{2m} + 2mx^{2m-2} + mn - 3m + 2mnx^2 - 4mx^2 \\
&= 2m[x^2 + m + x^{2m} + x^{2m-2} - 2 - 2x^2] + mn[1+2x^2] - m
\end{aligned}$$

□

Theorem 2.2. *Let G be the semitotal-line graph of a Dutch Windmill graph. Then*

$$
\begin{aligned}
HM(G) &= 32m^4 + 160m^3 + 80m^2 - 240m + 136mn \\
ReZG_1(G) &= \frac{2m^3}{m^2+2m+1} + m^2[\frac{2}{m^2+2m+1} - \frac{1}{m^2+2m+1} + \frac{2}{m^2+2m+1} \\
&\quad \frac{1}{2(m+1)}] + m[\frac{2}{m+1} + \frac{4}{m^2+2m+1} - \frac{1}{m^2+2m+1} + \frac{3}{2(m+1)} \\
&\quad - \frac{9}{2}] + \frac{1}{m+1} + 2mn \\
ReZG_2(G) &= \frac{2m^4}{m+1} + m^3\left[\frac{4}{2m+1} + \frac{4}{m+1} - \frac{1}{m+1}\right] + m^2\left[\frac{4}{m+2} - \frac{8}{m+3} + \frac{4}{2m+1}\right] \\
&\quad + m\left[\frac{4}{m+2} + \frac{8}{m+3} - \frac{34}{3} - \frac{1}{m+1}\right] + \frac{14}{3} \\
ReZG_3(G) &= 32m^5 + 112m^4 + 144m^3 + 176m^2 - 80m + 224mn \\
{}^{m}M_2(G) &= \frac{m^2}{2(m+1)} + m[\frac{1}{2m+1} - \frac{1}{4m+4} + \frac{1}{m+2} + \frac{1}{m+3} - \frac{25}{24}] + \frac{11mn}{24} \\
RM_2(G) &= 8m^4 + 12m^3 + 14m^2 - 30m + 15mn \\
RRR(G) &= 2m^2\sqrt{4m^2+4m+1} + m[2\sqrt{4m^2-1} - 2\sqrt{4m^2+4m+1} + 2\sqrt{2m+1} \\
&\quad + 2\sqrt{6m+3} - 9 - 4\sqrt{3}] + mn(3+2\sqrt{3}) \\
GO_1(G) &= 8m^4 + 28m^3 + 52m^2 - 88m + 52mn \\
GO_2(G) &= 32m^5 + 112m^4 + 144m^3 + 176m^2 - 80m + 224mn \\
HGO_1(G) &= 32m^6 + 176m^5 + 488m^4 + 640m^3 + 752m^2 - 2048m + 968mn \\
HGO_2(G) &= 20992mn - 58368m + 2m[(16m^3+24m^2+8m)^2 + (8m^2+24m+16)^2 \\
&\quad + (16m^2+64m+48)^2] + m(2m-1)(16m^3+48m^2+48m+16)^2 \\
PGO(G) &= \frac{2m^2}{\sqrt{4m^3+12m^2}+36m+12} + mn[\frac{1}{\sqrt{128}} + \frac{1}{\sqrt{12}}] + m[\frac{1}{2\sqrt{4m^3+6m^2+2m}} \\
&\quad - \frac{1}{\sqrt{16m^3+48m^2+48m+16}} + \frac{2}{\sqrt{2m^2+6m+4}} + \frac{1}{\sqrt{4m^2+20m+12}} \\
&\quad - \frac{3}{\sqrt{128}} - \frac{1}{\sqrt{3}}] \\
SGO(G) &= \frac{2m^2}{\sqrt{4m^2+12m+18}} + m[\frac{2}{\sqrt{4m^2+8m+2}} - \frac{1}{\sqrt{4m^2+12m+8}} \\
&\quad + \frac{2}{6m+8} + \frac{2}{10m+14} - \frac{3}{24} - \frac{4}{14}] + mn[\frac{1}{24} + \frac{2}{14}] \\
F(G) &= 16m^4 + 56m^3 + 48m^2 - 120m + 72mn
\end{aligned}
$$

Proof. Apply Formulas (5), (19), (20), (21), (25), (9), (10), (11),(13), (16), (17), (1), (15) and (6) to edge partition shown in Table 1 to get the required results. □

Theorem 2.3. *Let G be the semitotal-line graph of a Dutch Windmill graph. Then M-polynomial and certain topological in-*

dices in terms of M-polynomial are as under

$$M(G;x,y) = 2mx^{2m}y^{2m+2} + mx^{2m+2}y^{2m+2}(2m-1) + 2mx^{2m+2}y^{2}(1+y^{2}) + mx^{4}y^{4}(n-3) + mx^{4}y^{2}(2n-4)$$

$$M_1(G) = 8m^3 + 20m^2 - 28m + 20mn$$

$$M_2(G) = 8m^4 + 20m^3 + 24m^2 - 60m + 32mn$$

$${}^{m}M_2(G) = \frac{1}{2(m+1)} + \frac{m(2m-1)}{(2m+2)(2m+2)} + \frac{m}{2m+2} + \frac{m}{2(2m+2)} + \frac{m(n-3)}{16} + \frac{m(2n-4)}{8}$$

$$R_\alpha(G) = \frac{2m}{(2m+2)^\alpha(2m)^\alpha} + \frac{m(2m-1)}{(2m+2)^\alpha(2m+2)^\alpha} + \frac{2m}{(2)^\alpha(2m+2)^\alpha} + \frac{2m}{(4)^\alpha(2m+2)^\alpha} + \frac{m(n-3)}{(4)^\alpha(4)^\alpha} + \frac{m(2n-4)}{(2)^\alpha(4)^\alpha}$$

$$RR_\alpha(G) = 2m(2m)^\alpha(2m+2)^\alpha + m(2m-1)(2m+2)^\alpha(2m+2)^\alpha + 2m(2m+2)^\alpha(2)^\alpha + 2m(2m+2)^\alpha(4)^\alpha + m(n-3)(4)^\alpha(4)^\alpha + m(2n-4)(2)^\alpha(4)^\alpha$$

$$SDD(G) = 6m^2 + 7mn - 9m + 3$$

$$H(G) = \frac{4m}{4m+2} + \frac{4m}{4m+4} - \frac{7m}{12} + \frac{11m}{24}$$

$$I(G) = \frac{(8m^3+8m^2)}{4m+2} + \frac{(2m^2-m)(2m+2)^2}{4m+4} + \frac{(8m^2+8m)}{2m+4} + \frac{(16m^2+16m)}{2m+6} + \frac{(16mn-48m)}{8} + \frac{8m}{6}$$

$$A(G) = \frac{2m(4m^2+4m)^3}{(4m^3)} + \frac{m(2m-1)(2m+2)^6}{(4m+2)^3} + \frac{2m(4m+4)^3}{(2m+2)^3} + \frac{2m(8m+8)^3}{(2m+4)^3} + \frac{4096m(n-3)}{(6)^3} + \frac{512m(2n-4)}{64}$$

Proof. Apply Formulas (22), (23), (24), (25), (26), (27), (28), (29), (30) and (31) to the edge partition shown in the Table 1 to get the required results. □

3 Certain Topological Indices and polynomials of $L(Q(D_n^m))$

In this section, we compute Zagreb polynomials, hyper Zagreb index, Redefined Zagreb indices, modified first Zagreb index, Reduced second Zagreb index, Reduced Reciprocal Randić index, 1^{st} Gourava index, 2^{nd} Gourava index, 1^{st} hyper Gourava index, 2^{nd} hyper Gourava index, Product connectivity Gourava index, Sum connectivity Gourava index, Forgotten index, Forgotten polynomials, M-polynomials and some topological indices in terms of M-polynomials *i.e.* 1^{st} Zagreb index , 2^{nd} Zagreb index, Modified 2^{nd} Zagreb index, Randić index, Reciprocal Randić index, Symmetric division index, Harmonic index, Inverse Sum indeg index, Augmented Zagreb index for the line graph of semitotal-line graph of Dutch windmill graph. As the line graph of semitotal-line graph of Dutch windmill graph is shown in figure 2.

Theorem 3.1. *Let G be the line graph of semitotal-line graph of a Dutch windmill graph i.e. $L(Q(D_n^m))$. Then the forgotten polynomial and Zagreb polynomials for $n > 4$ are given by*

$$
\begin{aligned}
F(G,x) &= (4m^3-6m^2+2m)x^{2(4m+2)^2}+(4m^2-2m)(x^{20m^2+16m+16}+2x^{20m^2+32m+20}) \\
&+2m(x^{20m^2+16m+16}+2x^{4m^2+16m+32}+x^{8m^2+24m+20}+x^{4m^2+16m+52}+x^{20m^2+8m+4} \\
&+x^{4m^2+8m+20})+(2m^2-m)x^{32m^2}-m(4x^{72}+5x^{32}+12x^{52})+mn(x^{72}+2x^{32}+4x^{52}) \\
ZG_1(G,x) &= (4m^3-6m^2+2m)x^{8m+4}+(4m^2-2m)(x^{8m+2}+x^{6m+6}+x^{6m+4}) \\
&+(4mn-12m)x^{10}+2m(x^{6m+4}+x^{4m+6}+x^{6m+2}+2x^{2m+8}+x^{2m+6}+x^{2m+10}) \\
&+(2m^2-m)x^{8m}+(2mn-5m)x^{8}+(mn-4m)x^{12} \\
ZG_2(G,x) &= (4m^3-6m^2+2m)x^{16m^2+16m+4}+(4m^2-2m)(x^{16m^2+8m}+x^{8m^2+20m+8} \\
&+x^{8m^2+12m+4})+(4mn-12m)x^{24}+2m(x^{8m^2+16m}+2x^{8m+16}+x^{4m^2+12m+8}+x^{8m+8} \\
&+x^{8m^2+8m}+x^{12m+24})+(2m^2-m)x^{16m^2}+(2mn-5m)x^{16}+(mn-4m)x^{36} \\
ZG_3(G,x) &= 4m^3-4m^2-8m+3mn+(4m^2-2m)(x^2+x^{2m-2}+x^2m)+(4mn-12m)x^2 \\
&+2m(x^{4-2m}+2x^{2m}+x^2+x^{2-2m}+2x^{2m-2})
\end{aligned}
$$

Proof. Consider the line graph of semitotal-line of Dutch windmill graph, denoted by $L(Q(D_n^m))$. Where $2m^2+3mn-2m$ are the total number of vertices and $4m^3+8m^2-12m+7mn$ are the total number of edges in $L(Q(D_n^m))$. The edges partition of $L(Q(D_n^m))$ is given as $E_{(d_u,d_v)}$, where $uv \in E(Q(D_n^m))$. The edge partition $E_{(4m+2,4m+2)}$ contains $2m(2m-1)(m-1)$ edges for $d_u = 4m+2$ and $d_v = 4m+2$, the edge partition $E_{(4m+2,4m)}$ contains $4m^2-2m$ edges for $d_u = 4m+2$ and $d_v = 4m$, the edge partition $E_{(4m+2,2m+4)}$ contains $4m^2-2m$ edges for $d_u = 4m+2$ and $d_v = 2m+4$, the edge partition $E_{(4m+2,2m+2)}$ contains $4m^2-2m$ edges for $d_u = 4m+2$ and $d_v = 2m+2$, the edge partition $E_{(2m+4,4m)}$ contains $2m$ edges for $d_u = 2m+4$ and $d_v = 4m$, the edge partition $E_{(2m+4,2m+4)}$ contains m edges for $d_u = 2m+4$ and $d_v = 2m+4$ only for $n = 3$, the edge partition $E_{(2m+4,4)}$ contains $4m$ edges for $d_u = 2m+4$ and $d_v = 4$, the edge partition $E_{(2m+4,2m+2)}$ contains $2m$ edges for $d_u = 2m+4$ and $d_v = 2m+2$, the edge partition $E_{(2m+4,6)}$ contains $2m$ edges for $d_u = 6$ and $d_v = 4$ only for $n \geq 4$, the edge partition $E_{(2m+2,4m)}$ contains $2m$ edges for $d_u = 2m+2$ and $d_v = 4m$, the edge partition $E_{(2m+2,4)}$ contains $2m$ edges for $d_u = 2m+2$ and $d_v = 4$, the edge partition $E_{(4m,4m)}$ contains $2m^2 - m$ edges for $d_u = 4m$ and $d_v = 4m$, the edge partition $E_{(6,6)}$ contains $mn - 4m$ edges for $d_u = 6$ and $d_v = 6$ only for $n > 4$, the edge partition $E_{(4,4)}$ contains $(2mn - 5m)$ edges for $d_u = 4$ and $d_v = 4$ and the edge partition $E_{(6,4)}$ contains $4mn-12m$ edges for $d_u = 6$ and $d_v = 4$ shown in Table 2.

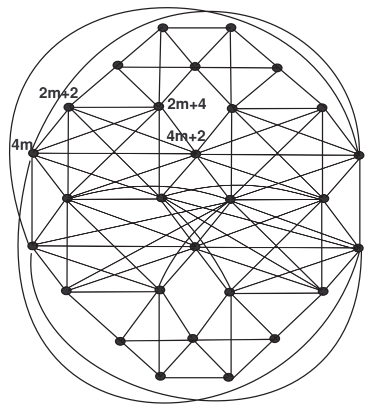

Figure 2: Line graph of Semitotal-line of Dutch windmill graph *i.e.* $L(Q(D_3^2))$

$$\mathbf{F(G,x)} = \sum_{\mathbf{uv \in E(G)}} \mathbf{x^{(d_u)^2+(d_v)^2}}$$

$$= (4m^3 - 6m^2 + 2m)x^{2(4m+2)^2} + (4m^2 - 2m)(x^{(4m+2)^2+(4m)^2} + x^{(4m+2)^2+(2m+4)^2}$$
$$+ x^{(4m+2)^2+(2m+2)^2}) + 2m(x^{(2m+4)^2+(4m)^2} + x^{(2m+4)^2+(4)^2} + x^{(2m+4)^2+(2m+2)^2} + x^{(2m+4)^2+(6)^2}$$
$$+ x^{(2m+2)^2+(4m)^2} + x^{(2m+2)^2+(4)^2}) + (2m^2 - m)x^{(4m)^2+(4m)^2} + (mn - 4m)x^{(6)^2+(6)^2}$$
$$+ (2mn - 5m)x^{(4)^2+(4)^2} + (4mn - 12m)x^{(6)^2+(4)^2}$$
$$F(G,x) = (4m^3 - 6m^2 + 2m)x^{2(4m+2)^2} + (4m^2 - 2m)(x^{20m^2+16m+16} + 2x^{20m^2+32m+20})$$
$$+ 2m(x^{20m^2+16m+16} + 2x^{4m^2+16m+32} + x^{8m^2+24m+20} + x^{4m^2+16m+52} + x^{20m^2+8m+4}$$
$$+ x^{4m^2+8m+20}) + (2m^2 - m)x^{32m^2} - m(4x^{72} + 5x^{32} + 12x^{52}) + mn(x^{72} + 2x^{32} + 4x^{52})$$

$$\mathbf{MG_1(G,x)} = \sum_{\mathbf{uv \in E(G)}} \mathbf{x^{d_u+d_v}}$$

Edges of type	Number of edges
$E_{(4m+2,4m+2)}$	$2m(2m-1)(m-1)$
$E_{(4m+2,4m)}$	$4m^2-2m$
$E_{(4m+2,2m+4)}$	$4m^2-2m$
$E_{(4m+2,2m+2)}$	$4m^2-2m$
$E_{(2m+4,4m)}$	$2m$
$E_{(2m+4,2m+4)}$	m only for $n=3$
$E_{(2m+4,4)}$	$4m$
$E_{(2m+4,2m+2)}$	$2m$
$E_{(2m+4,6)}$	$2m$ for $n\geq 4$
$E_{(2m+2,4m)}$	$2m$
$E_{(2m+2,4)}$	$2m$
$E_{(4m,4m)}$	$2m^2-m$
$E_{(6,6)}$	$mn-4m$ for $n>4$
$E_{(4,4)}$	$2mn-5m$
$E_{(4,4)}$	$4mn-12m$

Table 2: Edge partition of $L(Q(D_n^m))$

$$\begin{aligned}
&= (4m^3-6m^2+2m)x^{4m+2+4m+2} + (4m^2-2m)(x^{4m+2+4m} + x^{4m+2+2m+4} + x^{6m+4}) \\
&+ 2m(x^{2m+4+4m} + x^{2m+4+2m+2} + x^{2m+2+4m} + 2x^{2m+4+4} + x^{2m+2+4} + x^{2m+4+6}) \\
&+ (2m^2-m)x^{4m+4m} + (2mn-5m)x^{4+4} + (mn-4m)x^{6+6} + (4mn-12m)x^{6+4} \\
MG_1(G,x) &= (4m^3-6m^2+2m)x^{8m+4} + (4m^2-2m)(x^{8m+2} + x^{6m+6} + x^{6m+4}) \\
&+ (4mn-12m)x^{10} + 2m(x^{6m+4} + x^{4m+6} + x^{6m+2} + 2x^{2m+8} + x^{2m+6} + x^{2m+10}) \\
&+ (2m^2-m)x^{8m} + (2mn-5m)x^{8} + (mn-4m)x^{12}
\end{aligned}$$

$$\mathbf{MG_2(G,x)} = \sum_{\mathbf{uv\in E(G)}} \mathbf{x^{d_u d_v}}$$

$$\begin{aligned}
&= (4m^3-6m^2+2m)x^{(4m+2)(4m+2)} + 2m(x^{4m(2m+4)} + 2x^{4(2m+4)} + x^{(2m+4)(2m+2)} \\
&+ x^{6(2m+4)} + x^{4m(2m+2)} + x^{4(2m+2)}) + (4mn-12m)x^{(6)(4)} + (4m^2-2m)(x^{(4m+2)4m} \\
&+ x^{(4m+2)(2m+4)} + x^{(4m+2)(2m+2)}) + (2m^2-m)x^{(4m)(4m)} + (2mn-5m)x^{(4)(4)} \\
&+ (mn-4m)x^{(6)(6)} \\
MG_2(G,x) &= (4m^3-6m^2+2m)x^{16m^2+16m+4} + (4m^2-2m)(x^{16m^2+8m} + x^{8m^2+20m+8} \\
&+ x^{8m^2+12m+4}) + (4mn-12m)x^{24} + 2m(x^{8m^2+16m} + 2x^{8m+16} + x^{4m^2+12m+8} + x^{8m+8} \\
&+ x^{8m^2+8m} + x^{12m+24}) + (2m^2-m)x^{16m^2} + (2mn-5m)x^{16} + (mn-4m)x^{36}
\end{aligned}$$

$$\mathbf{MG_3(G,x)} = \sum_{\mathbf{uv \in E(G)}} \mathbf{x}^{|\mathbf{d_u - d_v}|}$$

$$\begin{aligned}
&= (4m^3 - 6m^2 + 2m)x^{4m+2-4m-2} + (4mn - 12m)x^2 \\
&+ (4m^2 - 2m)(x^{4m+2-4m} + x^{4m+2-2m-4-2} + x^{4m+2-2m-2}) + 2m(x^{2m+4-4m} \\
&+ 2x^{2m+4-4} + x^{2m+4-2m-2} + x^{2m+4-6} + x^{2m+2-4m} + x^{2m+2-4}) \\
MG_3(G,x) &= 4m^3 - 4m^2 - 8m + 3mn + (4m^2 - 2m)(x^2 + x^{2m-2} + x^2 m) + (4mn - 12m)x^2 \\
&+ 2m(x^{4-2m} + 2x^{2m} + x^2 + x^{2-2m} + 2x^{2m-2})
\end{aligned}$$

□

Theorem 3.2. *Let G be the line graph of semitotal line graph of Dutch Windmill graph. Then for $n > 4$*

$$HM(G) = 256m^5 + 544m^4 + 288m^3 + 352m^2 - 1616m + 672mn$$

$$ReZG_1(G) = \frac{1}{12m(2m+4)(2m+1)^2(m+1)}\Big(768m^6 + 2304m^5 + 2112m^4 + 264m^3 - 348m^2 - 204m - 24 + (2m+1)^2(24m^4 + 204m^3 + 108m^2 + 24) + (2m+4)(2m+1)^2 (6m^3 + 32m^2n - 48m^2)\Big) + \frac{9m^2n + 8mn + 67m - 35m^2}{3(m+2)}$$

$$ReZG_2(G) = 8m^4 - 8m^3 - 2m^2 + 2m + \frac{4m^2 - 2m}{36m^3 + 69m^2 + 39m + 6}(168m^4 + 467m^3 + 400m^2 + 136m + 14) + \frac{1}{5}(20m^3 - 10m^2 - 194m + 248mn) + \frac{2}{18m^6 + 261m^5 + 1417m^4 + 2943m^3 + 4859m^2 + 2142m + 360}(66m^8 + 1358m^7 + 10224m^6 + 34144m^5 + 63568m^4 + 52910m^3 + 10692m^2 + 1608m)$$

$$ReZG_3(G) = 512m^6 + 1152m^5 + 672m^4 + 710m^3 + 972m^2 - 4160m + 1648mn$$

$$^mM_2(G) = \frac{2m^3 - 3m^2 + m}{4m+2} + \frac{21mn - 10m - 43}{40} + \frac{(2m^2 - m)(33m^2 + 41 + 11)}{(4m+1)(3m+3)(3m+2)} + \frac{264m^6 + 2725m^5 + 10427m^4 + 17034m^3 + 11856m^2 + 2736m}{(3m+2)(4m+8)(m+4)(2m+3)(3m+1)(m+6)} + \frac{8m^3n + 28m^2n + 40mn - 32m^3 - 76m^2 - 124m}{12(8m^2 + 28m + 40)}$$

$$RM_2(G) = 64m^5 + 96m^4 + 124m^3 + 84m^2 - 260m + 94mn$$

$$RRR(G) = 11mn - 35m + (4mn - 12m)\sqrt{15} + (2m^2 - m)\sqrt{16m^2 - 1} + (4m^3 - 6m^2 + 2m)\sqrt{16m^2 + 8m + 1} + (4m^2 - 2m)[\sqrt{16m^2 - 1} + 2m[\sqrt{8m^2 + 14m - 4} + \sqrt{12m + 18} + \sqrt{4m^2 + 8m + 3} + \sqrt{10m + 15} + \sqrt{8m^2 + 2m - 1} + \sqrt{6m + 3}]$$

$$GO_1(G) = 64m^5 + 160m^4 + 136m^3 + 160m^2 - 520m + 232mn$$

$$GO_2(G) = 512m^6 + 1152m^5 + 672m^4 + 710m^3 + 972m^2 - 4160m + 1648mn$$

$$HGO_1(G) = 1024m^7 + 3584m^6 + 3872m^5 + 3616m^4 + 3673m^3 + 3259m^2 - 22080m + 8080mn$$

$$HGO_2(G) = 65536m^9 + 313344m^8 + 305664m^7 + 31792m^6 + 364692m^5 + 88064m^4 + 168960m^3 + 140032m^2 - 1334272m + 482560mn$$

$$PGO(G) = \frac{(4m^3 - 6m^2 + 2m)}{\sqrt{128m^3 + 192m^2 + 96m + 16}} + (4m^2 - 2m)\Big[\frac{1}{\sqrt{128m^3 + 96m^2 + 16m}} + \frac{1}{\sqrt{48m^3 + 168m^2 + 168m + 48}} + \frac{1}{\sqrt{48m^3 + 104m^2 + 72m + 16}}\Big] + 2m\Big[\frac{1}{\sqrt{48m^3 + 128m^2 + 64m}} + \frac{2}{\sqrt{16m^2 + 96m + 128}} + \frac{1}{\sqrt{16m^3 + 72m^2 + 104m + 48}} + \frac{1}{\sqrt{24m^2 + 168m + 240}} + \frac{1}{\sqrt{48m^3 + 16m^2}} + \frac{1}{\sqrt{16m^2 + 64m + 48}}\Big] + (2m^2 - m)\frac{1}{\sqrt{128m^3}} + (mn - 4m)\frac{1}{\sqrt{432}} + (2mn - 5m)\frac{1}{\sqrt{128}} + (4mn - 12m)\frac{1}{\sqrt{240}}$$

$$SGO(G) = \frac{4m^3 - 6m^2 + 2m}{\sqrt{16m^2 + 24m + 8}} + (4m^2 - 2m)\Big[\frac{1}{\sqrt{16m^2 + 16m + 2}} + \frac{1}{\sqrt{8m^2 + 26m + 14}} + \frac{1}{\sqrt{8m^2 + 18m + 8}}\Big] + 2m[\frac{1}{\sqrt{8m^2 + 22m + 4}} + \frac{2}{\sqrt{10m + 24}} + \frac{1}{\sqrt{4m^2 + 16m + 14}}$$

Proof. Apply Formulas (5), (19), (20), (21), (25), (9), (10), (11),(13), (16), (17), (1), (15) and (6) to the edge partitions shown in Table 2 to get the required results. □

Theorem 3.3. *Let G be the line graph of semitotal-line graph of a Dutch Windmill graph. Then M-polynomial and certain topological indices in terms of M-polynomial for $n > 4$ are given by*

$$\begin{aligned}
M(G;x,y) &= (4^3 - 6m^2 + 2m)x^{4m+2}y^{4m+2} + (4m^2 - 2m)(x^{4m}y^{4m+2} + x^{2m+4}y^{4m+2} \\
&\quad + x^{2m+2}y^{4m+2}) + 2m(x^{4m}y^{2m+4} + 2x^4y^{2m+4} + x^{2m+2}y^{2m+4} + x^6y^{2m+4} \\
&\quad + x^{2m+2}y^{4m} + x^4y^{2m+2}) + (2m^2 - m)x^{4m}y^{4m} + (mn - 4m)x^6y^6 \\
&\quad + (2mn - 5m)x^4y^4 + (4mn - 12m)x^4y^6 \\
M_1(G) &= 32m^4 + 64m^3 + 40m^2 - 136m + 68mn \\
M_2(G) &= 64m^5 + 128m^4 + 72m^3 + 120m^2 - 384m + 164mn \\
{}^{m}M_2(G) &= \frac{2m^3 - 3m^2 + m}{8m^2 + 8m + 2} + \frac{(2m^2 - m)(20m^2 + 36m + 8)}{(2m+1)(8m^2 + 16m)(2m+2)} \\
&\quad + \frac{64m^2 + 136m + 24}{(m+2)(96m+96)} + \frac{16m + 16}{(4m+4)^2} + \frac{2m - 1}{16m}
\end{aligned}$$

$$
\begin{aligned}
&+\frac{mn-4m}{36}+\frac{2mn-5m}{16}+\frac{mn-3m}{6}\\
R_\alpha(G) &= \frac{4m^3-6m^2+2m}{(4m+2)^{2\alpha}}+(4m^2-2m)\Big(\frac{1}{(16m^2+8m)^\alpha}+\frac{1}{(8m^2+20m+8)^\alpha}\\
&+\frac{1}{(8m^2+12m+4)^\alpha}\Big)+2m\Big(\frac{1}{(8m^2+16m)^\alpha}+\frac{2}{(8m+16)^\alpha}+\frac{1}{(4m^2+12m+8)^\alpha}\\
&+\frac{1}{(12m+24)^\alpha}\Big)+2m\Big(\frac{1}{(8m^2+8m)^\alpha}+\frac{1}{(8m+8)^\alpha}\Big)+\frac{(2m^2-m)}{(16m^2)^\alpha}+\frac{(mn-4m)}{(36)^\alpha}\\
&+\frac{(2mn-5m)}{(16)^\alpha}+\frac{(4mn-12m)}{(24)^\alpha}\\
RR_\alpha(G) &= (4m^3-6m^2+2m)(4m+2)^{2\alpha}+(4m^2-2m)\Big((16m^2+8m)^\alpha+(8m^2+20m+8)^\alpha\\
&+(8m^2+12m+4)^\alpha\Big)+2m\Big((8m^2+16m)^\alpha+2(8m+16)^\alpha+(4m^2+12m+8)^\alpha\\
&+(12m+24)^\alpha\Big)+2m\Big((8m^2+8m)^\alpha+(8m+8)^\alpha\Big)+(2m^2-m)(16m^2)^\alpha\\
&+(2mn-5m)(16)^\alpha+(4mn-12m)(24)^\alpha+(mn-4m)(36)^\alpha\\
SDD(G) &= 8m^3-8m^2-46m+\frac{44mn}{3}+(4m^2-2m)\Big(\frac{8m^2+4m+1}{4m^2+2m}+\frac{5m62+8m+5}{2m^2+5m+2}\\
&+\frac{5m^2+6m+2}{2m^2+3m+1}\Big)+2m\Big(\frac{5m62+4m+4}{4m^2+4m}+\frac{m^2+4m+5}{m+2}+\frac{2m^2+6m+5}{m^2+3m+2}\\
&+\frac{m^2+4m+13}{3m+6}+\frac{5m^2+2m+1}{2m^2+2m}+\frac{m^2+2m+5}{2m+2}\Big)\\
H(G) &= \frac{(4m^3-6m^2+2m)}{(4m+2)}+(4m^2-2m)\Big(\frac{1}{4m+1}+\frac{1}{3m+3}+\frac{1}{3m+2}\Big)\\
&+2m\Big(\frac{1}{3m+2}+\frac{2}{m+4}+\frac{1}{2m+3}+\frac{1}{m+3}+\frac{1}{m+5}+\frac{1}{3m+1}\Big)+\frac{2m^2-m}{4m}\\
&+\frac{mn-4m}{6}+\frac{2mn-5m}{4}+\frac{4mn-12m}{5}\\
I(G) &= \frac{(64m^5-32m^4-48m^3+8m^2+8m)}{8m+4}+\frac{(64m^4-16m^2}{8m+2}\\
&+\frac{(32m^4+64m^3-8m^2-16m)}{6m+6}+\frac{(32m^4+32m^3-8m^2-8m)}{6m+4}\\
&+\frac{(16m^3+32m^2)}{6m+4}+\frac{(32m^2+64m)}{2m+8}+\frac{(8m^3+24m^2+16m)}{4m+6}\\
&+\frac{(24m^2+48m)}{2m+10}+\frac{(16m^3+16m^2)}{6m+2}+\frac{(16m^2+16m)}{2m+6}
\end{aligned}
$$

Proof. Apply Formulas (22), (23), (24), (25), (26), (27), (28), (29), (30) and (31) to edge partition shown in the Table 2 to get the required results. □

References

[1] M. R. Farahani, M. K. Jamil, S. Wang, W. Gao and B. Wei, The hosoya, schultz and modified schultz polynomials of a class of Dutch windmill graph $D_n^{(m)}$, *Communications in Applied Analysis* **22**(1): 43-62 (2017).

[2] M. R. Kanna, R. Pradeep Kumar and R. Jagadeesh, Computation of topological indices of Dutch windmill graph, *Open Journal of Discrete Mathematics* **6**: 74-81 (2016).

[3] J. Devillers and A. T(Eds.). Balaban, Topological indices and related descriptors in QSAR and QSPR, *Gordon and Breach, Amsterdam, The Netherlands*, (1999).

[4] I. Gutman, N. Trinajstić and C. F. Wilcox, Graph theory and molecular orbits. Total π-electron energy of alternate hydrocarbons, *Chem. Phys. Lett.* **17**: 535-538 (1972).

[5] M. H. Khalifeh, H. Yousefi-Azari and A. R. Ashrafi, The first and second Zagreb indices of some graph operations, *Discrete. Appl. Math.* **157**: 804-811 (2009).

[6] H. Deng, D. Sarala, S. K. Ayyaswamy and S. Balachandran, The Zagreb index of four operations on graphs, *Appl. Math. and comput.* **275**: 422-431 (2016).

[7] A. E. Nabeel, Zagreb polynomials of certain families of dendrimer nanostars, *Tikrit Journal of Pure Science* **20**(4): (2015).

[8] J. Hao, Theorems about Zagreb indices and modified Zagreb indices, *MATCH Commun. Math. Comput. Chem.* **65**: 659-670 (2011).

[9] G.H. Shirdel, H. Rezapour and A. M. Syadi, The hyper-Zagreb index of graph operations, *Iranian J. Math.Chem.* **4**(2): 213-220 (2013).

[10] B. Furtula and I. Gutman, A forgotten topological index, *J. Math. Chem.* **53**: 1184-1190 (2015).

[11] M. Imran and S. Akhter, The sharp bounds on general sum-connectivity index of four operations on graphs, *J. Inequal. Appl.* **241** (2016).

[12] I. Gutman, B. Furtula and C. Elphick, Three new/old vertex-degree based topological indices, *MATCH Commun. Math. Comput. Chem.* **72**(24): 617-632 (2014).

[13] V. R. Kulli, The Gourava indices and coindices of graphs, *Annals of Pure and Applied Mathematics* **14**(1): 33-38 (2017).

[14] V. R. Kulli, The product connectivity Gourava index, *J. Comp. ✠Math. Sci.* **8**(6): 235-242 (2017).

[15] V. R. Kulli, On the sum connectivity Gourava index, *International Journal of Mathematical Archive* **8**(6): 211-217 (2017).

[16] V. R. Kulli, On hyper-Gourava indices and coindices, *International Journal of Mathematical Archive* **8**(12): 116-120 (2017).

[17] P. S. Ranjini, V. Lokesha and A. Usha, Relation between phenylene and hexagonal squeeze using harmonic index. *Int. J. Graph Theory* **1**: 116-21 (2013).

[18] M. S. Ahmad, W. Nazeer, S. M. Kang and C. Y. Jung, M-polynomials and degree based topological indices for the line graph of Firecracker graph, *Global Journal of Pure and Applied Mathematics* **13**(6): 2749-2776 (2017).

[19] M. F. Nadeem, S. Zafar and Z. Zahid, On the certain topological indices of the line graph of subdivision graphs, *Appl. Math. Comput.* **217**: 790-794 (2015).

[20] Y. Gao, M. K. Jamil, A. Aslam and M. R. Farahani, Computation of some new/old vertex-degree-based topological indices of line graph of subdivision graph of some nanosrtuc-

tures, *Journal of Optoelectronics and Biomedical Materials* **9**(3): 135-142 (2017).

[21] M. R. Farahani, M. K. Jamil, M. R. Rajesh Kanna and S. M. Hosamani, The wiener index and hosaya polynomial of the Subdivision Graph of wheel $S(W_n)$ and the line graph subdivision of the wheel $L(S(W_n))$, *Appl. Math.* **6**(2): 21-24 (2016).

[22] M. R. Farahani and M. K. Jamil, The schultz and modified schultz polynomials of the certain subdivision and line subdivision graphs, *J. Chem. Pharm. Res.* **8**(3): 51-57 (2016).

[23] G. Mahanappriya and D. Vijayalakshmi, Topological indices of the total graph of subdivision graph, *Annals of Pure and Applied Mathematics* **14**(2): 231-235 (2017).

[24] M. Ajmal, W. Nazeer, W. Khalid and S. M. Kang, Forgotten polynomial and forgotten index for the line graphs of Banana tree graph, Firecracker graph and subdivision graphs, *Global Journal of Pure and Applied Mathematics* **13**(6): 2673-2682 (2017).

[25] G. Srinivasa and K. Asha, Atom-bond connectivity index of subdivision graphs of some special graphs, *International Journal of Advance Research in Computer Science* **8**(6): (2017).

[26] S. H. Bertz, The bond graph, *J. C. S. Chem. Commun.*: 818-820 (1981).

[27] I. Gutman and E. Estrada, Topological indices based on the line graph of molecular graph, *J. Chem. Inf. Comput. Sci.* **36**: 541 (1996)

[28] M. R. Farahani, M. F. Nadeem, S. Zafar and M. N. Husin, Study of the topological indices of the line graphs of H-pantacenic nanotubes, *New FRONT. Chem.* **26**(1): 31-38 (2017).