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ORDINARY MODE IN AN INHOMOGENEOUS THERMAL PLASMA IN THE PRESENCE OF MILDLY SUPRATHERMAL ELECTRONS

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Abstract—The perpendicularly propagating ordinary mode is investigated in an inhomogeneous thermal plasma containing a component of mildly suprathermal electrons. Expressions for the real and imaginary parts of the refractive index of the ordinary mode are obtained. Using some typical plasma parameters graphs are obtained exhibiting the behaviour of the real part of the refractive index and the penetration character of the ordinary mode into the plasma as a function of ω^2.

BORNATICI and ENGLEMANN (1978) have shown the effect of runaways on electron cyclotron radiation in a cold inhomogeneous plasma, from the macroscopic point of view. The purpose of this note is to investigate the above mentioned situation for a hot plasma.

As in BORNATICI and ENGELMANN (1978) we consider the plasma in a slab geometry, having a density gradient in the x-direction and an externally applied uniform magnetic field $\mathbf{B}_0$ in the z-direction. The geometric optics approximation is used to describe waves propagating in the x-direction in a frame of reference with respect to which particles of species α move along $\mathbf{B}_0$ with uniform streaming velocity $\mathbf{v}_{0,\alpha}$. All spatial and temporal variations are taken in the form:

$$\exp i\left\{\frac{\omega}{c}\int_0^x N\,\mathrm{d}x - \omega t\right\}$$

where $N = N(x) = k(c/\omega)$ is the slowly varying index of refraction.

Equations used to describe the Fourier analysed components of the wave electric field $\mathbf{E} = \mathbf{E}(k, \omega)$, the plasma dielectric tensor ϵ, the fluctuating current density $\mathbf{j}$, and the fluctuating number density $n_\alpha(k, \omega)$ are those described by BORNATICI and ENGELMANN (1978) and are given by:

$$\left(N^2 - i\frac{c}{\omega}\frac{\mathrm{d}N}{\mathrm{d}x}\right)(\hat{y}E_y + \hat{z}E_z) - \epsilon\cdot E = 2i\frac{c}{\omega}N\left(\hat{y}\frac{\mathrm{d}E_y}{\mathrm{d}x} + \hat{z}\frac{\mathrm{d}E_z}{\mathrm{d}x}\right) \tag{1}$$

$$\epsilon\cdot\mathbf{E} = \mathbf{E} + \frac{4\pi}{\omega}i\mathbf{j} \tag{2}$$

$$\mathbf{j} = \sum_\alpha q_\alpha(n_{0,\alpha}\mathbf{v}_\alpha(k, \omega) + \hat{z}v_{0,\alpha}n_\alpha(k, \omega)) \tag{3}$$

$$\frac{n_\alpha(k, \omega)}{n_{0,\alpha}} = \left(1 - \frac{i}{kL_\alpha}\right)\frac{kv_{\alpha,x}}{\omega} - \frac{i}{\omega}\frac{\mathrm{d}v_{\alpha,x}}{\mathrm{d}x} \tag{4}$$

where $\hat{y}$, $\hat{z}$ are unit vectors in the y-and z-directions respectively and $n_{0,\alpha}$ and $\mathbf{v}_{0,\alpha}$ are the unperturbed number density and streaming velocity of particles of species α respectively and

$$L=\left(\frac{1}{n_{0,\alpha}}\frac{\mathrm{d}n_{0,\alpha}}{\mathrm{d}x}\right)^{-1}$$

The fluctuating velocity $\mathbf{v}_\alpha=\mathbf{v}_\alpha(k,\omega)$ is obtained from the linearized equation of motion in the collisionless limit, with an isotropic pressure tensor and is given by the expression:

$$\frac{\partial\mathbf{v}_\alpha}{\partial t}=\frac{q_\alpha}{m_\alpha}\mathbf{E}_1+\omega_{c,\alpha}(\mathbf{v}_\alpha\times\hat{z})+\frac{q_\alpha}{m_\alpha c}\mathbf{v}_{0,\alpha}\times\mathbf{B}_1-\frac{KT_\alpha}{m_\alpha n_{0,\alpha}}\nabla n_\alpha \tag{5}$$

where K is Boltzmann's constant, $\omega_{c,\alpha}=(q_\alpha B_0/m_\alpha c)$ the cyclotron frequency, T_α and m_α are the temperature and mass of particles of species α, $\mathbf{E}_1$ and $\mathbf{B}_1$ are the fluctuating electric and magnetic fields respectively.

From Maxwell's equations and differentiating equation (5) with respect to time we get the following expressions for $\mathbf{n}_\alpha$

$$v_{\alpha,x}=\frac{iq_\alpha}{m_\alpha\omega}E_x-i\frac{\omega_{c,\alpha}}{\omega}v_{\alpha,y}+\frac{ic}{\omega}\frac{q_\alpha}{m_\alpha}\frac{v_{0,\alpha}}{c^2}NE_z\left(1-\frac{i}{kE_z}\right)+\frac{v^2_{Th,\alpha}}{c}\frac{Nn_\alpha}{n_{0,\alpha}} \tag{6}$$

$$v_{\alpha,y}=i\frac{q_\alpha}{m_\alpha\omega}E_y+i\frac{\omega_{c,\alpha}}{\omega}v_{\alpha,x} \tag{7}$$

$$v_{\alpha,z}=i\frac{q_\alpha}{m_\alpha\omega}E_z \tag{8}$$

where $v_{th,\alpha}=\sqrt{(KT_\alpha/m_\alpha)}$ is the thermal velocity of particles of species α.

From equations (4), (6) and (7) we find that the fluctuating velocity $v_{\alpha,x}$ is governed by the following first order first degree differential equation:

$$\frac{\mathrm{d}v_{\alpha,x}}{\mathrm{d}x}+P_\alpha(x)v_{\alpha,x}+Q_\alpha(x)=0 \tag{9}$$

$$P_\alpha(x)=i\frac{\omega}{c}\frac{c^2}{v^2_{th,\alpha}}\frac{1}{N}\left(1-\frac{\omega^2_{c,\alpha}}{\omega^2}\right)+i\left(1-\frac{i}{kL_\alpha}\right)k \tag{10}$$

$$Q_\alpha(x)=i\frac{\omega}{c}\frac{c^2}{v^2_{th,\alpha}}\frac{1}{N}\left[i\frac{q_\alpha}{m_\alpha\omega}E_x+\frac{\omega_{c,\alpha}}{\omega^2}\frac{q_\alpha}{m_\alpha}E_y+\right.$$
$$\left.+i\frac{c}{\omega}\frac{q_\alpha}{m_\alpha}\frac{v_{0,\alpha}}{c^2}NE_z\left(1-\frac{i}{kE_z}\frac{\mathrm{d}E_z}{\mathrm{d}x}\right)\right] \tag{11}$$

The formal solution of equation (9) can be written as (Ross 1964)

$$v_{\alpha,x} = \exp\left\{-\int_0^x P_\alpha(x')\,dx'\right\}\left[C - \int_0^x Q_\alpha(x')\exp\left\{\int_0^{x'} P_\alpha(x'')\,dx''\right\}dx'\right] \tag{12}$$

where C is given by the boundary value.

In order to derive an expression for the dispersion relation, we neglect ion-dynamics and concern ourselves only with high frequency waves. Following Bornatici and Engelmann (1978) we separate the motion of the electrons into the motion of their centre of mass at velocity

$$\mathbf{v}_{CM} = \frac{1}{n_0}\sum_\alpha n_{0,\alpha}\mathbf{v}_{0,\alpha},$$

$\sum_\alpha$ indicates summation over electrons only—the bulk electrons and mildly suprathermal electrons and the relative velocity $\mathbf{u}_\alpha = \mathbf{v}_{0,\alpha} - \mathbf{v}_{CM}$. We shift to a comoving frame of reference in which the ordinary mode becomes purely transverse ($E_x = E_y = 0$). It may be noted that the extra-ordinary mode is not affected by the runaways and thus is not investigated. Using equations (2), (3), (4) and (12) we obtain

$$\epsilon_{zz} = 1 - \sum_\alpha \frac{\omega_{p,\alpha}^2}{\omega^2} + \sum_\alpha \frac{\omega_{p,\alpha}^2}{\omega^2}\frac{v_{0,\alpha}^2}{v_{th,\alpha}^2}\left[-i\frac{\omega}{c}\frac{c^2}{v_{th,\alpha}^2}\frac{1}{N}\left(1 - \frac{\omega_{c,\alpha}^2}{\omega^2}\right)\cdot\right.$$
$$\left.\cdot\exp\left\{-\int_0^x P_\alpha(x')\,dx'\right\}\cdot\int_0^x \exp\left\{\int_0^{x'} P_\alpha(x'')\,dx''\right\}dx' + 1\right] \tag{13}$$

where $\omega_{p,\alpha} = \sqrt{(4\pi n_{0,\alpha}e^2/m)}$ is the plasma frequency of the electrons.

We assume two components of electrons—the bulk electrons with a uniform streaming velocity $\mathbf{v}_{0,1}$ and the mildly suprathermal electrons with a uniform streaming velocity $\mathbf{v}_{0,2} \gg \mathbf{v}_{0,1}$. Their unperturbed number densities are $n_{0,1}$ and Δn respectively, and L the scale length is taken to be the same for both components. Equation (13) can now be written as

$$\epsilon_{zz} = 1 - \frac{\omega_p^2}{\omega^2} + i\frac{\omega}{c}\frac{\omega_p^2}{\omega^2}\frac{1}{N}\left(1 - \frac{\omega_c^2}{\omega^2}\right)\left[\frac{u_1^2c^2}{v_{th,1}^2}\exp\left\{-\int_0^x p_1\,dx'\right\}\cdot\right.$$
$$\cdot\int_0^x \exp\left\{\int_0^{x'} P_1\,dx''\right\}dx' + \frac{u_2^2c^2}{v_{th,2}^2}\exp\left\{-\int_0^x P_2\,dx'\right\}$$
$$\left.\cdot\int_0^x \exp\left\{\int_0^{x'} P_2\,dx''\right\}dx'\right] + \frac{\omega_p^2}{\omega^2}\frac{u_2^2}{v_{th,2}^2} \tag{14}$$

where $\omega_p = \sqrt{(4\pi n_0 e^2/m)}$ is the total electron plasma frequency, ω_c is the electron-cyclotron frequency, and $u_1^2 = (n_{0,1}/n_0)v_{0,1}^2$, $u_2^2 = (\Delta n_0/n_0)v_{0,2}^2$. The suffixes 1, 2 correspond to the bulk and mildly suprathermal components respectively.

In order to yield an expression for the dispersion relation we substitute equation (14) into equation (1) and differentiate with respect to x. We also take into account that the electrons are only mildly suprathermal, thus $P_l - P_2$ is very small. We separate N into real and imaginary parts by substituting

$$N = N_r + i\frac{c}{\omega}k_I, \quad N_r > \frac{c}{\omega}|k_I| \tag{15}$$

The resulting expression is in various powers of c/ω. Collecting leading-order terms from the real and imaginary parts we obtain

$$N_r^4X^2 - \{a(X-Y) + X - (1-b)\}N_r^2X + \\ + a(X-Y)(X-1) = 0 \tag{16}$$

$$k_I - \frac{4N_r^2X - \{a(X-Y) + X - (1-b)\}}{2N_r[2N_r^2X - \{a(X-Y) + X - (1-b)\}]} \cdot \frac{dN_\gamma}{dx} = \\ = \frac{1}{2}\frac{(1-b)}{2N_r^2X - \{a(X-Y) + X - (1-b)\}} \cdot \frac{1}{L} \equiv k_i \tag{17}$$

where $X = \omega^2/\omega_p^2$, $Y = \omega_c^2/\omega_p^2$, $a = c^2/v_{th}^2$, $b = u^2/v_{th}^2$. It may be noted that both a, b correspond to the mildly suprathermathermal component.

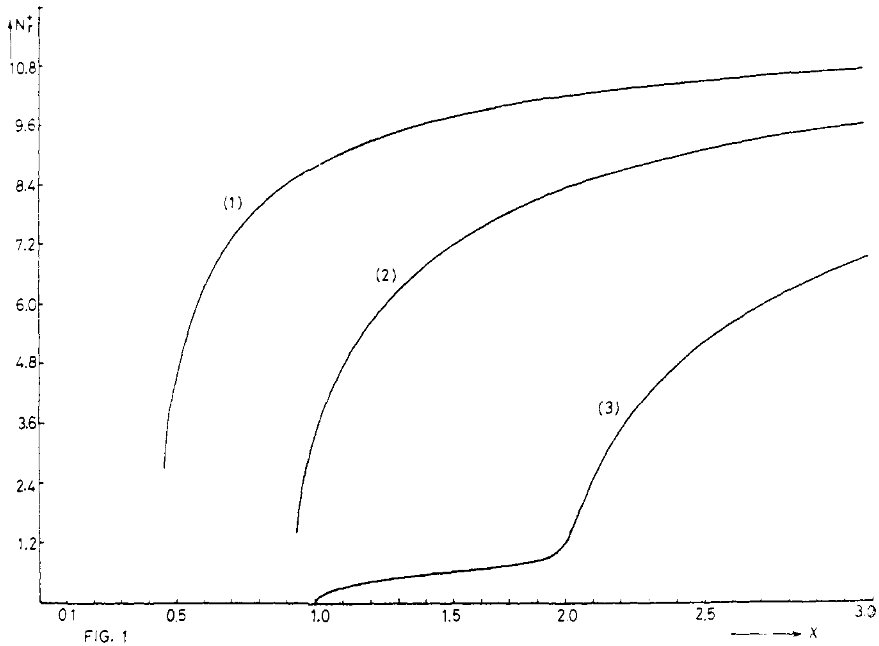

FIG. 1

FIG. 1.—The N_r^+ mode as a function of $X(=\omega^2/\omega_p^2)$. Curves (1), (2) and (3) correspond to $Y = \omega_c^2/\omega_p^2 = 1/2$, 1, 2 respectively. $a = 130$, $b = 10$, $\omega_p^2 = 1.9 \times 10^{23}$ (rad s^{-1})^2.

From equations (16) and (17) we note that the real part of the refractive index is affected by the relative motion only, whereas the imaginary part depends both on the relative motion and the slow spatial variation of the real part of the refractive index.

Propagation characteristics can be investigated by looking at the solution to the biquadratic equation (16). In general there are two modes of propagation corresponding to the two solutions $N_r^{\pm 2}$. The penetration parameter is determined from equation (17), and is given by $\tau^{\pm} = 2k_i^{\pm}L$. Figures 1–4, exhibit the behaviour of $N_r^{\pm}$ and $\tau^{\pm}$ as functions of ω^2 and the parameters are fixed in the following manners: $a = 130$, $b = 10$, $\omega_p^2 = 1.9 \times 10^{23}$ (rad $s^{-1})^2$. The curves (1), (2) and (3) correspond to $Y = 1/2$, $Y = 1$ and $Y = 2$, respectively.

When $Y = 1/2$, 1 the N_r^+ mode begins continuous propagation just below ω_c^2. On the other hand the N_r^- mode begins continuous propagation when $\omega^2 > \omega_p^2$ and has a narrow propagation band just below ω_c^2. At the points $\omega^2 = \omega_c^2$ and $\omega^2 = \omega_p^2$. $N_r^- = 0$ which are reflection points.

When $Y = 2$, the N_r^+ mode exists for $\omega^2 > \omega_p^2$, the increase is at first gradual, but then rises steeply from around the point $\omega^2 = \omega_c^2$. This mode has a reflection point at $\omega^2 = \omega_p^2$. The N_r^- mode begins propagation when $\omega^2 > \omega_c^2$ and has a reflection point at $\omega^2 = \omega_c^2$.

In general the same behaviour pattern is exhibited when electron cyclotron frequency is smaller or larger than the electron plasma frequency corresponding to $Y \leq 1$ and $Y > 1$ respectively. We also see that the N_r^+ mode corresponds to subluminous propagation, except in the range $\omega_p^2 < \omega^2 < \omega_c^2$, when $Y > 1$. On the other hand the propagation of the N_r^- mode is mainly superluminous.

For $L > 0$, the penetration parameter τ^+ has negative values, whereas τ^- has

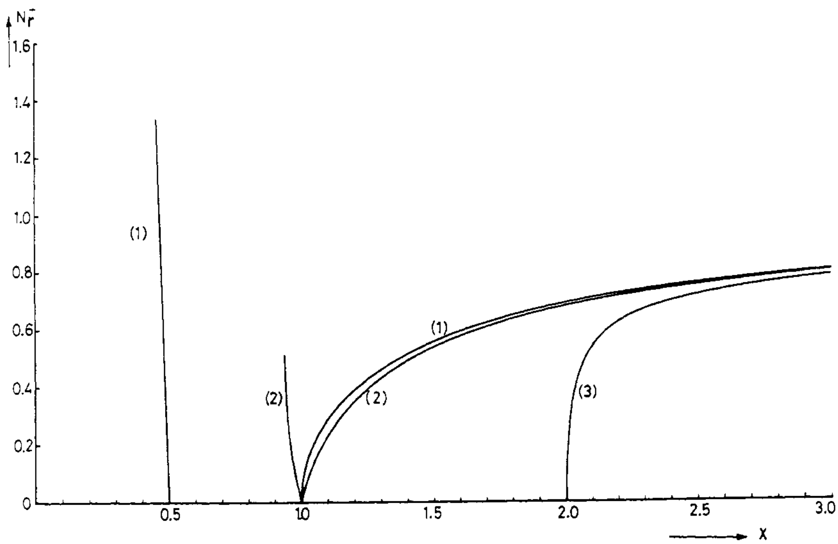

FIG. 2.—The N_r^- mode as a function of $X(=\omega^2/\omega_p^2)$. Curves (1), (2) and (3) correspond to $Y = \omega_c^2/\omega_p^2 = 1/2$, 1, 2 respectively $a = 130$, $b = 10$, $\omega_p^2 = 1.9 \times 10^{23}$ (rad $s^{-1})^2$.

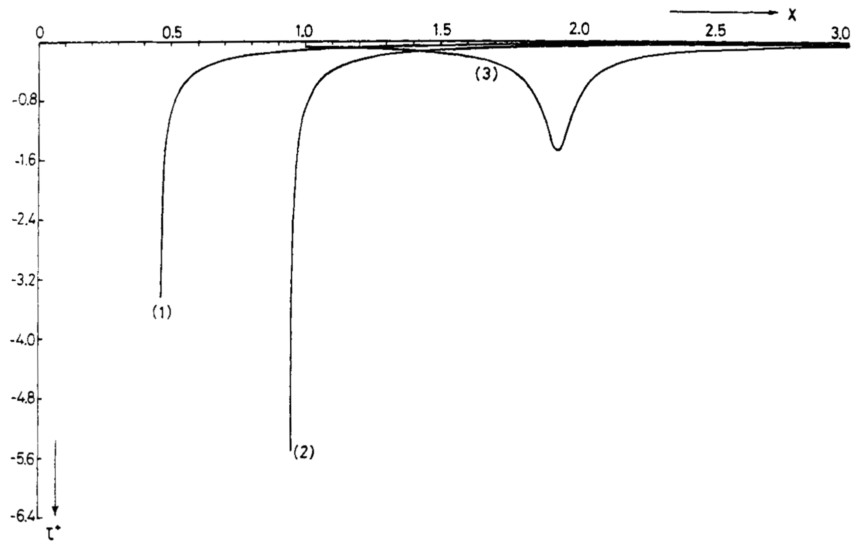

FIG. 3.—The penetration parameter τ^+ as a function of $X(=\omega^2/\omega_p^2)$. Curves (1), (2) and (3) correspond to $Y=\omega_c^2/\omega_p^2=1/2$, 1, 2 respectively. $a=130$, $b=10$, $\omega_p^2=1.9\times10^{23}$ $(\text{rad s}^{-1})^2$.

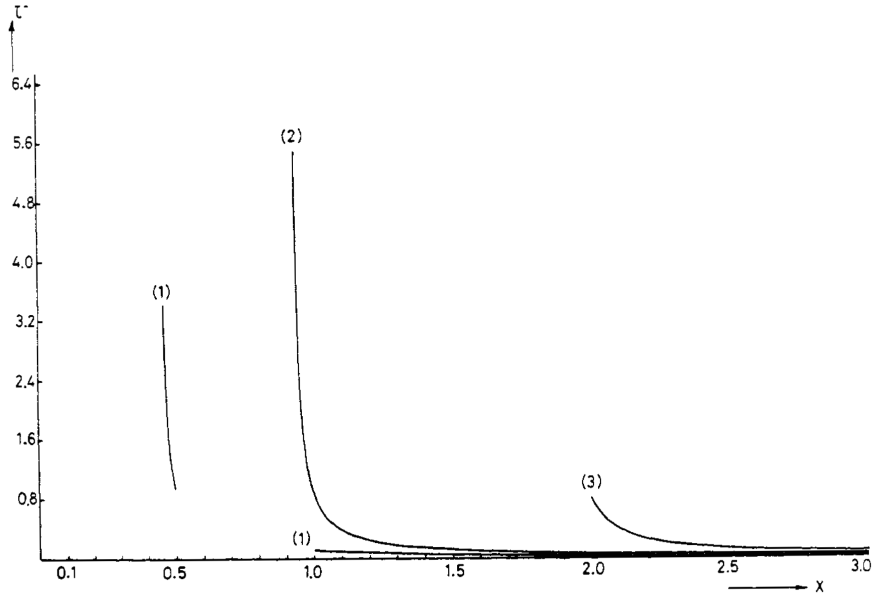

FIG. 4.—The penetration parameter τ^- as a function of $X(=\omega^2/\omega_p^2)$. Curves (1), (2) and (3) correspond to $Y=\omega_c^2/\omega_p^2=1/2$, 1, 2 respectively. $\alpha=130$, $b=10$, $\omega_p^2=1.9\times10^{23}$ (rad s^{-1}).

positive values. This in fact tells whether the energy is transferred from the electrons to the wave or vice versa. We also see that maximum values of the penetration parameter occur near the electron cyclotron frequency, showing that it is here that a maximum exchange of energy between the particles and wave takes place. This can be shown analytically also and the point where a maximum exchange takes place is given by the formula

$$X_{max} = \frac{a}{a-1} - \frac{[(a-1)+b(a+1)]}{(a-1)^2} \approx Y^2 \tag{18}$$

It may be noted that the cold approximation (BORNATICI and ENGELMANN (1978)) did not yield this result, since the approximation broke down around the said value.

When thermal effects are neglected, we obtain the same result as that by BORNATICI and ENGELMANN (1978), who investigated the cold collisionless case.

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