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Modulational stability of coupled non-linear helicon–acoustic waves in a piezoelectric semiconductor plasma

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Abstract. In the present paper we investigate the behaviour and subsequent modulational instability or stability of coupled non-linear helicon–acoustic waves in a piezoelectric semiconductor plasma. We use the Krylov–Bogoluibov–Mitropolski technique of multiple-scale analysis to arrive at the non-linear Schrödinger equation, which governs the behaviour of the above-mentioned waves. Physical parameters of InSb are used to investigate numerically whether the wave is modulationally stable or unstable.

1. Introduction

Since the discovery of helicon wave propagation in semiconductors, the interactions between it and other modes—acoustic as well as electromagnetic—of the propagating medium have received considerable attention. There exists a vast literature on helicon waves, including review articles and monographs on the propagation characteristics, experimental properties and applications of these waves (see, e.g., Morgan (1967), Kaner and Skobkov (1968), Maxfield (1969), Baynham and Boardman (1970), Petrashov (1984) and Platzman and Wolff (1973)). Most of the above-mentioned studies deal with the linear behaviour of these waves. However, in the past decade or so, non-linear propagation characteristic of helicon and other electromagnetic modes as well as acoustic modes have become the subject of investigation, in semiconductors as well as in piezoelectric semiconductors. Piezoelectric semiconductors are of importance not only because of the applications of such materials in various solid state and ultrasonic devices, but also because such materials present the opportunity to investigate the coupling between mechanical and electromagnetic effects.

Non-linear processes in semiconductors and piezoelectric semiconductor plasmas have been investigated for the past two decades. Progress in this area has received considerable impetus from corresponding developments in non-linear physics, plasma physics and computational methods. Pawlik and Rowland (1975) presented a non-linear theory of the propagation of acoustic waves in a piezoelectric semiconductor plasma, based on an asymptotic expansion in multiple time and space scales. The envelope of the wave amplitude is found to satisfy a non-linear Schrödinger (NLS) equation with complex coefficients. The solution of this equation is obtained using a perturbation technique. This results in a solitary wave and is used to study the saturation of the instability. It is found that the saturation mechanism is a local change in the DC electric field and carrier concentration.

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The non-linear effects investigated thus far also include work by Guha and Sen (1979), who studied the modulational instability of a laser beam via the non-linear interaction of an external laser beam with a low-frequency transverse acoustic wave in the presence of a transverse magnetostatic field in a heavily doped piezoelectric semiconductor and showed that the instability results in the amplification of acoustic waves. Sen (1980) has studied the phenomenon of stimulated Brillouin scattering in a one-component (electron) homogeneous piezoelectric semiconductor subject to a large transverse magnetostatic field when the semiconductor is irradiated with a spatially uniform laser beam. Sen *et al* (1980) have investigated the phenomenon of Raman instability in an n-type piezoelectric semiconductor in the presence of a large transverse magnetostatic field. The above-mentioned workers have shown that a large transverse magnetostatic field reduces the threshold value of the pump amplitude and increases the growth rate of the unstable Raman mode at an electric field amplitude greater than the threshold value. Sen and Sen (1982) have reported the results of the analytical investigation of a stimulated Brillouin instability in a magnetoactive piezoelectric semiconductor plasma under various geometrical configurations of the electric field and discussed the possibilities of obtaining maximum growth of the unstable Brillouin mode in the crystal.

Ghosh and Khan (1986) have studied the excitation of acoustic–helicon waves and the subsequent parametric amplification of acoustic waves in piezoelectric semiconducting crystals under the influence of an external magnetic field. Shah *et al* (1991) have investigated the non-linear behaviour of coupled helicon–acoustic waves in a piezoelectric semiconductor plasma via a Korteveg deVries (KdV) type of equation. In later work, Shah *et al* (1993) studied the propagation of non-linear helicon waves in a layered structure. The reductive perturbation method was used to derive the non-linear evolution equation. This equation has a one-soliton solution and this solution has been derived. More recently Anwar *et al* (1995) have considered parametric instability in a piezoelectric semiconductor plasma. In this work the three interacting waves were an ordinary wave, an extraordinary wave and a coupled upper hybrid acoustic wave.

In the present work we shall be considering the non-linear propagation of coupled helicon–acoustic waves in a piezoelectric semiconductor plasma via the solutions of the NLS equation. We shall use the Krylov–Bogoluibov–Mitropolski (KBM) technique (Kakutani and Sugimoto 1974) to arrive at the above-mentioned non-linear evolution equation. The linear dispersion relation is obtained in the lowest order of KBM expansion. In the next order of the KBM method (section 3) we establish a relationship between the group velocity and the frame of reference of the coupled helicon–acoustic waves. In section 4 we obtain the NLS equation and in section 5 we numerically investigate modulational stability–instability and on this basis discuss the possible types of solution of the NLS equation.

2. Mathematical formulation

We consider an n-type piezoelectric semiconductor plasma in order to investigate the non-linear behaviour of coupled helicon–acoustic waves. The set of equations necessary for deriving the NLS equation for the above-mentioned waves for the one-dimensional case is given below:

$$\partial_t n + \partial_t n v_z = 0 \tag{1}$$

$$\partial_t v_\pm + v_z \partial_z v_\pm = -\frac{e}{m} E_\pm \mp \frac{\mathrm{i}e}{m}(v_z B_\pm - \omega_c v_\pm) \tag{2}$$

$$\partial_t v_z + v_z \partial_z v_z = -\frac{e}{m}(v_x B_y - v_y B_x) - \frac{v_T^2}{n}\partial_z n \tag{3}$$

$$(\rho \partial_{tt} - c_e \partial_{zz}) u_\mp = \beta \partial_z E_\pm \tag{4}$$

$$\partial_z E_\pm = \pm \mathrm{i} \partial_t B_\pm \tag{5}$$

$$\partial_{zz} E_\pm = \mu_0 [\partial_{tt} D_\pm + \partial_t j_\pm] \tag{6}$$

$$D_\pm = \varepsilon E_\pm - \beta \partial_z u_\mp \tag{7}$$

$$j_\pm = -n e v_\pm. \tag{8}$$

Equation (1) is a continuity equation of the electrons; equations (2) and (3) are the electron equations of motion in the perpendicular and parallel directions, respectively. Equation (4) is the lattice equation of motion, and equations (5)–(8) are Maxwell's equations. Since helicon waves are circularly polarized waves, the fluctuating quantities have all been expressed in the form $\phi_\pm = \phi_x \pm \mathrm{i}\phi_y$, where $\pm$ corresponds to the right and left circularly polarized waves, respectively. The variables u, v, n, E and B are the lattice displacement, the electron velocity, the electron number density, the electric field and the magnetic field, respectively. The quantities μ, ε and ω_c are the magnetic susceptibility, the dielectric constant and the electron cyclotron frequency, respectively.

In order to derive the NLS equation we use the KBM technique (Kakutani and Sugimoto 1974). The parameters n and v_z do not contribute to the linear dispersion relation for the helicon waves; thus the fluctuations in the leading order terms would be of order ε^4. Thus an expansion is made in the following way:

$$n = n_0 + \varepsilon^4 n_1$$
$$v_z = \varepsilon^4 v_{z1} + \cdots.$$

The other variables are expanded in the following manner:

$$\phi_\pm = \varepsilon \phi_{\pm 1} + \varepsilon^3 \phi_{\pm 2} \ldots . \tag{9}$$

Here ε is the perturbation parameter. The quantities n_1, $v_{\pm 1}$, $u_{\mp 1}$, $E_{\pm 1}$, $B_{\pm 1}, \ldots, n_2$, $v_{\pm 2}$, $u_{\mp 2}$, $E_{\pm 2}$, $B_{\pm 2}, \ldots$ depend on x and t through a and $\bar{a}$ (the complex amplitude and its complex conjugate) and ψ (phase factor) which is given by $\psi = kx - \omega t$. The complex amplitude a is a slowly varying function of x and t of the form

$$\begin{aligned} \partial_t a &= \varepsilon \mathcal{A}_1(a, \bar{a}) + \varepsilon^2 \mathcal{A}_2(a, \bar{a}) + \cdots \\ \partial_x a &= \varepsilon \mathcal{B}_1(a, \bar{a}) + \varepsilon^2 \mathcal{B}_2(a, \bar{a}) + \cdots . \end{aligned} \tag{10}$$

In the light of equation (10) we may rewrite ∂_t and ∂_x as

$$\begin{aligned} \partial_t &= -\omega \partial_\psi + \varepsilon^2 (\mathcal{A}_1 \partial_a + \mathrm{CC}) + \varepsilon^4 (\mathcal{A}_2 \partial_a + \mathrm{CC}) + \cdots \\ \partial_x &= k \partial_\psi + \varepsilon^2 (\mathcal{B}_1 \partial_a + \mathrm{CC}) + \varepsilon^4 (\mathcal{B}_2 \partial_a + \mathrm{CC}) + \cdots . \end{aligned} \tag{11}$$

The unknown functions $\mathcal{A}_1$, $\mathcal{B}_1$; $\mathcal{A}_2$, $\mathcal{B}_2$; ... are arbitrary at this stage and are determined later when the solution is made free from secular terms.

The variables u, v_ρ, v_s, n, E and B have been normalized in the following manner: $u = u\omega_P/v_T$, $v_\rho = v_\rho/v_T$, $v_s = v_s/v_T$, $n = n/n_0$, $E = E/v_T B_0$ and $B = B/B_0$. We further note here that the wavenumber k and frequency ω are also normalized. This normalization has been done in the following way: $k = k v_T/\omega_P$ and $\omega = \omega/\omega_P$.

We now substitute the expanded quantities and operators in the set of equations (1)–(8) and collect terms in different orders of ε. From the first order in ε we eliminate $E_{\pm 1}$, $u_{\mp 1}$ and $B_{\pm 1}$ and arrive at the following differential equation in $v_{\pm 1}$:

$$\left[\omega \partial_{\psi\psi} \pm \mathrm{i}\omega_c \partial_\psi + \frac{1}{\zeta c^2}\right] v_{\pm 1} = 0 \tag{12}$$

where

$$\zeta = \frac{\omega}{c^2} - \frac{k^2}{\omega} - \frac{\alpha k^2 v_\rho \omega}{\omega^2 - k^2 v_s^2}$$
$$\alpha = \mu_0 \beta v_T / B_0.$$

Assuming a plane-wave solution of the form

$$v_\pm = a\exp(1\psi) + \mathrm{CC} \tag{13}$$

and substituting in equation (12) we get the linear dispersion relation

$$(\omega \pm \omega_c)\zeta - \frac{1}{c^2} = 0. \tag{14}$$

From equations of order ε we can obtain expressions for the other unknown quantities and these are given by

$$E_{\pm 1} = \frac{\mathrm{i}}{\zeta\omega_{c^2}}[a\exp(\mathrm{i}\psi) + \mathrm{CC}] \tag{15}$$

$$u_{\mp 1} = \frac{kV\rho}{\zeta\omega_c c^2(\omega^2 - k^2 v_s^2)}[a\exp(\mathrm{i}\psi) + \mathrm{CC}] \tag{16}$$

$$B_{\pm 1} = \mp\frac{k}{\zeta\omega_c c^2 \omega}[a\exp(\mathrm{i}\psi) + \mathrm{CC}]. \tag{17}$$

The group velocity of the coupled helicon–acoustic wave is obtained from equation (14) and has the following form:

$$V_g = \frac{(2/\zeta c^2)\{k/\omega + \alpha k v_\rho \omega/(\omega^2 - k^2 v_s^2) + \alpha k v_\rho^3 \omega v_s^2/(\omega^2 - k^2 v_s^2)^2\}}{\zeta + (1/\zeta c^2)\{1/c^2 + k^2/\omega^2 - \alpha k^2 v_\rho/(\omega^2 - k^2 v_s^2) + 2\alpha k^2 \omega^2 v_\rho/(\omega^2 - k^2 v_s^2)^2\}}. \tag{18}$$

3. Terms of order ε^3

In this section we collect third-order terms in ε from the initial set of equations (1–8). This results in a set of equations relating together $u_{\mp 2}$, $E_{\pm 2}$, $B_{\pm 2}$ and cross products between terms with subscript 1. Using the results of the previous section and eliminating $u_{\mp 2}$, $E_{\pm 2}$ and $B_{\pm 2}$, we obtain the following differential equation for $v_{\pm 2}$:

$$\left[\omega\partial_{\psi\psi} \pm \mathrm{i}\omega_c\partial_\psi + \frac{1}{\zeta c^2}\right]v_{\pm 2} = -\partial_\psi f_3 + \frac{\omega_c}{\zeta}\left[f_4 - \frac{k}{\omega}f_1 + \frac{\alpha k\omega f_2}{\omega^2 - k^2 v_s^2}\right] \tag{19}$$

where

$$f_1 = -\frac{\mathrm{i}}{\zeta\omega_c c^2}\left[\frac{k}{\omega}(\mathcal{A}_1 + \mathrm{CC}) + (\mathcal{B}_1 + \mathrm{CC})\right][\exp(\mathrm{i}\psi) + \mathrm{CC}]$$

$$f_2 = \frac{2\mathrm{i}\omega k v_\rho}{\zeta\omega_c c^2(\omega^2 - k^2 v_s^2)}(\mathcal{A}_1 + \mathrm{CC})[\exp(\mathrm{i}\psi) + \mathrm{CC}]$$
$$+ \frac{\mathrm{i}v_\rho}{\zeta\omega_c c^2}(\mathcal{B}_1 + \mathrm{CC})\left\{1 + \frac{2k^2 v_s^2}{\omega^2 - k^2 v_s^2}\right\}(\exp(\mathrm{i}\psi) + \mathrm{CC})$$

$$f_3 = -(\mathcal{A}_1 + \mathrm{CC})[\exp(\mathrm{i}\psi) + \mathrm{CC}]$$

$$f_4 = \frac{\mathrm{i}k}{\zeta\omega_c c^2}\left[\frac{1}{\omega} + \frac{\alpha v_\rho \omega}{\omega^2 - k^2 v_s^2}\right][\exp(\mathrm{i}\psi) + \mathrm{CC}](\mathcal{B}_1 + \mathrm{CC})$$
$$- \frac{\mathrm{i}}{\zeta\omega_c c^2}\left[-\frac{1}{c^2} + \frac{\alpha k^2 v_\rho}{\omega^2 - k^2 v_s^2}\right](\mathcal{A}_1 + \mathrm{CC})[\exp(\mathrm{i}\psi) + \mathrm{CC}].$$

We see from the above expression that the terms proportional to exp(iψ) and its complex conjugate are secular terms, which cause a divergence in the solution. Such terms are removed by putting them equal to zero (Kakutani and Sugimoto 1974). This results in the following:

$$\mathcal{A}_1 + V_g\mathcal{B}_1 = 0. \tag{20}$$

We note here that

$$\mathcal{A}_1 = \partial_{t1}a \qquad \mathcal{B}_1 = \partial_{x1}a. \tag{21}$$

Equation (20) shows that the amplitude remain constant in a frame of reference moving with the group velocity of the wave.

Expressions for $E_{\pm2}$, $u_{\mp2}$ and $B_{\pm2}$ can now be obtained and these are given by

$$E_{\pm2} = \frac{1}{\omega_c}\{v_g\mathcal{B}_1 + \mathrm{i}(\omega \pm \omega_c)C_1\}[\exp(\mathrm{i}\psi) + \mathrm{CC}] + C_2(a,\bar{a}) \tag{22}$$

$$u_{\mp2} = \frac{1}{\omega^2 - k^2v_s^2}\left\{\frac{-\mathrm{i}}{\zeta\omega_c c^2}\left[1 + \frac{2k^2v_s^2}{\omega^2 - k^2v_s^2} - \frac{2k\omega v_g}{\omega^2 - k^2v_s^2}\right]\mathcal{B}_1 - \frac{kv_\rho}{\omega_c}[\mathrm{i}v_g\mathcal{B}_1 - (\omega \pm \omega_c)C_1]\right\}[\exp(\mathrm{i}\psi) + \mathrm{CC}] + C_3(a,\bar{a}) + C_4(a,\bar{a}) \tag{23}$$

$$B_{\pm2} = \frac{1}{\pm\mathrm{i}\omega}\left\{\frac{-1}{\zeta\omega c^2\omega_c}(\omega - kv_g)\mathcal{B}_1 - \frac{k}{\omega_c}[v_g\mathcal{B}_1 + \mathrm{i}(\omega \pm \omega_c)C_1]\right\} \times(\exp(\mathrm{i}\psi) + \mathrm{CC}) + C_5(a,\bar{a}). \tag{24}$$

From terms of order ε^4 we obtain an expression relating the parallel velocity fluctuations to the perpendicular velocity fluctuations. This is given by the following equation:

$$v_{z1} = \frac{-k}{2\zeta c^2(\omega^2 - k^2v_T^2)}|v|^2 \tag{25}$$

where

$$|v|^2 = v_x^2 + v_y^2.$$

4. The non-linear Schrödinger equation

In this section we collect the next order terms, i.e. terms of order ε^5 from the set of equations (1)–(8). We generate a differential equation in $v_{\pm3}$ using the same argument as used for the ε^3-order case by eliminating all other unknowns. This differential equation has terms proportional to exp(iψ) on the right-hand side. The terms proportional to exp(iψ) and its complex conjugate are secular terms; these are set equal to zero in order to remove secularity in the differential equation. When these secular terms are put equal to zero, the following expression is obtained:

$$\mathrm{i}(\mathcal{A}_2 + V_g\mathcal{B}_2) + P(\mathcal{B}_1\partial_a\mathcal{B}_1 + \mathrm{CC}) + Q|a^2|\bar{a} = 0 \tag{26}$$

where P and Q are given by

$$P = -\left\{\left\{-\frac{1}{\zeta c^2}\left[\frac{3\alpha k^2v_\rho\omega}{(\omega^2 - k^2v_s^2)^2} - \frac{4\alpha k^2v_\rho\omega^3}{(\omega^2 - k^2v_s^2)^3} - \frac{k^2}{\omega^3}\right] + \left[\frac{1}{c^2} + \frac{k^2}{\omega^2} - \frac{\alpha k^2v_\rho}{\omega^2 - k^2v_s^2} + \frac{2\alpha k^2v_\rho\omega^2}{(\omega^2 - k^2v_s^2)^2}\right]\right\}V_g^2 + 2\left\{\frac{1}{\zeta c^2}\left[\frac{\alpha kv_\rho}{\omega^2 - k^2v_s^2} - \frac{2\alpha kv_\rho\omega^2}{(\omega^2 - k^2v_s^2)^2} + \frac{\alpha k^3v_\rho v_s^2}{(\omega^2 - k^2v_s^2)^2} - \frac{k}{\omega^2}\,\frac{-4\alpha k^3v_\rho v_s^2\omega^2}{(\omega^2 - k^2v_s^2)^3}\right]\right.$$

$$-\left[\frac{k}{\omega}+\frac{\alpha k v_\rho \omega}{(\omega^2-k^2v_s^2)}+\frac{\alpha k^3 v_\rho v_s^2\omega}{(\omega^2-k^2v_s^2)^2}\right]\Bigg\}V_g$$
$$+\frac{1}{\zeta c^2}\left[\frac{1}{\omega}+\frac{\alpha v_\rho\omega}{\omega^2-k^2v_s^2}+\frac{5\alpha k^2 v_\rho v_s^2\omega}{(\omega^2-k^2v_s^2)^2}+\frac{4\alpha k^4 v_\rho v_s^4\omega}{(\omega^2-k^2v_s^2)^3}\right]\Bigg\}$$
$$\Bigg/\left\{\zeta+\frac{1}{\zeta c^2}\left[\frac{1}{c^2}+\frac{k^2}{\omega^2}-\frac{\alpha k^2 v_\rho}{\omega^2-k^2v_s^2}+\frac{\alpha k^2 v_\rho\omega^2}{(\omega^2-k^2v_s^2)^2}\right]\right\}$$
$$Q=-\frac{k^2}{2\zeta c^2(\omega^2-k^2v_T^2)}\Bigg/\left\{\zeta+\frac{1}{\zeta c^2}\left[\frac{1}{c^2}+\frac{k^2}{\omega^2}-\frac{\alpha k^2 v_\rho}{\omega^2-k^2v_s^2}+\frac{2\alpha k^2 v_\rho\omega^2}{(\omega^2-k^2v_s^2)^2}\right]\right\}.$$

Equation (26) can be rewritten in a standard form by doing a reverse coordinate transformation (Kakutani and Sugimoto 1974) and we obtain the standard form of the NLS equation:

$$\mathrm{i}\partial_\tau a+P\partial_{\zeta\zeta}a+Q|a^2|\bar{a}=0. \tag{27}$$

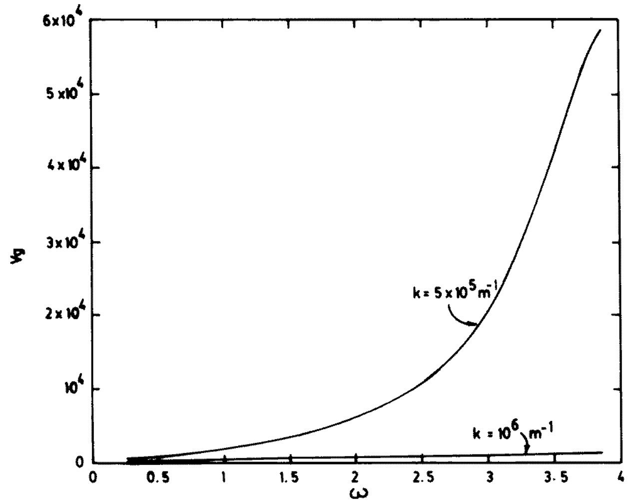

Figure 1. Plots of group velocity v_g against the normalized propagation frequency for $n_0 = 10^{23}$ m^{-3} and $B_0 = 11.3$ T.

5. Results and discussion

In the present section we use typical piezoelectric semiconductor plasma parameters and investigate the modulational stability–instability of the coupled helicon–acoustic waves. It has been shown (see, e.g., Hasegawa (1975)) that the sign of the ratio P/Q determines whether the wave described by the NLS equation (27) is modulationally stable or unstable. We note that P is the group dispersion parameter and Q is the non-linearity parameter. If $P/Q < 0$, then the wave is modulationally unstable and this leads to the formation of the envelope solitons, in which case equation (27) has known analytical solutions. In the other case, i.e. when $P/Q > 0$, then the wave is modulationally stable and this in turn implies a solution for the absent region of the wave field and is also known as an envelope hole (Hasegawa 1975). In this case, equation (27) also has a known analytical solution. However, the solution in this case has an additional parameter which describes the depth of

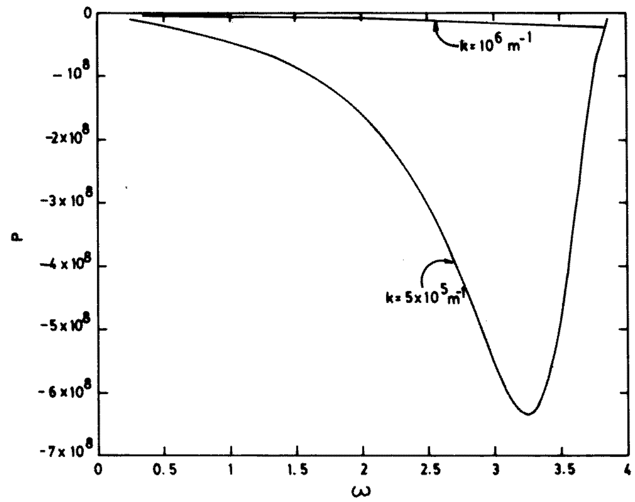

Figure 2. Plots of the group dispersion parameter P versus the normalized propagation frequency for $n_0 = 10^{23}\ \mathrm{m}^{-3}$ and $B_0 = 11.3$ T.

the modulation. For a particular value of this additional parameter, we can obtain solutions which represent the formation of an envelope soliton. We have used the following typical InSb parameters (Sen and Sen 1985) to investigate the modulational stability–instability of the coupled helicon–acoustic waves: $m = 0.014m_0$, $\varepsilon = 15.8$, $\beta = 0.054\ \mathrm{cm}^{-2}$, $\rho = 5.8 \times 10^3\ \mathrm{kg\ m}^{-3}$, $v_s = 0.795$ (normalized by v_T) and $v_T = 5.03 \times 10^3\ \mathrm{m\ s}^{-1}$.

In figure 1 we have plotted the group velocity v_g (given by equation (18)) against ω for $n_0 = 10^{23}\ \mathrm{m}^{-3}$ and $B_0 = 11.3$ T. We see that for lower values of k the group velocity is higher than for larger values of k, but in general v_g increases with increasing propagation frequency ω. In figures 2 and 3 we have plotted P and Q, respectively, against ω. Both P and Q vary in a complicated fashion with respect to ω. In figures 4–6 we have obtained graphs for P/Q against the normalized propagation frequency for different values of the normalized wavenumber k. We have taken a fixed value of magnetic field (in our case $B_0 = 11.3$ T). Figures 4–6 correspond to different values of the background electron number density (see figure captions). We see from figures 4–6 that, for lower values of the wavenumber, P/Q tends to have positive values in general. However, for lower values of ω, P/Q has negative values but becomes positive as ω increases. This implies that there are regions where the non-linear wave is modulationally unstable and regions where it is modulationally stable, i.e. for the same wavenumber there will be a region where there are envelope soliton solutions and regions where soliton hole solutions exist. Soliton hole solutions are characterized by an additional parameter which is known as the modulational parameter, and this defines the depth of the modulation. If this additional modulation parameter is equal to unity, then shock wave solutions are obtained (Hasegawa 1975). We further note that, as the number density increases, the regions where the soliton solutions exist become larger. We have also checked that by changing the value of the background magnetic field there is no significant change in the trend of the graphs. We note here that in all the figures, i.e. figures 1–6, the values of k have been stated in dimensional terms although while obtaining these graphs the dimensionless values of the wavenumber has been used $(k = kv_T/\omega_P)$.

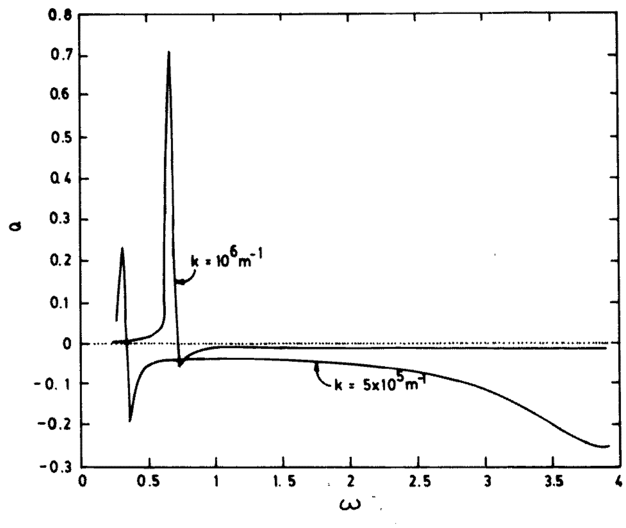

Figure 3. Plots of the non-linearity parameter Q versus the normalized propagation frequency ω for $n_0 = 10^{23}$ m^{-3} and $B_0 = 11.3$ T.

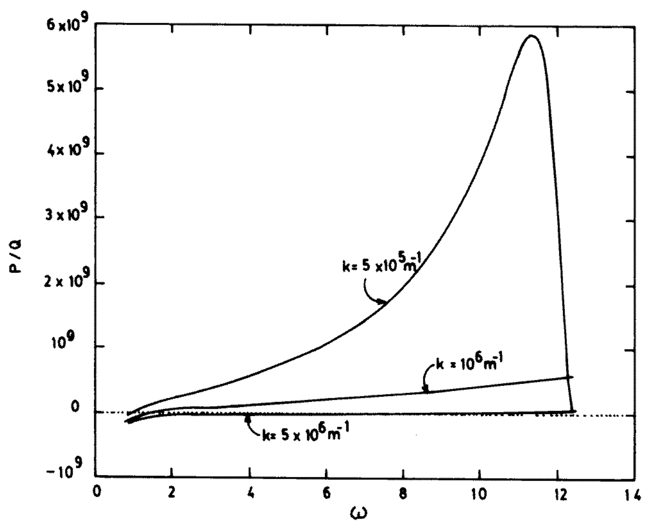

Figure 4. Plots of the modulational stability–instability parameter P/Q versus the normalized frequency ω for three different values of the wavenumber k, for $n_0 = 10^{22}$ m^{-3} and $B_0 = 11.3$ T.

Thus our numerical analysis shows that coupled helicon–acoustic waves exhibit a complicated behaviour and the type of semiconductor used would have its own regions of envelope soliton propagation and regions of soliton hole propagation. In previous work in this direction by Pawlik and Rowland (1975) and Abdullah *et al* (1988) the NLS equation was derived for acoustic waves in a piezoelectric semiconductor plasma. Pawlik and Rowland (1975) used a multiple-scale analysis in order to derive the NLS equation for the 'White' equations; however, no numerical analysis was carried out to discuss the

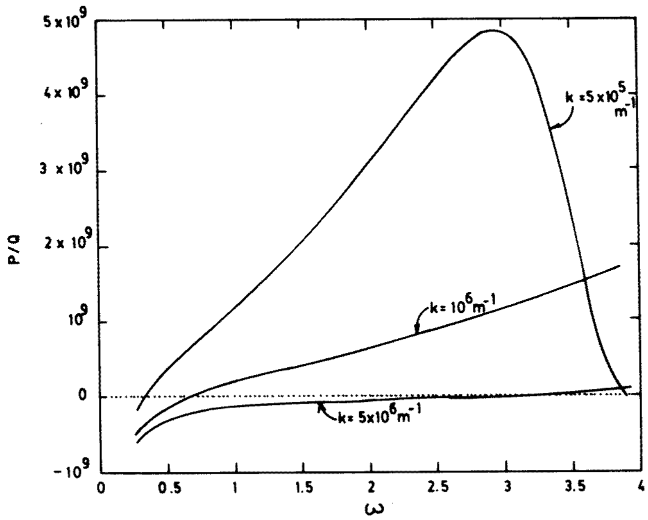

Figure 5. Plots of the modulational stability–instability parameter P/Q versus the normalized frequency ω for three different values of the wavenumber k, for $n_0 = 10^{23}$ m^{-3} and $B_0 = 11.3$ T.

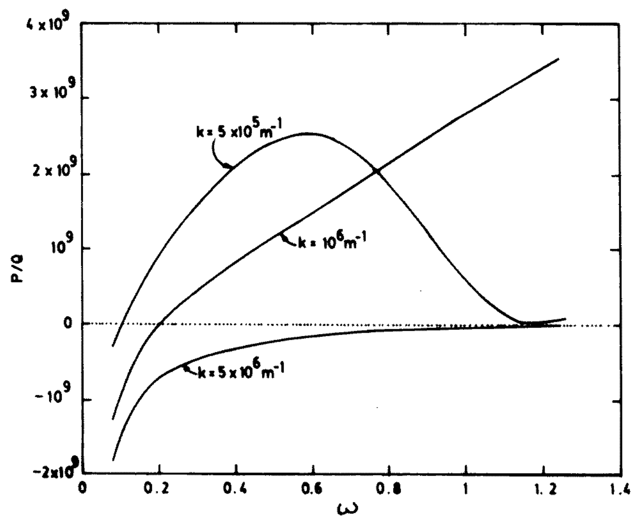

Figure 6. Plots of the modulational stability–instability parameter versus the normalized frequency ω for three different values of the wavenumber k, for $n_0 = 10^{24}$ m^{-3} and $B_0 = 11.3$ T.

modulational instability of the acoustic waves. Abdullah *et al* (1988) used the KBM method to derive the NLS equation for acoustic waves coupled to electron plasma waves, but again a modulational stability analysis was not carried out. Thus in the present work in comparison with the above-mentioned papers we have considered coupled helicon–acoustic waves and have derived the NLS equation; our numerical analysis shows that regions of modulational stability and instability exist which correspond to envelope soliton and soliton hole solutions, respectively. We have used physically relevant parameters which can be of use to experimental situations.

Finally we note that we have not included collisions in our initial set of equations. If collisions had been included, then a background DC electric field would also have been necessarily included. This would have led to the appearance of a linear interaction term in equation (27). This term can be removed by a change in variables and the NLS equation would again have assumed the form of equation (27). This would have complicated the algebra but not necessarily have given us qualitatively different results. Thus in the interests of keeping the problem mathematically more tractable we considered a collisionless piezoelectric semiconductor plasma.

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