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# Density-wave propagation within layered high-temperature superconducting plasmas

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**Abstract.** In the present work we theoretically investigate density-wave propagation in a superconducting medium, consisting of a finite number of layers. An electromagnetic wave interacts with superconducting electrons to set up charge-density gradients within the superconducting electron plasma. We use the London equations and a two fluid approach along with a Kronig–Penney model to describe the layered medium, in order to investigate the density wave behaviour by deriving a linear dispersion relation. It is shown that the charge density wave dissipates gradually. We numerically investigate the dependence of the complex Bloch-wave number on the propagation frequency using the standard boundary conditions of the Kronig–Penney model. Expressions of reflectivity and transmissivity are derived for a periodic layered structure consisting of a finite number of superconducting layers; these quantities are investigated numerically for a high temperature superconductor and their dependence on background parameters is discussed.

## 1. Introduction

In the absence of a complete and satisfactory theory of superconductivity, phenomenological theories provide useful insights into superconductivity as these theories are based on fundamental physical principles e.g. the London theory is based on Maxwell's equations. Since these theories are useful in predicting the behaviour of superconductors (rather than providing a complete description of its occurrence), they can be applied to devices which use superconductors. The London model has been used earlier by Kogan (1981) and Fabrizio (1991) to investigate the properties of charge density waves. Later Bunch and Grow (1997) used the London equations along with a two fluid model (Gorter and Casimir model) to investigate the propagation of charge density waves in superconductors. The two fluid model takes losses into account, since one fluid is related to the superconducting electrons and other to the semiconducting electrons. This approach is further justified since high $T_c$ ceramics exhibit metallic properties along the superconducting plane (*a–b* plane) and semiconductor characteristics along the *c* plane. Bunch and Grow (1997) have noted that this approach may have applications in the use of superconductors in travelling wave devices. The advantages of such a device are that no electron focusing structure is needed and no cathode is necessary. This makes possible the fabrication of millimetre and infrared devices since HTSs are compatible with such fabrication (as opposed to conventional electron devices which are difficult to size down at high frequencies) along with the fact these would be moderate power devices which are high quality with low noise. The microwave and infrared properties are of importance in superconductors because of the existence of the energy gap, which implies that photons of energy less than the energy gap are not absorbed. For superconductors the frequencies of interest are those which lie below the electron plasma frequency, that is the microwave

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and infrared frequency range. Bunch and Grow (1997) have investigated the interaction of a guided slow electromagnetic wave with a d.c. superconducting electron current, which in turn sets up charge density gradients within the superconducting electron plasma. The relevant frequency range of interest for the propagation density waves is the microwave and infrared frequency domain. We note here that charge density waves have been of interest in metals (Peierls instability—see Kittel 1996) and a comprehensive review of these is given by Wilson *et al* (1975) for the case of metallic layered structures.

With the advances of new technology and experimental techniques in solid state physics and material science, the fabrication of more and more artificial materials with special structures and properties is taking place. Such materials include metal–semiconductor or semiconductor–insulator periodic layers etc. The theoretical work of Baynham and Boardman (1968, 1969) is considered a watershed for the description of theoretical work in the area of semiconductor periodic multilayer structures. Since then a lot of theoretical work has appeared investigating both linear and nonlinear properties of wave propagation in periodic multilayer media (see e.g. Kushwaha and Halevi 1987, Shah *et al* 1993, Ali and Shah 1997). For the case of a periodic medium consisting of a finite number of layers, the transfer matrix method has been used extensively to investigate the propagation of waves in such media. The transfer matrix approach was developed by Abeles (1950) for work in the field of optics and later was further developed in the works by Del Castillo-Mussot *et al* (1988).

In the present work we investigate the propagation of density waves in a periodic superconducting medium (each layer is say a YBCO superconductor described by the London and two fluid models). We neglect interlayer transfer (within the YBCO layers) of carriers since carriers are confined mainly to the $CuO_2$ layers (Tachiki *et al* 1994 and references therein for experimental evidence) and this makes transport through the semiconducting layers weak. Thus in our case the periodic medium is described using the Kronig–Penney model and each layer is taken to have bulk superconducting properties. This allows us to take into account the boundary conditions of each bulk layer and its respective thickness. Bunch and Grow (1997) used thin film parameters in their work, but did not account for its finite dimensions. Our model overcomes this shortcoming. Additionally from the point of view of applications to devices, which use different conducting and superconducting layers, our approach is potentially more useful. We note here that Tachiki *et al* (1994) have used a similar approach to investigate electromagnetic phenomena related to a low frequency plasma in cuprate superconductors, but their case is limited to investigating the case of a single layer (thin film) only, taking its finite dimensions into account.

The layout of the paper is as follows. In the next section we give a mathematical formulation of the problem and derive the linear dispersion relation in an infinite periodic superconducting medium consisting of two alternating layers of superconductors. In section 3, we derive expressions for the reflectivity and transmissivity for a medium consisting of finite number of superconducting layers. The transfer matrix method is used to obtain the above mentioned expressions. In section 4, we present a numerical analysis of the results of the previous sections and in section 5 we give a conclusion of our investigations.

## 2. Linear dispersion relation

In the present section we give the mathematical formulation of the problem and derive the linear dispersion relation for density waves propagating in a layered superconducting medium. Figure 1 gives a schematic representation of the layered medium, which consists of two alternating layers having thicknesses $d_1$ and $d_2$ respectively. The layers have number densities $n_1$ and $n_2$, both of which have a superconducting component $n_s$ and a normal state component

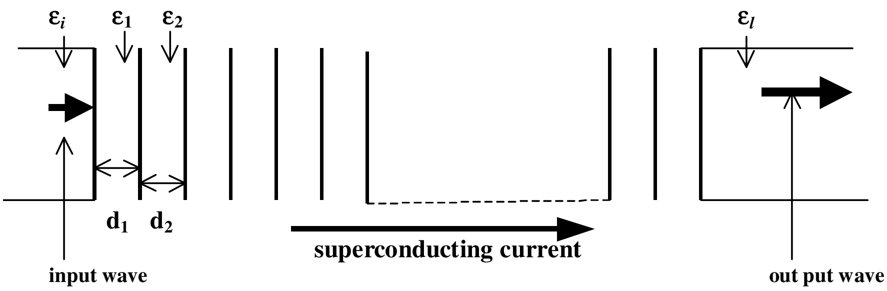



**Figure 1.** Schematic diagram of the periodic superconducting medium. $d_1$ and $d_2$ and $\varepsilon_1$ and $\varepsilon_2$ are the thicknesses and dielectric constants of the alternating layers respectively. $\varepsilon_i$ and $\varepsilon_l$ are the dielectric constants of the first and the last medium.

$n_n$. Each layer is described by a set of equations which consist of Maxwell's equations, the London equations (London and London 1935, Bunch and Grow 1997 etc) and an equation of motion for the superconducting electrons. These equations are as follows

$$\nabla \times E = -\mu_\alpha \frac{\partial H}{\partial t} \tag{1}$$

$$\nabla \times H = J_\alpha + \varepsilon_\alpha \frac{\partial E}{\partial t} \tag{2}$$

$$\nabla \cdot E = e\frac{n_\alpha}{\varepsilon_\alpha} \tag{3}$$

$$\nabla \cdot H = 0 \tag{4}$$

$$\nabla \times \left(\frac{m_\alpha}{e^2 n_{s\alpha}} J_{s\alpha}\right) = -\mu_\alpha H \tag{5}$$

$$J_\alpha = j_{s\alpha} + j_{n\alpha} \tag{6}$$

where $j_{s\alpha} = e n_{s\alpha} v_{s\alpha}$ and $j_{n\alpha} = b_\alpha \sigma_\alpha E$ are the superconducting and the normal electron current densities respectively.

$$m_\alpha \frac{\mathrm{d}v_{s\alpha}}{\mathrm{d}t} = eE \tag{7}$$

$$n_{s\alpha} = (1 - b_\alpha) n_\alpha. \tag{8}$$

Equations (1)–(4) are the Maxwell equations; here $\varepsilon_\alpha = \varepsilon_{r\alpha}\varepsilon_0$, where $\varepsilon_{r\alpha}$ is the relative dielectric constant and $\varepsilon_0$ is the permittivity of free space. Equation (5) is the relevant London equation. Equation (6) is the expression for the current density and equation (7) is the equation of motion of the superconducting electrons. Equation (8) gives the relation between the superconducting electrons $n_{s\alpha}$ and $n_\alpha$ (the total electron number density); the parameter $b_\alpha$ is a fraction which gives the number of normal state electrons and $v_{s\alpha}$ is the superconducting electron velocity. Equations (7) and (8) come from the Gorter and Casimir model—for a discussion see Portis (1990). The subscript $\alpha$ ($\alpha = 1, 2$) in the above set of equations denotes the layers of the multilayered periodic medium. A slow electromagnetic wave (Bunch and Grow 1997) propagates close to the surface of the layered superconducting medium, which interacts with the superconducting electrons in each layer; this in turn sets up the density wave in each layer of the medium.

We first of all give a brief derivation of the density wave in each layer of the superconducting medium. We note in advance that this result is essentially the same as the linear dispersion relation derived by Bunch and Grow (1997). We assume that all the fluctuating quantities are small in comparison with the background values or the d.c. values, so that all $a(z,t)$ are of the form

$$a(z,t) = a_0 + a_1 e^{-ikz+i\omega t} \tag{9}$$

where $a_0$ are the background or d.c. values and $a_1$ are the fluctuating or a.c. values. By eliminating all the variables in favour of one of the variables we arrive at the following differential equation, for density waves in each layer propagating in the $z$ direction only:

$$(1-b_\alpha)(1+i\delta_\alpha)\frac{\partial^2}{\partial z^2} j_{\alpha 1} + 2i\beta_\alpha\left(1+\frac{\gamma_\alpha}{2}-\frac{b_\alpha}{2}(1+\gamma_\alpha+i\delta_\alpha)\right)\frac{\partial}{\partial z} j_{\alpha 1}$$
$$-\beta_\alpha^2(1-\gamma_\alpha-i\delta_\alpha) j_{\alpha 1} = 0. \tag{10}$$

Here $\beta_\alpha = \omega/v_{s0\alpha}$ is the propagation constant of the superconducting electrons in layer $\alpha$ and $v_{s0\alpha}$ is the d.c. superconducting electron velocity set up by an external source, and

$$\delta_\alpha = \frac{\sigma_\alpha b_\alpha}{\omega\varepsilon_\alpha} \tag{11}$$

where $\sigma_\alpha$ is the conductivity of the normal state electrons of the $\alpha$th layer and

$$\gamma_\alpha = \frac{\omega_{sp\alpha}^2}{\omega^2} \tag{12}$$

is the ratio of the superconducting electron plasma frequency $\omega_{sp\alpha} = (e^2 n_{s0\alpha}/m\varepsilon_\alpha)^{1/2}$ and the frequency of the density wave in the $\alpha$th layer.

Using (9) we obtain the linear dispersion relation

$$k_\alpha^2(1-b_\alpha)(1+i\delta_\alpha) - 2k_\alpha\beta_\alpha\left(1+\frac{\gamma_\alpha}{2}-\frac{b_\alpha}{2}(1+\gamma_\alpha+i\delta_\alpha)\right) + \beta_\alpha^2(1-\gamma_\alpha-i\delta_\alpha) = 0. \tag{13}$$

We further note that the linear dispersion relation (13) is a complex quadratic equation, implying that within each layer there will be two modes of propagation which will either grow or be damped. A numerical analysis of equation (13) is given in section 4, where the results of analysis are presented graphically.

Since we wish to investigate the propagation of the density waves in a periodic superconducting medium, we follow the standard procedure developed by Baynham and Boardman (1968, 1969) (see also Bass *et al* 1989). The two periodically alternating layers have thicknesses $d_1$ and $d_2$ respectively and $d = d_1 + d_2$ is the period of the medium. The solution within each layer is given by

$$\begin{aligned} A_1(z) &= X_1 e^{ik_1 z} + X_2 e^{-ik_1 z} \\ A_2(z) &= Y_1 e^{ik_2 z} + Y_2 e^{-ik_2 z}. \end{aligned} \tag{14}$$

We introduce the boundary conditions which are used in the standard treatment of layered media, having a Kronig–Penney type of structure. We assume that the fields (and their derivatives) of the two layers are connected to one another at the boundary of the two layers in the following way:

$$\begin{aligned} & X_1 e^{ik_1 d_1} + X_2 e^{-ik_1 d_1} = Y_1 e^{ik_2 d_1} + Y_2 e^{-ik_2 d_1} \\ & k_1(X_1 e^{ik_1 d_1} - X_2 e^{-ik_1 d_1}) = k_2(Y_1 e^{ik_2 d_1} - Y_2 e^{-ik_2 d_1}) \\ & X_1 + X_2 = e^{i\bar{q}d}(Y_1 e^{ik_2 d} + Y_2 e^{-ik_2 d}) \\ & k_1(X_1 - X_2) = k_2 e^{i\bar{q}d}(Y_1 e^{ik_2 d} - Y_2 e^{-ik_2 d}) \end{aligned} \tag{15}$$

where $\bar{q}$ is the analogue of the Bloch wave vector. Solving the set of equations (15) we obtain the linear dispersion relation for density waves in a periodic superconducting medium.

$$\cos\bar{q}d = \cos k_1 d_1 \cos k_2 d_2 - \frac{1}{2}\left(\frac{k_1}{k_2}+\frac{k_2}{k_1}\right)\sin k_1 d_1 \sin k_2 d_2 \tag{16}$$

where $k_1$ and $k_2$ are given by equation (13).

As noted earlier that the linear dispersion relation within each layer (equation (13)) has complex coefficients, this implies that $k$ is complex and can be expressed as

$$k_\alpha = k_{r,\alpha} + \mathrm{i}k_{i,\alpha}. \tag{17}$$

This in turn implies that Bloch wave vector $\bar{q}$ is also complex and therefore is written as

$$\bar{q} = \bar{q}_r + \mathrm{i}\bar{q}_i. \tag{18}$$

Substituting (17) and (18) into (16) we obtain, after separating into real and imaginary parts, expressions for the real and imaginary parts of the Bloch wave number; these are given by the following two expressions respectively:

$$\bar{q}_r = \frac{1}{d}\sin^{-1}\left(\frac{g}{\pm\sqrt{-l \pm (l^2+4g^2)^{1/2}/2}}\right) \tag{19}$$

$$\bar{q}_i = \frac{1}{d}\sinh^{-1}\left(\pm\sqrt{\frac{-l \pm (l^2+4g^2)^{1/2}}{2}}\right) \tag{20}$$

where

$$l = 1 - g^2 - f^2 \tag{21}$$

$$\begin{aligned} f = {} & \cos k_{r1}d_1 \cosh k_{i1}d_1 \cos k_{r2}d_2 \cosh k_{i2}d_2 - \sin k_{r1}d_1 \sinh k_{i1}d_1 \sin k_{r2}d_2 \sinh k_{i2}d_2 \\ & -\frac{1}{2}\left[\frac{k_{r1}}{k_{r2}} + \frac{k_{r2}}{k_{r1}}\right][\sin k_{r1}d_1 \cosh k_{i1}d_1 \sin k_{r2}d_2 \cosh k_{i2}d_2 \\ & - \cos k_{r1}d_1 \sinh k_{i1}d_1 \cos k_{r2}d_2 \sinh k_{i2}d_2] - \frac{1}{2}\left[\left(\frac{k_{r1}k_{i2}}{k_{r2}^2} - \frac{k_{i1}}{k_{r2}}\right)\right. \\ & \left. + \left(\frac{k_{r2}k_{i1}}{k_{r1}^2} - \frac{k_{i2}}{k_{r1}}\right)\right][\sin k_{r1}d_1 \cosh k_{i1}d_1 \cos k_{r2}d_2 \sinh k_{i2}d_2 \\ & + \cos k_{r1}d_1 \sinh k_{i1}d_1 \sin k_{r2}d_2 \cosh k_{i2}d_2] \end{aligned} \tag{22}$$

and

$$\begin{aligned} g = {} & \cos k_{r1}d_1 \cosh k_{i1}d_1 \sin k_{r2}d_2 \sinh k_{i2}d_2 + \sin k_{r1}d_1 \sinh k_{i1}d_1 \cos k_{r2}d_2 \cosh k_{i2}d_2 \\ & +\frac{1}{2}\left[\frac{k_{r1}}{k_{r2}} + \frac{k_{r2}}{k_{r1}}\right][\sin k_{r1}d_1 \cosh k_{i1}d_1 \cos k_{r2}d_2 \sinh k_{i2}d_2 \\ & + \cos k_{r1}d_1 \sinh k_{i1}d_1 \sin k_{r2}d_2 \cosh k_{i2}d_2] - \frac{1}{2}\left[\left(\frac{k_{r1}k_{i2}}{k_{r2}^2} - \frac{k_{i1}}{k_{r2}}\right)\right. \\ & \left. + \left(\frac{k_{r2}k_{i1}}{k_{r1}^2} - \frac{k_{i2}}{k_{r1}}\right)\right][\sin k_{r1}d_1 \cosh k_{i1}d_1 \sin k_{r2}d_2 \cosh k_{i2}d_2 \\ & - \cos k_{r1}d_1 \sinh k_{i1}d_1 \cos k_{r2}d_2 \sinh k_{i2}d_2] \end{aligned} \tag{23}$$

We note here that in the above equations we have taken $k_{r,\alpha} \gg k_{i,\alpha}$, because we begin by considering equation (13), which is the linear dispersion relation of the density waves within each layer. Since this is a quadratic equation it will have two roots for $k_\alpha$ for the upper and lower signs respectively. For the parameter values for a typical HTS, $YBa_2Cu_3O_7$ (YBCO), as given by Bunch and Grow (1997) we take conductivity $\sigma_\alpha$ between $10^4$ and $10^7$ $\Omega^{-1}$ m$^{-1}$, superconducting electron velocity $v_{s0\alpha}$ of the order of $10^5$ m s$^{-1}$ and superconducting electron number density $n_{s0\alpha} \sim 10^{-27}$ m$^{-3}$, see also figure captions. We note that $\gamma \gg 1$ and

$\gamma > \delta \gg 1$ for microwave and infrared frequencies. Using this we see that for the root with lower sign we have approximately the following:

$$k_{\alpha r} = -\frac{\beta_\alpha}{1 - b_\alpha} \tag{24}$$

$$k_{\alpha i} = -\frac{\beta_\alpha}{(1 - b_\alpha)\delta_\alpha} \tag{25}$$

which shows that $k_r \gg k_i$. For the root with upper sign it can be shown that $k_r \sim k_i$, thus this root is heavily damped and therefore is not of further interest.

## 3. Reflectivity and transmissivity through a finite layer periodic superconductor

So far we have not imposed any restriction on the number of layers in the superconducting medium and the linear dispersion relation (equation (16)) is valid for a medium consisting of an infinite number of layers. In order to incorporate finite number of layers it is convenient to use the transfer matrix approach, the details of which are available in many texts (e.g. Abeles 1950, Born and Wolf 1989, Del Castillo-Mussot *et al* 1988 etc). The essence of the transfer matrix method is to connect the field at, say $z = 0$, to the point $z = Nd$ by multiplying $N$ matrices, where $N$ is the number of layers. Without going into details of the derivation which are present in the references mentioned above we write the transfer matrix for $N$ layers

$$m_N = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \tag{26}$$

where

$$\begin{aligned} M_{11} &= m_{11}\frac{\sin N\varphi}{\sin\varphi} - \frac{\sin(N-1)\varphi}{\sin\varphi} \\ M_{12} &= m_{12}\frac{\sin N\varphi}{\sin\varphi} \\ M_{21} &= m_{21}\frac{\sin N\varphi}{\sin\varphi} \\ M_{22} &= m_{22}\frac{\sin N\varphi}{\sin\varphi} - \frac{\sin(N-1)\varphi}{\sin\varphi} \end{aligned} \tag{27}$$

where $\varphi$ is the linear dispersion relation and is given by

$$\varphi = \cos\bar{q}d$$

and $m_{11}$, $m_{12}$, $m_{21}$ and $m_{22}$ are the matrix elements of a $2 \times 2$ matrix $m$ of transformation for a structure of period 1. We note that both the matrices $m$ and $m^N$ are unimodular and are related to one another via the relationship

$$m_N = (m)^N.$$

In obtaining expressions (26) and (27) we have followed the treatment of Bass *et al* (1989), where Sylvester's formula (Gantmacher 1967) is used which allows us to express a matrix in the form of an interpolational polynomial. For the sake of completeness the elements of the matrix $m$ are given below:

$$\begin{aligned} m_{11} &= \cos k_1d_1 \cos k_2d_2 - \frac{k_2}{k_1}\sin k_1d_1 \sin k_2d_2 \\ m_{12} &= -k_2^{-1}\cos k_1d_1 \sin k_2d_2 - k_1^{-1}\sin k_1d_1 \cos k_2d_2 \\ m_{21} &= k_1 \sin k_1d_1 \cos k_2d_2 + k_2 \cos k_1d_1 \sin k_2d_2 \\ m_{22} &= \cos k_1d_1 \cos k_2d_2 - \frac{k_1}{k_2}\sin k_1d_1 \sin k_2d_2. \end{aligned} \tag{28}$$

It should be noted that

$$\cos\bar{q}d = \tfrac{1}{2}(m_{11} + m_{22}).$$

We note here that the linear dispersion relation (equation (16)) can be derived via this method also and it remains independent of the number of layers.

We can now obtain expressions for the coefficients of reflection and transmission for density wave incident normally upon $N$ number of layers, where the layers are bounded on each side by media having dielectric constants given by $\varepsilon_i$ and $\varepsilon_l$ (correspondingly having wave vectors $k_i$ and $k_l$) respectively. In general the reflection and transmission coefficients are given by $r$ and $t$ respectively (Born and Wolf 1989)

$$r = \frac{R}{I_0} = \frac{(M_{11} + \mathrm{i}M_{12}k_l)\mathrm{i}k_0 - (M_{21} + \mathrm{i}M_{22}k_l)}{(M_{11} + \mathrm{i}M_{12}k_l)\mathrm{i}k_0 + (M_{21} + \mathrm{i}M_{22}k_l)} \tag{29}$$

$$t = \frac{T}{I_0} = \frac{2\mathrm{i}k_0}{(M_{11} + \mathrm{i}M_{12}k_l)\mathrm{i}k_0 + (M_{21} + \mathrm{i}M_{22}k_l)} \tag{30}$$

where $I_0$, $R$ and $T$ denote the amplitudes of the incident, reflected and transmitted waves and $k_0$ and $k_l$ are the wave numbers of the initial and final media having dielectric constants $\varepsilon_i$ and $\varepsilon_l$ respectively. Using expressions (27) and (28) the coefficients of reflection and transmission are given by

$$\begin{aligned} r = \Bigg[&\frac{\sin N\varphi}{\sin\varphi}\Bigg\{\mathrm{i}\cos k_1d_1\cos k_2d_2(k_0 - k_l) - \mathrm{i}\sin k_1d_1\sin k_2d_2\left(\frac{k_0k_2}{k_1} - \frac{k_lk_1}{k_2}\right) \\ &+ \cos k_1d_1\sin k_2d_2\left(\frac{k_lk_0}{k_2} - k_2\right) + \sin k_1d_1\cos k_2d_2\left(\frac{k_lk_0}{k_1} - k_1\right)\Bigg\} \\ &-\mathrm{i}\frac{\sin(N-1)\varphi}{\sin\varphi}(k_0 - k_l)\Bigg]\Bigg[\frac{\sin N\varphi}{\sin\varphi}\Bigg\{\mathrm{i}\cos k_1d_1\cos k_2d_2(k_0 + k_l) \\ &-\mathrm{i}\sin k_1d_1\sin k_2d_2\left(\frac{k_0k_2}{k_1} + \frac{k_lk_1}{k_2}\right) + \cos k_1d_1\sin k_2d_2\left(\frac{k_lk_0}{k_2} + k_2\right) \\ &+ \sin k_1d_1\cos k_2d_2\left(\frac{k_lk_0}{k_1} + k_1\right)\Bigg\} - \mathrm{i}\frac{\sin(N-1)\varphi}{\sin\varphi}(k_0 + k_l)\Bigg]^{-1} \end{aligned} \tag{31}$$

$$\begin{aligned} t = 2\mathrm{i}k_0\Bigg[&\frac{\sin N\varphi}{\sin\varphi}\Bigg\{\mathrm{i}\cos k_1d_1\cos k_2d_2(k_0 - k_l) - \mathrm{i}\sin k_1d_1\sin k_2d_2\left(\frac{k_0k_2}{k_1} - \frac{k_lk_1}{k_2}\right) \\ &+ \cos k_1d_1\sin k_2d_2\left(\frac{k_lk_0}{k_2} - k_2\right) + \sin k_1d_1\cos k_2d_2\left(\frac{k_lk_0}{k_1} + k_1\right)\Bigg\} \\ &-\mathrm{i}\frac{\sin(N-1)\varphi}{\sin\varphi}(k_0 - k_l)\Bigg]^{-1}. \end{aligned} \tag{32}$$

In terms of $r$ and $t$, the reflectivity and transmissivity are

$$R_{ref} = |r|^2 \tag{33}$$

$$T_{trn} = \frac{k_l}{k_0}|t|^2. \tag{34}$$

The complete expressions for $R_{ref}$ and $T_{trn}$ are easily derived from equations (31) and (32) and the results of their numerical investigations are given in the following section. For the case of a single layer the above expressions reduce to the standard ones (e.g. Born and Wolf 1989).

## 4. Numerical analysis

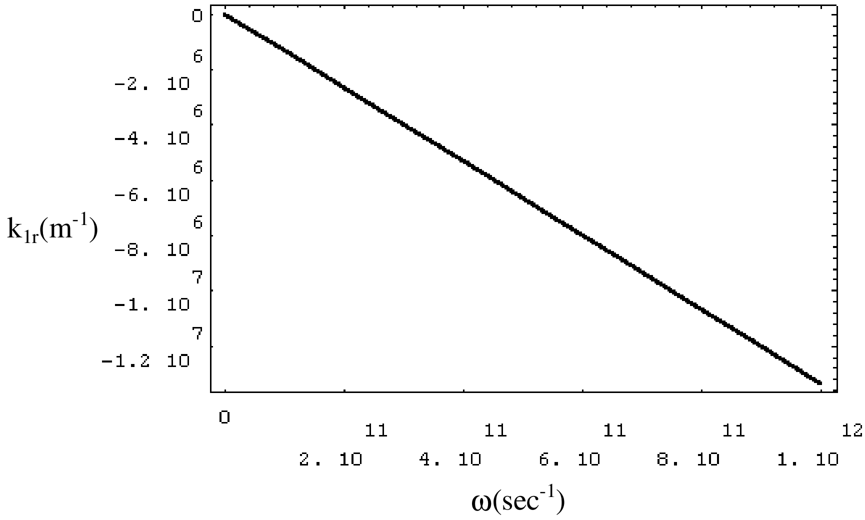
In this section we present a numerical analysis of the results obtained in the previous two sections. We have followed Bunch and Grow (1997) in taking the conductivity to be frequency independent; they have used the following relationship between conductivity and surface resistance $R_s$ (Laderman *et al* 1991)

$$\sigma = \frac{2R_s}{\omega^2\mu_0^2\lambda^3}.$$

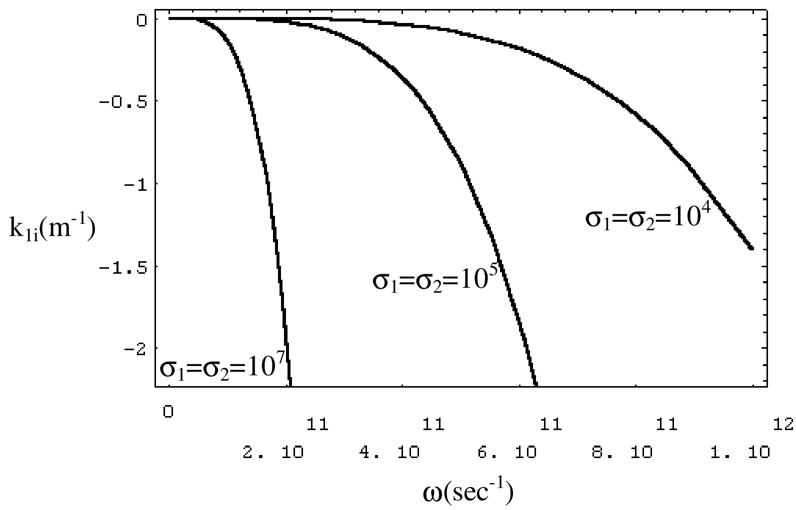
The fact that $R_s$ is directly proportional to $\omega^2$ is used and has been verified experimentally (see e.g. Laderman *et al* 1991, Lyons and Withers 1990), where $\lambda$ is the London penetration depth and we note here that for thin film YBCO, it has a value of $1.6 \times 10^{-7}$ m (Bunch and Grow 1997) which is much smaller than the layer thicknesses (see figure captions). An alternative approach could have been made to use a frequency dependent conductivity at high frequencies given by Tinkham (1975). However we feel that for the purposes of the present analysis it is sufficient to use a constant conductivity for the reasons given above.

As discussed in section 2, the root of equation (13) with lower sign is investigated numerically and its results are presented graphically in figures 2(a) and (b). The numerical values used are given in the figure captions. The real and imaginary parts of $k_1$ are presented in figures 2(a) and (b) respectively. We first of all note that this solution of $k_1$ corresponds to a backward propagating density wave, and has similar values and characteristics in each of the two layers, thus for the sake of brevity figures 2(a) and (b) show results for $\alpha = 1$ only. We further note that we have taken graphs for different values of the conductivity. The real part of the wave number $k_{1r}$ does not change with a change in conductivity; however the imaginary part $k_{1i}$ shows significant changes for different conductivities (see figure 2(b)). We see that as conductivity decreases, the damping of the density waves occurs as the wave travels towards higher frequency regions. We also note that if the component of the normal state electrons increases, the damping also increases (not presented in the graphs). The real part of the wave number $k_{\alpha r}$ is sensitive to the magnitude of the superconducting velocity $v_{s0\alpha}$. As $v_{s0\alpha}$ increases the wave number $k_{\alpha r}$ becomes smaller i.e. the waves become longer wavelength waves (see equation (24)). The effective mass values of electrons are assumed to be $m_1 = 15\,m_0$ and $m_2 = 10\,m_0$ respectively and the values of the relative dielectric constant are considered as $\varepsilon_{r1} = 3$ and $\varepsilon_{r2} = 4$ (Aarnink 1992). All other parameter values used in the numerical analysis are those given in the figure captions and those for a typical YBCO superconducting thin film (Bunch and Grow 1997).

We further investigate the behaviour of the Bloch wave number $\bar{q}$ which describes the propagation characteristics of the density waves in a periodic structure. Both the real ($\bar{q}_r$) and imaginary ($\bar{q}_i$) parts of the Bloch wave number are investigated using equations (19) and (20). We have chosen different values for the layer thicknesses, the component of the normal state electron densities etc (see figure captions). We note that for most cases a band gap exists for frequencies near $\omega = 10^9$ s$^{-1}$ and then there is a continuous propagation band, as the frequency increases to $10^{12}$ s$^{-1}$. We note here that for the Bloch wave number a positive imaginary $\bar{q}_i$ corresponds to damping (due to the manner in which the sign has been chosen in the exponential of equation (15)). Figures 3(a) and (b) show the real and imaginary parts of $\bar{q}$. As seen for the case of $k_\alpha$ the real part shows no variation for different conductivity values, but the imaginary part shows shifts to the right as conductivity values decrease (not shown in the figures); figure 3(b) shows the value of $\bar{q}_i$ for conductivities $\sigma_1 = \sigma_2 = 10^7$ for a layered medium. In figures 3(c) and (d) we have obtained graphs for a higher value of normal state electron number density in layer 2 (by increasing the value of $b$: see equation (8)). We



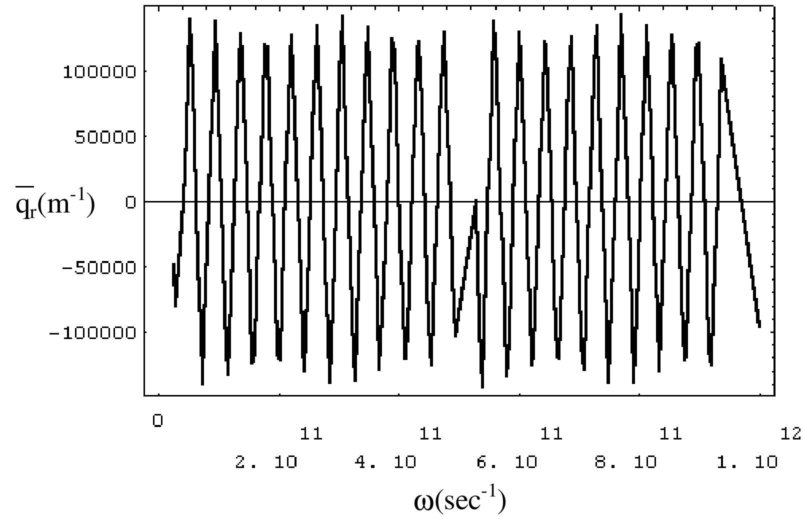

(a)



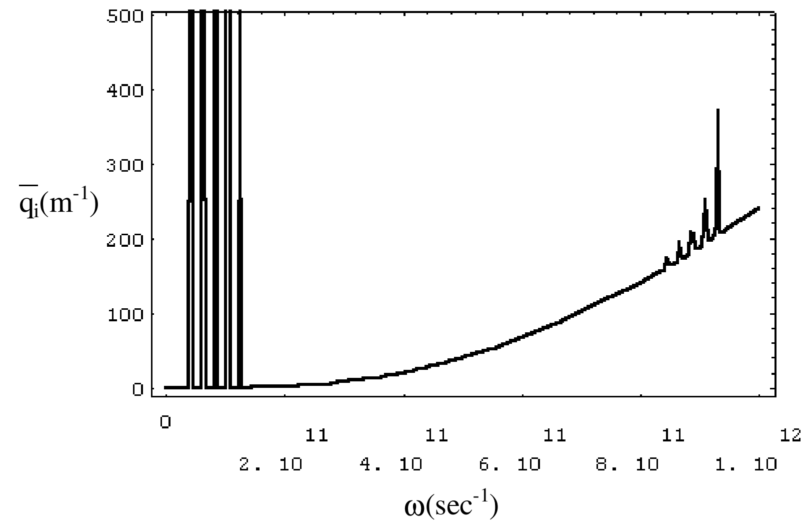

(b)

**Figure 2.** (a) $k_{1r}$ versus frequency $\omega$; (b) $k_{1i}$ versus frequency $\omega$, with varying conductivities $\sigma_1 = \sigma_2 = 10^7\ \Omega^{-1}\ \mathrm{m}^{-1}$; $\sigma_1 = \sigma_2 = 10^5\ \Omega^{-1}\ \mathrm{m}^{-1}$ and $\sigma_1 = \sigma_2 = 10^4\ \Omega^{-1}\ \mathrm{m}^{-1}$. Other parameters are $n_{s01} = 1.4 \times 10^{27}\ \mathrm{m}^{-3}$, $n_{s02} = 1.2 \times 10^{27}\ \mathrm{m}^{-3}$, $\varepsilon_{r1} = 3$, $\varepsilon_{r2} = 4$, $d_1 = 10^{-5}$ m, $d_2 = 10^{-6}$ m, $v_{s01} = v_{s02} = 10^5\ \mathrm{m\ s}^{-1}$, $m_1 = 15\,m_0$, $m_2 = 10\,m_0$ and $b_1 = 0.25$, $b_2 = 0.35$.

see that not only the behaviour of $\bar{q}_i$ changes but also its magnitude shows a marked increase, which implies that the density wave is heavily damped, although the corresponding $k_{\alpha i}$ remains relatively lightly damped (not shown graphically). We see from the graphical analysis that the propagation characteristics of the density wave in the layered superconducting medium are significantly more sensitive to the numerical values of the parameters, and if travelling wave devices are to be fabricated using thin film layers then this sensitivity must be taken into account.



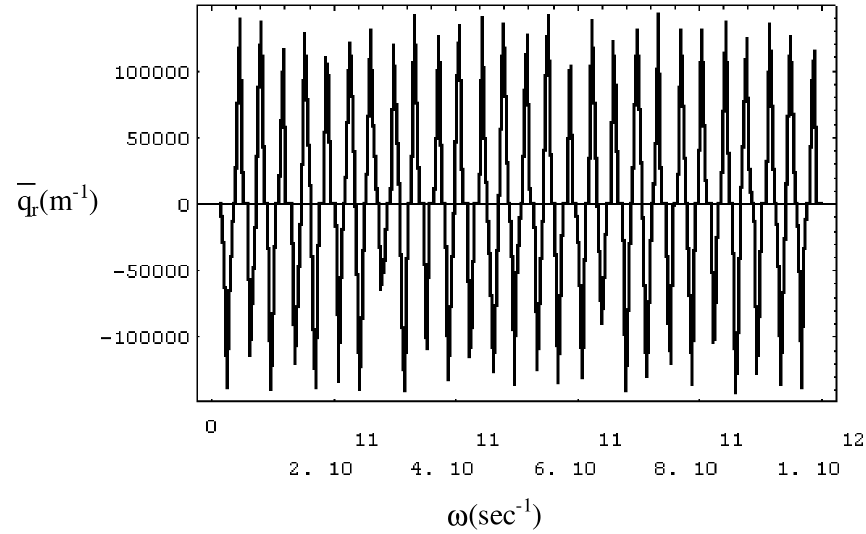

(a)



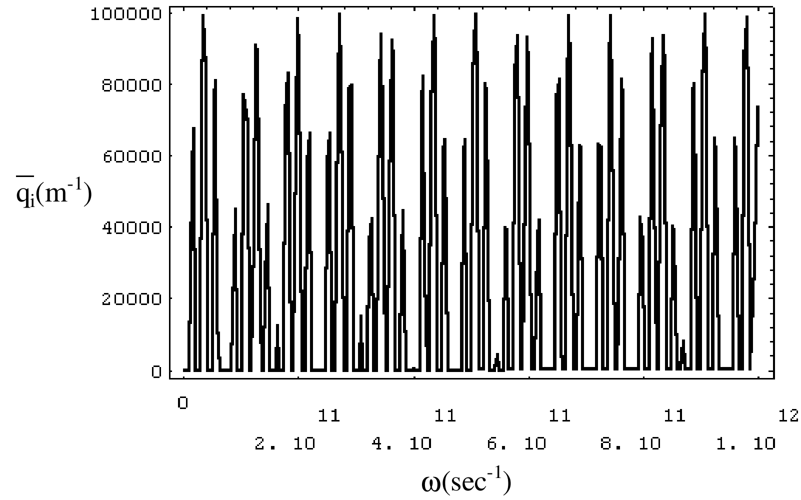

(b)

**Figure 3.** (a) Real Bloch wave vector $\bar{q}_r$ versus frequency $\omega$, with $b_1 = 0.25$, $b_2 = 0.35$. (b) Imaginary Bloch wave vector $\bar{q}_i$ versus frequency $\omega$, with $b_1 = 0.25$, $b_2 = 0.35$. (c) Real Bloch wave vector $\bar{q}_r$ versus frequency $\omega$, with $b_1 = 0.25$, $b_2 = 0.75$. (d) Imaginary Bloch wave vector $\bar{q}_i$ versus frequency $\omega$, with $b_1 = 0.25$; $b_2 = 0.75$. $\sigma_1 = 10^7\ \Omega^{-1}\ \mathrm{m}^{-1}$, $\sigma_2 = 10^6\ \Omega^{-1}\ \mathrm{m}^{-1}$, $n_{s01} = 1.4 \times 10^{27}\ \mathrm{m}^{-3}$, $n_{s02} = 1.2 \times 10^{27}\ \mathrm{m}^{-3}$, $\varepsilon_{r1} = 3$, $\varepsilon_{r2} = 4$, $d_1 = 10^{-5}$ m, $d_2 = 10^{-6}$ m, $v_{s01} = v_{s02} = 10^5\ \mathrm{m\ s}^{-1}$ and $m_1 = 15\, m_0$, $m_2 = 10\, m_0$.

Figures 4(a) and (b) show the dependence of the reflectivity and transmissivity on the propagation frequency of the density wave. We have assumed that the medium through which the density wave is incident on the layered structure has the same properties as the final medium i.e. $\varepsilon_i = \varepsilon_l = \varepsilon_0$. The equations themselves impose no such restriction and different types of $\varepsilon_i$ and $\varepsilon_l$ can be used in principle. Figures 4(a) and (b) show that reflectivity is




(c)




(d)

**Figure 3.** (Continued)

small at lower frequencies and at higher frequencies it goes up to almost its maximum value of 1. Transmissivity shows the opposite trend, in that that it is large at low frequencies and decreases at higher frequencies. This shows that the medium has positive absorptance $A$ (where $A = 1 - (R_{ref} + T_{trn})$ for low frequencies, which implies that wave gives up some energy to medium. Figures 4(c) and (d) show plots for reflectivity and transmissivity for relatively thick structure of alternating layers. We see that reflectivity and transmissivity of the wave strongly depend on the frequency and thicknesses of layers because when we take relatively thick layers, the reflectivity increases and transmissivity diminishes. For the purposes of the above numerical analysis we have taken the number of layers $N = 10$. We also see from figure 4 that both reflectivity and transmissivity are modified by the periodicity of the medium—i.e.

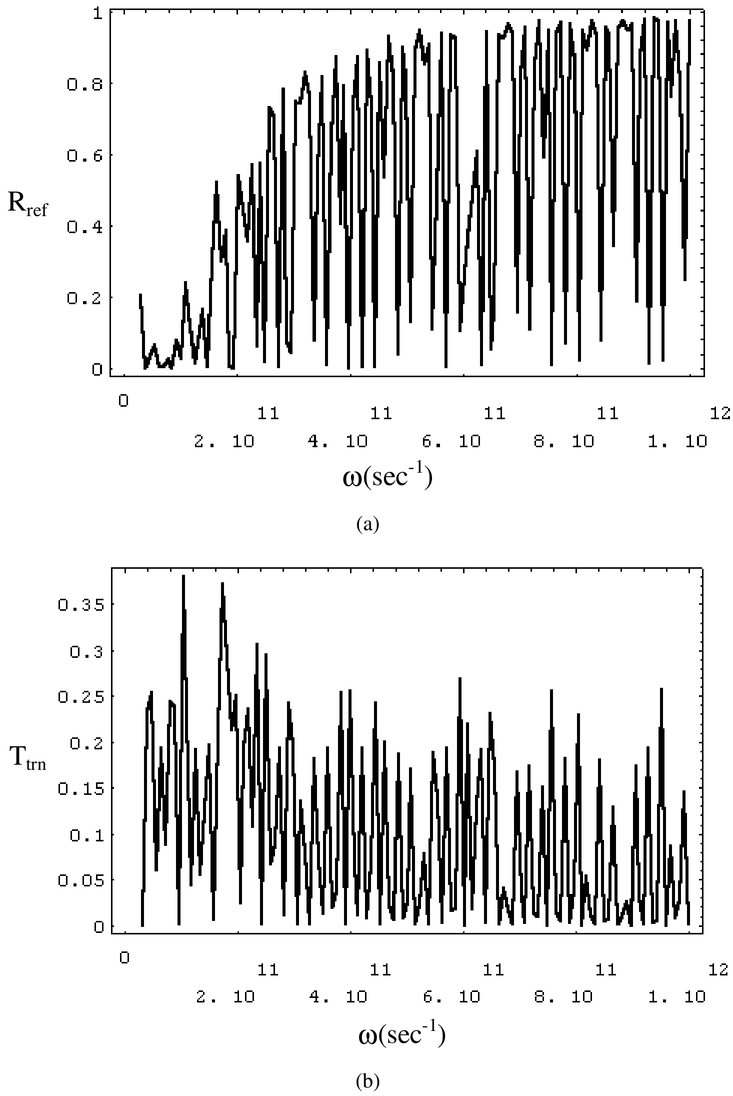
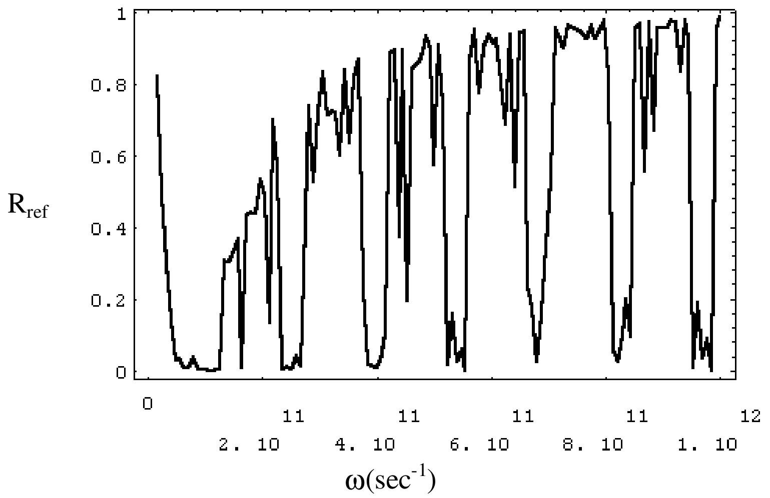



**Figure 4.** (a) Reflectivity $R_{ref}$ versus frequency $\omega$, with $d_1 = 10^{-5}$ m, $d_2 = 10^{-6}$ m. (b) Transmissivity $T_{trn}$ versus frequency $\omega$, with $d_1 = 10^{-5}$ m, $d_2 = 10^{-6}$ m. (c) Reflectivity $R_{ref}$ versus frequency $\omega$, with $d_1 = 10^{-3}$ m, $d_2 = 10^{-4}$ m. (d) Transmissivity $T_{trn}$ versus frequency $\omega$, with $d_1 = 10^{-3}$ m, $d_2 = 10^{-4}$ m. $b_1 = 0.25$, $b_2 = 0.35$, $\sigma_1 = 10^7\ \Omega^{-1}\ \text{m}^{-1}$, $\sigma_2 = 10^6\ \Omega^{-1}\ \text{m}^{-1}$, $n_{s01} = 1.4 \times 10^{27}\ \text{m}^{-3}$, $n_{s02} = 1.2 \times 10^{27}\ \text{m}^{-3}$, $\varepsilon_{r1} = 3$, $\varepsilon_{r2} = 4$, $v_{s01} = v_{s02} = 10^5\ \text{m s}^{-1}$, $m_1 = 15\,m_0$, $m_2 = 10\,m_0$ and $N = 10$.

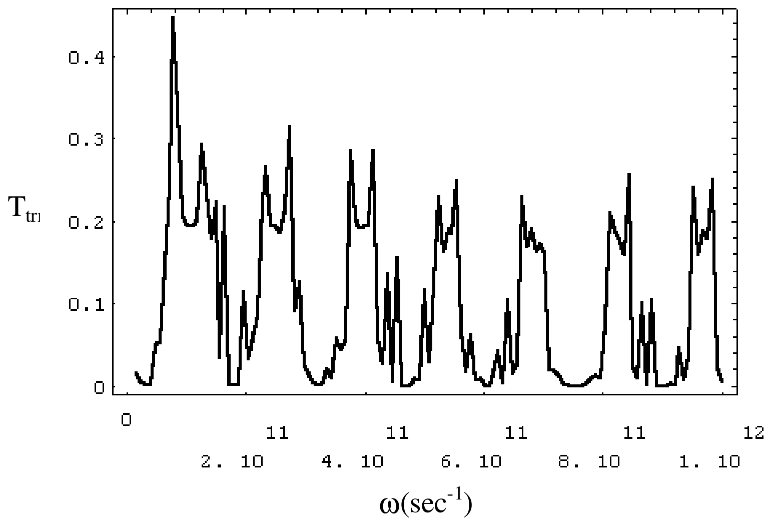
for the case of thicker layers the periodic variations of both reflectivity and transmissivity are larger.

## 5. Conclusion

In the present work we have considered a periodic layered structure consisting of two alternating superconducting layers. For this medium we have investigated the propagation characteristics




(c)




(d)

**Figure 4.** (Continued)

of density waves which arise as a consequence of the interaction of a slow electromagnetic wave (external) with the superconductor. We have derived linear dispersion relation for the density waves for the layers individually and the layered structure as a whole. The layered medium is described by using the Kronig–Penney model. We see that the linear dispersion relations are complex and the density waves are damped. A numerical investigation of these linear dispersion relations has been carried out and its results presented in the previous section. We see that the real and imaginary parts of $k_\alpha$ and $\bar{q}$ are sensitive to the values of the superconducting velocity $v_{s0\alpha}$ and the parameter $b_\alpha$ respectively. The Bloch wave number $\bar{q}$ is modified by the period of the superconducting medium, figures 3(a) and (b). We also note that $\bar{q}$ shows a propagation gap around $\omega = 10^9\ \mathrm{s}^{-1}$.

We have further considered the case of the layered medium when it consists of a finite number of superconducting layers. By making use of the transfer matrix approach we have derived expressions for the reflectivity and transmissivity of the density wave as it propagates through the periodic medium. In the section on numerical analysis we have presented graphical results of these quantities for different background parameter values, which are given in the figure captions. We see that both reflectivity and transmissivity are sensitive to the thickness of the layers. The results also show that the medium has a positive absorptance, implying that the density wave loses some of its energy to the periodic medium. We would like to note here that we have related our work to high temperature superconductors although the analysis here remains valid for ordinary superconductors as well. The reasons for this is that from the point of view of exploiting superconductors in travelling wave devices for the purposes of amplification, HTSs are more appropriate from the fabrication point of view.

In view of the recent idea of fabricating travelling wave devices consisting of superconductors (see references in section 1) we feel that the work presented here is of current relevance. From the point of view of device fabrication involving superconductors, such as amplifiers etc, the work presented here can have potential applications. Furthermore the analysis presented here can easily be extended to the case where the periodic medium consists of alternating superconducting and insulating layers.

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