

**Longitudinal oscillations and linear Landau damping in quark-gluon plasma**G. Murtaza,<sup>1,\*</sup> N. A. D. Khattak,<sup>1,†</sup> and H. A. Shah<sup>2</sup><sup>1</sup>*Salam Chair in Physics, G C University, Lahore 54000, Pakistan*<sup>2</sup>*Department of Physics, G C University, Lahore 54000, Pakistan*

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On the basis of the semiclassical kinetic Vlasov equation for quark-gluon plasma and the Yang-Mills equation in covariant gauge, linear Landau damping for electrostatic perturbations such as Langmuir waves is investigated for the extreme-relativistic and strongly relativistic cases. It has been observed that for the extreme-relativistic case, wherein the thermal speed of the particles exceeds the phase velocity of the perturbations, the linear Landau damping is absent as has been reported in the literature. However, a departure from extreme-relativistic case generates an imaginary component of the frequency giving rise to linear Landau damping effect. The relevant integral for the conductivity tensor has been evaluated and the dispersion relation for the longitudinal part of the oscillation was obtained. Further, it is also noted that both the real part of the oscillation frequency and the damping rate are sensitive to the choice of the wave number  $k$  and the Debye length  $\lambda_D$  associated with quark-gluon plasma.

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**I. INTRODUCTION**

For the last two decades, theoretical and experimental efforts are continuing in order to understand the characteristics of quark-gluon plasma (QGP) [1–4]. In QGP, the protons and neutrons lose their identity, and the nucleus turns into a soup of strongly interacting quarks and gluons with properties different from the normal nuclear matter. The matter in the early Universe was almost certainly QGP until the temperature fell below a few trillion degrees, a millionth of a second after the big bang. A similar state of matter is also supposed to exist in the core of the neutron stars. The QGP signals are being probed in the relativistic heavy-ion collision (RHIC) experiments, at Brookhaven National Laboratory and the Large Hadron Collider (LHC) at CERN [5]. In these experiments heavy ions such as lead (Pb) or gold (Au) are accelerated to very high energies and are made to collide, compressing the nuclear matter to extreme energy densities (greater than 1 GeV/fm<sup>3</sup>), creating conditions akin to that of the early Universe. These experiments provide unique opportunities to probe into a highly excited dense nuclear matter under controlled laboratory conditions. QGP may be regarded as the quantum chromodynamics (QCD) analog of the ordinary plasma phase of matter. However, unlike the ordinary plasma, the deconfined quanta of QGP are not directly observable because of the fundamental confining properties of the QCD vacuum.

It is known that, in spite of the nonperturbative nature of QCD at large distances and at high temperatures, quark-gluon matter becomes similar to an electron-ion plasma, due to the screening of the color field. The colored charged quarks (and antiquarks) interact with the colored field particle gluons. As the effective coupling constant  $g(T)$  decreases with increasing temperature, the hadronic matter for larger temperature is expected to be in the state of a weakly

interacting gas of quarks and gluons. We therefore expect to be able to describe this deconfined phase by a method similar to those for an ordinary plasma in electromagnetic fields [6,7]. The applicability of the plasma description is governed by the plasma parameter  $\Lambda_{QCD} = 12\pi n\lambda_d^3$ , which should be large compared to unity (here  $\lambda_d$  is the Debye length and  $n$  is the plasma density). For quark-gluon matter with very large density and zero chemical potential, the plasma description of the deconfined phase appears to be justified as long as one does not come close to the phase transition point [8].

The knowledge about the spectrum of the color fluctuations in the plasma is an important ingredient to our understanding of its color conduction properties and of the relevance of color degree of freedom to the hadron formation out of the plasma, with reference to the on-going experiments on heavy-ion collisions (LHC and RHIC) and the matter in the early Universe (microsecond after the big bang). The first systematic application of field theoretical techniques to calculate the nonabelian plasma properties, and in particular its excitation spectrum, was done in the beginning of the 1980s [7,9–12]. On the perturbative level, it was noticed by Klimov [13] and by Weldon [11,12] that the leading term in high temperature expansion of the one loop polarization tensor was gauge invariant. It was later on recognized by Heinz [8] that this result could be obtained from the classical color kinetic theory in the linear response function approximation. It was shown that, within the one loop approximations, the dispersion relation for gluonic excitations has two branches [10,11], in correspondence with the analogous longitudinal and transverse modes of the electromagnetic waves in ordinary plasmas. It has also been observed that almost all the results obtained from the hard thermal loop approximations (with certain limits) can also be described in terms of simple semiclassical physics [15,16]. They can indeed be obtained from a set of equations which generalize to non-abelian plasmas, by using the coupled Maxwell and Vlasov equations which are widely used in ordinary plasmas.

Heinz and Seimens [17] carried out an analysis of the colored collective modes in a QGP on the basis of “quark-gluon transport theory” near equilibrium. They found that

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two optical modes (longitudinal and transverse) exist starting for  $k=0$  at the plasma frequency, while there is no acoustic mode starting at  $\omega=0$ . An important conclusion made here was that linear Landau damping is absent in QGP due to the contribution of massless gluon in the collective modes, similar to the situation we have for the case of extreme-relativistic electron plasma [18]. Further, Markov and Markova [19] developed the theory of nonlinear damping on the basis of hard loop approximation. Linear Landau damping was argued to be absent on the basis of an earlier work on this subject [17]. However, it was shown that nonlinear effects play a role in damping the longitudinal waves. It was also shown that the Landau damping rate so obtained is gauge invariant [20].

In this paper we examine the linear Landau damping for electrostatic oscillations (such as Langmuir waves) for extreme-relativistic and strongly relativistic velocities on the basis of semiclassical kinetic Vlasov equation for the QGP and the Yang-Mills equation in covariant gauge. As it has been reported earlier that for an extreme-relativistic case in which the thermal speed of the particles is fixed at the speed of light, the linear Landau damping vanishes. Evidently here the thermal speed exceeds the phase velocity of the perturbations and thus the two speeds fail to resonate. However, in the strongly relativistic case in which the thermal speed departs from the speed of light, the imaginary part of the perturbation shows up and thus the Landau damping effect can be observed which is sensitive to the choice of wave number  $k$  and Debye length  $\lambda_D$ .

The plan of the paper is as follows. In Sec. II we develop the linearized Vlasov kinetic equation for the perturbed distributions for the quark and gluon plasma and derive a relation for the dielectric response function in terms of the polarization tensor using the Yang-Mills equation. The integrals of the polarization tensor are evaluated for the electrostatic perturbations for both the extreme-relativistic and strongly relativistic cases. In Sec. III the plasma dispersion relation is developed and discussed for two limiting cases, i.e.,  $k^2 \ll k_D^2$  and  $k_D^2 \ll k^2$ .

## II. LINEARIZED VLASOV THEORY OF QUARK-GLUON PLASMA

In the ultrarelativistic high temperature collisionless quark-gluon plasma, the plasma species quarks, antiquarks, and gluons are supposed to be in thermal equilibrium and behave like free gas particles obeying Fermi-Dirac and Bose-Einstein statistics, respectively. In order to consider the problem of linear Landau damping for such a phase, we need to solve the Boltzmann-Vlasov kinetic equations. These equations in relativistic notations can be expressed [2,3,14,17] as

$$p^\mu D_\mu f_{q,\bar{q}} \pm \frac{1}{2} g p^\mu \left\{ F_{\mu\nu}, \frac{\partial f_{q,\bar{q}}}{\partial p_\nu} \right\} = 0,$$

$$p^\mu \tilde{D}_\mu f_g + \frac{1}{2} g p^\mu \left\{ \tilde{F}_{\mu\nu}, \frac{\partial f_g}{\partial p_\nu} \right\} = 0. \quad (1)$$

The force term  $F_{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu) - ig[A_\mu, A_\nu]$  satisfies Maxwell's equation in relativistic notation,

$$D_\mu F^{\mu\nu}(x) - \zeta^{-1} \partial^\nu \partial^\mu A_\mu(x) = -J^\nu(x),$$

where  $\zeta$  is a gauge parameter,  $g$  is the coupling parameter,  $A$  is the field potential, and  $D_\mu$  and  $\tilde{D}_\mu$  are the covariant derivatives, respectively, which act as

$$D_\mu = \partial_\mu - ig[A_\mu(x), \cdot],$$

$$\tilde{D}_\mu = \partial_\mu - ig[\tilde{A}_\mu(x), \cdot],$$

where  $[\cdot]$  denotes the commutator. The generators of the group are denoted by  $t^a$  and  $T^a$  for the fundamental and adjoint representations, respectively. Thus for the fundamental representation the color matrix is expressed as  $A_\mu = A_\mu^a t^a$  and the field tensor as  $F_{\mu\nu} = F_{\mu\nu}^a t^a$ , and similarly for the adjoint representation we have  $\tilde{A}_\mu = A_\mu^a T^a$  and  $\tilde{F}_{\mu\nu} = F_{\mu\nu}^a T^a$  with

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc} A_\mu^b A_\nu^c.$$

Here  $J^\nu$  is the color current density given by

$$J^\nu = g t^a \int d^4 p p^\nu [\text{Tr} t^a (f_q - f_{\bar{q}}) + \text{Tr}(T^a f_g)]. \quad (2)$$

We also note that  $\text{Tr}(t^a t^b) = \delta^{ab}$ ,  $\text{Tr}(T^a T^b) = N_c \delta^{ab}$ , and  $[t^a, t^b] = i f^{abc} t^c$ , where the structure constants  $f^{abc} = i(T^a)^{bc}$ . Here  $\mu, \nu$  are the Minkowski indices which vary from 0 to 3; and  $a, b, c, \dots$  are the color indices which run from 1 to  $N-1$  of  $SU(N)$  gauge group with  $N_f$  flavors of quarks.

We now decompose the distribution functions  $f_s$  into regular and random (turbulent) parts,  $f_s = f_s^R + f_s^T$ , where the subscript  $s = q, \bar{q}, g$  specifies the species (i.e., quarks, antiquarks, and gluons, respectively), such that the average  $\langle \rangle$  of the statistical ensembles yields

$$\langle f_s \rangle = f_s^R, \quad \langle f_s^T \rangle = 0.$$

In a similar way the four-vector field potential  $A_\mu$  can also be decomposed into regular and turbulent parts:

$$A_\mu = A_\mu^R + A_\mu^T.$$

For simplicity, the regular part of the field  $A_\mu^R$  is assumed to be zero (field-free case) and also the average of the turbulent part  $\langle A_\mu^T \rangle = 0$ .

The field tensor  $F_{\mu\nu}$  can be decomposed into its linear and nonlinear parts. In the absence of external force (field-free case), the regular part can be taken as zero and thus only the perturbed part of the force will contribute. We therefore have

$$F_{\mu\nu} = (F_{\mu\nu}^T)_L + (F_{\mu\nu}^T)_{NL}.$$

Now substituting the values of the distribution functions  $f_s$  in the Vlasov equation for the quarks and antiquarks, we get

$$\begin{aligned}
p^\mu \partial_\mu (f_{q,\bar{q}}^R + f_{q,\bar{q}}^T) &= igp^\mu [A_\mu^T, f_{q,\bar{q}}^R] + igp^\mu [A_\mu^T, f_{q,\bar{q}}^T] \\
&\mp \frac{1}{2} gp^\mu \left\langle (F_{\mu\nu}^T)_L, \frac{\partial f_{q,\bar{q}}^R}{\partial p_\nu} \right\rangle \\
&\mp \frac{1}{2} gp^\mu \left\langle (F_{\mu\nu}^T)_L, \frac{\partial f_{q,\bar{q}}^T}{\partial p_\nu} \right\rangle \\
&\mp \frac{1}{2} gp^\mu \left\langle (F_{\mu\nu}^T)_{NL}, \frac{\partial f_{q,\bar{q}}^R}{\partial p_\nu} \right\rangle \\
&\mp \frac{1}{2} gp^\mu \left\langle (F_{\mu\nu}^T)_{NL}, \frac{\partial f_{q,\bar{q}}^T}{\partial p_\nu} \right\rangle,
\end{aligned}$$

and after averaging the above equation, we obtain

$$\begin{aligned}
p^\mu \partial_\mu f_{q,\bar{q}}^R &= igp^\mu \langle [A_\mu^T, f_{q,\bar{q}}^T] \rangle \mp \frac{1}{2} gp^\mu \left\langle \left\langle (F_{\mu\nu}^T)_L, \frac{\partial f_{q,\bar{q}}^T}{\partial p_\nu} \right\rangle \right\rangle \\
&\mp \frac{1}{2} gp^\mu \left\langle \left\langle (F_{\mu\nu}^T)_{NL}, \frac{\partial f_{q,\bar{q}}^T}{\partial p_\nu} \right\rangle \right\rangle \\
&\mp \frac{1}{2} gp^\mu \left\langle \left\langle (F_{\mu\nu}^T)_{NL}, \frac{\partial f_{q,\bar{q}}^R}{\partial p_\nu} \right\rangle \right\rangle.
\end{aligned}$$

In a similar way, for the gluon component of the plasma, we have the relationship

$$\begin{aligned}
p^\mu \partial_\mu f_g^R &= igp^\mu \langle [\tilde{A}_\mu^T, f_g^T] \rangle - \frac{1}{2} gp^\mu \left\langle \left\langle (\tilde{F}_{\mu\nu}^T)_L, \frac{\partial f_g^T}{\partial p_\nu} \right\rangle \right\rangle \\
&- \frac{1}{2} gp^\mu \left\langle \left\langle (\tilde{F}_{\mu\nu}^T)_{NL}, \frac{\partial f_g^T}{\partial p_\nu} \right\rangle \right\rangle \\
&- \frac{1}{2} gp^\mu \left\langle \left\langle (\tilde{F}_{\mu\nu}^T)_{NL}, \frac{\partial f_g^R}{\partial p_\nu} \right\rangle \right\rangle.
\end{aligned}$$

These two equations represents the Vlasov equations involving regular parts of the distribution function. We can obtain the corresponding equations for the turbulent parts by subtracting the regular parts from the original distribution (containing both of the regular and turbulent parts). Thus for quarks and antiquarks, we obtain

$$\begin{aligned}
p^\mu \partial_\mu f_{q,\bar{q}}^T &= igp^\mu ([A_\mu^T, f_{q,\bar{q}}^T] - \langle [A_\mu^T, f_{q,\bar{q}}^T] \rangle) \\
&\mp \frac{1}{2} gp^\mu \left\langle \left\langle (F_{\mu\nu}^T)_L, \frac{\partial f_{q,\bar{q}}^R}{\partial p_\nu} \right\rangle \right\rangle \mp \frac{1}{2} gp^\mu \left\langle \left\langle (F_{\mu\nu}^T)_L, \frac{\partial f_{q,\bar{q}}^T}{\partial p_\nu} \right\rangle \right\rangle \\
&- \left\langle \left\langle \left\langle (F_{\mu\nu}^T)_L, \frac{\partial f_{q,\bar{q}}^T}{\partial p_\nu} \right\rangle \right\rangle \right\rangle \mp \frac{1}{2} gp^\mu \left\langle \left\langle (F_{\mu\nu}^T)_{NL}, \frac{\partial f_{q,\bar{q}}^R}{\partial p_\nu} \right\rangle \right\rangle \\
&- \left\langle \left\langle \left\langle (F_{\mu\nu}^T)_{NL}, \frac{\partial f_{q,\bar{q}}^R}{\partial p_\nu} \right\rangle \right\rangle \right\rangle \mp \frac{1}{2} gp^\mu \left\langle \left\langle (F_{\mu\nu}^T)_{NL}, \frac{\partial f_{q,\bar{q}}^T}{\partial p_\nu} \right\rangle \right\rangle \\
&- \left\langle \left\langle \left\langle (F_{\mu\nu}^T)_{NL}, \frac{\partial f_{q,\bar{q}}^T}{\partial p_\nu} \right\rangle \right\rangle \right\rangle,
\end{aligned} \tag{3}$$

and for gluons with turbulent distribution function  $f_g^T$ , we get

$$\begin{aligned}
p^\mu \partial_\mu f_g^T &= igp^\mu ([\tilde{A}_\mu^T, f_g^T] - \langle [\tilde{A}_\mu^T, f_g^T] \rangle) \\
&- \frac{1}{2} gp^\mu \left\langle \left\langle (\tilde{F}_{\mu\nu}^T)_L, \frac{\partial f_g^R}{\partial p_\nu} \right\rangle \right\rangle - \frac{1}{2} gp^\mu \left\langle \left\langle (\tilde{F}_{\mu\nu}^T)_L, \frac{\partial f_g^T}{\partial p_\nu} \right\rangle \right\rangle \\
&- \left\langle \left\langle \left\langle (\tilde{F}_{\mu\nu}^T)_L, \frac{\partial f_g^T}{\partial p_\nu} \right\rangle \right\rangle \right\rangle - \frac{1}{2} gp^\mu \left\langle \left\langle (\tilde{F}_{\mu\nu}^T)_{NL}, \frac{\partial f_g^R}{\partial p_\nu} \right\rangle \right\rangle \\
&- \left\langle \left\langle \left\langle (\tilde{F}_{\mu\nu}^T)_{NL}, \frac{\partial f_g^R}{\partial p_\nu} \right\rangle \right\rangle \right\rangle - \frac{1}{2} gp^\mu \left\langle \left\langle (\tilde{F}_{\mu\nu}^T)_{NL}, \frac{\partial f_g^T}{\partial p_\nu} \right\rangle \right\rangle \\
&- \left\langle \left\langle \left\langle (\tilde{F}_{\mu\nu}^T)_{NL}, \frac{\partial f_g^T}{\partial p_\nu} \right\rangle \right\rangle \right\rangle.
\end{aligned} \tag{4}$$

These Vlasov equations containing the perturbed part of the distribution function will be used to evaluate the dielectric response function.

#### A. Landau damping and dielectric response function

Now we can express the perturbed distribution function as

$$f_s^T = \sum_{n=1}^{\infty} f_s^{T(n)}, \quad s = q, \bar{q}, g,$$

where  $n$  represents the order of the perturbation. Now linearizing the Vlasov equations by collecting the first-order terms, and substituting them in Eqs. (3) and (4), we obtain

$$\begin{aligned}
p^\mu \partial_\mu f_{q,\bar{q}}^{T(1)} &= \mp \frac{1}{2} gp^\mu \left\langle \left\langle (F_{\mu\nu}^T)_L, \frac{\partial f_{q,\bar{q}}^R}{\partial p_\nu} \right\rangle \right\rangle, \\
p^\mu \partial_\mu f_g^{T(1)} &= -\frac{1}{2} gp^\mu \left\langle \left\langle (\tilde{F}_{\mu\nu}^T)_L, \frac{\partial f_g^R}{\partial p_\nu} \right\rangle \right\rangle.
\end{aligned}$$

The unperturbed equilibrium distribution functions for quarks (antiquarks) and gluons are

$$\begin{aligned}
f_{q,\bar{q}}^R &\equiv f_{q,\bar{q}}^0 = \frac{4N_f \theta(p_0) \delta(p^2)}{(2\pi)^3} \frac{1}{e^{(p\bar{u} - \mu)/T} + 1}, \\
f_g^R &\equiv f_g^0 = \frac{4\theta(p_0) \delta(p^2)}{(2\pi)^3} \frac{1}{e^{(p\mu)/T} - 1},
\end{aligned} \tag{5}$$

which are Fermi-Dirac and Bose-Einstein distribution functions, respectively. The factor  $u$  represents the hydrodynamic four-velocity, which in the plasma rest frame is  $(1,0,0,0)$ . Here  $T$  is the average plasma temperature and  $\mu$  is the chemical potential.

We rewrite the first-order Vlasov equation for quarks (and antiquarks) as

$$\begin{aligned}
p^\mu \partial_\mu f_{q,\bar{q}}^{T(1)} &= \mp \frac{1}{2} gp^\mu \left\langle \left\langle (F_{\mu\nu}^T)_L, \frac{\partial f_{q,\bar{q}}^R}{\partial p_\nu} \right\rangle \right\rangle \\
&= \mp gp^\mu [\partial_\mu A_\nu^T(x) - \partial_\nu A_\mu^T(x)] \left( \frac{\partial f_{q,\bar{q}}^R}{\partial p_\nu} \right)
\end{aligned}$$

and perform its Fourier transformation to obtain the perturbed distribution function for quarks and antiquarks:

$$f_{q,\bar{q}}^{T(1)}(k,p) = \mp g \frac{\chi^{\nu\lambda}(k,p)}{pk + i\epsilon p_0} \frac{\partial f_{q,\bar{q}}^{(0)}}{\partial p^\lambda} A_\nu^T(k),$$

where  $\chi^{\nu\lambda}(k,p) = [(pk)g^{\nu\lambda} - p^\nu k^\lambda]$ . Similarly, for the case of gluons we get

$$f_g^{T(1)}(k,p) = -g \frac{\chi^{\nu\lambda}(k,p)}{pk + i\epsilon p_0} \frac{\partial f_g^{(0)}}{\partial p^\lambda} \bar{A}_\nu^T(k).$$

Now using the above perturbed distribution functions along with Maxwell's equation, we calculate the color current density  $J_\mu$  [which is also decomposed into its regular and turbulent parts,  $J_\mu = J_\mu^R + J_\mu^T$ , such that  $\langle J_\mu \rangle = J_\mu^R$  and  $\langle J_\mu^T \rangle = 0$ ]. By applying perturbations in the form of  $J_\mu^T(x) = \sum_{n=1}^{\infty} J_\mu^{T(n)}(x)$ , we obtain

$$J_\mu^{T(n)}(x) = g t^a \int d^4 p p_\mu \{ \text{Tr} [t^a (f_q^{T(n)} - f_{\bar{q}}^{T(n)})] + \text{Tr} (T^a f_g^{T(n)}) \}.$$

Taking first-order perturbations in the above equation, we obtain

$$\begin{aligned} J^{T(1)\mu} &= g t^a \int d^4 p p^\mu \frac{-g \chi^{\nu\lambda}(k,p)}{pk + i\epsilon p_0} \\ &\times \text{Tr} \left[ t^a \left( \frac{\partial f_q^{(0)}}{\partial p^\lambda} A_\nu^T(k) - \frac{\partial f_{\bar{q}}^{(0)}}{\partial p^\lambda} A_\nu^T(k) \right) \right. \\ &\left. + T^a \left( \frac{\partial f_g^{(0)}}{\partial p^\lambda} \bar{A}_\nu^T(k) \right) \right]. \end{aligned} \quad (6)$$

In the above expression for the perturbed current density  $J^{T(1)\mu}$ , first-order perturbations of the distribution functions  $f_s^{T(1)}$  have been used. In a more standard form, the perturbed current density  $J^{T(1)\mu}$  can also be expressed as

$$J^{T(1)\mu}(k) = \Pi^{\mu\nu}(k) A_\nu^T(k),$$

where

$$\Pi^{\mu\nu}(k) = g^2 \int d^4 p \frac{p^\mu}{pk + i\epsilon p_0} \left( p^\nu k \cdot \frac{\partial}{\partial p} - p \cdot k \frac{\partial}{\partial p^\nu} \right) N_{eq} \quad (7)$$

is the polarization tensor and  $N_{eq} = 1/2(f_q^{(0)} + f_{\bar{q}}^{(0)}) + N_c f_g^{(0)}$  is the quark-gluons equilibrium number density. The above equation for the current density is the tensor analog of the generalized Ohm's law. To study the linear Landau damping in a QGP, we need to calculate  $\Pi^{\mu\nu}$ .

Now we define the permittivity tensor  $\epsilon^{\mu\nu}$  as

$$\epsilon^{\mu\nu} = \delta^{\mu\nu} - \frac{1}{\omega^2} \Pi^{\mu\nu}. \quad (8)$$

The tensor structure of the response function  $\epsilon^{\mu\nu}$  can be separated into longitudinal and transverse components as

$$\epsilon_L = 1 - \frac{1}{k^2} \Pi_{00} \quad (9)$$

and

$$\epsilon_T = 1 - \frac{1}{2\omega^2} \Pi_{\mu\mu} + \frac{1}{2k^2} \Pi_{00}, \quad (10)$$

respectively. The treatment extended here to the problem of Landau damping in QGP is almost classical and the only place where the quantum mechanics enters are the equilibrium distribution functions for quark and gluon species of the plasma.

### B. Longitudinal component of permittivity tensor

The longitudinal permittivity tensor  $\epsilon_L$  depends on the conductivity polarization tensor  $\Pi_{00}$ . Since the quarks and antiquarks are fermions, and gluons are bosons, their equilibrium distribution functions  $f_{q\bar{q}}$  and  $f_g$  are given by

$$f_{q\bar{q}}^- = \frac{1}{z^{\mp 1} \exp(cp/T) + 1}, \quad f_g = \frac{1}{z^{-1} \exp(cp/T) - 1}, \quad (11)$$

where  $cp/T$  is the kinetic energy of the plasma particles for a very high temperature relativistic case, normalized over temperature (in eV). We note that in the high temperature limit, rest mass energy is ignored in the relativistic energy equation. The fugacity number  $z = \exp(\mu/T)$  depends upon the nature and the chemical potential  $\mu$  of the particles in that phase. The conductivity tensor  $\Pi_{00}$  is given by [2,3]

$$\Pi_{00} = \frac{2g^2}{(2\pi)^3} \int \frac{(k \cdot v) \frac{\partial f}{\partial p}}{k \cdot v - \omega - i\epsilon} d^3 p. \quad (12)$$

The wave number  $k$  and frequency  $\omega$  correspond to the propagation of the oscillations in the QGP and  $v$  is the thermal speed of the plasma particles.

In spherical polar coordinates, after performing integration over  $\phi$ , the above equation can be expressed as

$$\begin{aligned} \Pi_{00} &= -\frac{2g^2 c}{zT(2\pi)^2} \int_0^\infty \left( \frac{N_f \exp(cp/T)}{[z^{-1} \exp(cp/T) + 1]^2} \right. \\ &\left. + \frac{z^2 N_f \exp(cp/T)}{[z \exp(cp/T) + 1]^2} + \frac{N_c \exp(cp/T)}{[z^{-1} \exp(cp/T) - 1]^2} \right) \\ &\times p^2 \left( \int_0^\pi \frac{kv \cos(\theta)}{kv \cos(\theta) - \omega - i\epsilon} \sin(\theta) d\theta \right) dp. \end{aligned}$$

Now using the well known Plemelj formula [18], the integration over  $\theta$  can be performed as

$$\begin{aligned} & \int_0^\pi \frac{kv \cos(\theta)}{kv \cos(\theta) - \omega - i\epsilon} \sin(\theta) d\theta \\ &= 2 + \frac{\omega}{kv} \left[ \ln \left( 1 - \frac{\omega}{kv} \right) - \ln \left( -1 - \frac{\omega}{kv} \right) \right] \\ & \quad + i\pi \frac{\omega}{kv} \Theta \left( 1 - \frac{\omega}{kv} \right), \end{aligned}$$

where  $\Theta(1 - (\omega/kv))$  is the Heaviside unit-step function. After straightforward algebraic manipulation, we obtain

$$\begin{aligned} & \int_0^\pi \frac{kv \cos(\theta)}{kv \cos(\theta) - \omega - i\epsilon} \sin(\theta) d\theta \\ &= 2 - \sum_{s=0}^{\infty} [1 + (-1)^s] \frac{\left(\frac{kv}{\omega}\right)^{s+1}}{s+1} + i\pi \frac{\omega}{kv} \Theta \left( 1 - \frac{\omega}{kv} \right). \end{aligned}$$

Thus the polarization tensor for longitudinal perturbation  $\Pi_{00}$  becomes

$$\begin{aligned} \Pi_{00} = & -\frac{4g^2c}{T(2\pi)^2} \int_0^\infty \left( \frac{N_f \exp(cp/T)}{[\exp(cp/T) + 1]^2} + \frac{N_f \exp(cp/T)}{[\exp(cp/T) + 1]^2} \right. \\ & + \left. \frac{N_c \exp(cp/T)}{[\exp(cp/T) - 1]^2} \right) p^2 \left[ -\frac{1}{3} \left( \frac{kv}{\omega} \right)^2 - \frac{1}{5} \left( \frac{kv}{\omega} \right)^4 \right. \\ & \left. - \frac{1}{7} \left( \frac{kv}{\omega} \right)^6 + \dots + i\pi \frac{\omega}{kv} \Theta \left( 1 - \frac{\omega}{kv} \right) \right] dp. \quad (13) \end{aligned}$$

### 1. Extreme or ultrarelativistic case (i.e., $v=c$ )

Here we discuss the extreme relativistic case when the particles thermal speed equals the velocity of light, i.e.,  $v=c$ . Thus for the fugacity number  $z=1$ , Eq. (13) becomes

$$\begin{aligned} \Pi_{00} = & -\frac{2g^2T^2}{c^2(2\pi)^2} \int_0^\infty \left( \frac{N_f \exp(x)}{[\exp(x) + 1]^2} + \frac{N_f \exp(x)}{[\exp(x) + 1]^2} \right. \\ & + \left. \frac{N_c \exp(x)}{[\exp(x) - 1]^2} \right) x^2 \left[ \left[ -\frac{1}{3} \left( \frac{kc}{\omega} \right)^2 - \frac{1}{5} \left( \frac{kc}{\omega} \right)^4 - \frac{1}{7} \left( \frac{kc}{\omega} \right)^6 \right. \right. \\ & \left. \left. + \dots \right] + i\pi \frac{\omega}{kc} \Theta \left( 1 - \frac{\omega}{kc} \right) \right] dx, \end{aligned}$$

where  $x=cp/T$ . Now we note that

$$\begin{aligned} & \int_0^\infty \left( \frac{N_f \exp(x)}{[\exp(x) + 1]^2} + \frac{N_f \exp(x)}{[\exp(x) + 1]^2} + \frac{N_c \exp(x)}{[\exp(x) - 1]^2} \right) x^2 dx \\ &= \frac{\pi^2}{6} (N_f + 2N_c) \end{aligned}$$

and using the definition for the analog of the plasma frequency [2,3]  $3\omega_p^2 = (g^2T^2/c^2)[\frac{1}{6}(N_f + 2N_c)]$ , the above expression for  $\Pi_{00}$  becomes

$$\begin{aligned} \Pi_{00} = & 3\omega_p^2 \left\{ \left[ \frac{1}{3} \left( \frac{kc}{\omega} \right)^2 + \frac{1}{5} \left( \frac{kc}{\omega} \right)^4 + \frac{1}{7} \left( \frac{kc}{\omega} \right)^6 + \dots \right] \right. \\ & \left. - i\pi \frac{\omega}{kc} \Theta \left( 1 - \frac{\omega}{kc} \right) \right\}. \quad (14) \end{aligned}$$

Calculating the longitudinal response function  $\epsilon_L$  and equating it to zero, we obtain the linear dispersion relation

$$\begin{aligned} 1 - \frac{3\omega_p^2}{c^2k^2} \left\{ \left[ \frac{1}{3} \left( \frac{kc}{\omega} \right)^2 + \frac{1}{5} \left( \frac{kc}{\omega} \right)^4 + \frac{1}{7} \left( \frac{kc}{\omega} \right)^6 + \dots \right] \right. \\ \left. - i\pi \frac{\omega}{kc} \Theta \left( 1 - \frac{\omega}{kc} \right) \right\} = 0. \end{aligned}$$

The imaginary component depends on the Heaviside unit-step function  $\Theta(1 - (\omega/kc))$  which leads to the conclusion that this term survives only if its argument is positive or that the phase velocity  $\omega/k < c$  (being the particle thermal speed). Therefore, Landau damping vanishes for the extreme-relativistic case. On the other hand the real part yields

$$\frac{3\omega_p^2}{c^2k^2} \left[ \frac{1}{3} \left( \frac{kc}{\omega} \right)^2 + \frac{1}{5} \left( \frac{kc}{\omega} \right)^4 + \frac{1}{7} \left( \frac{kc}{\omega} \right)^6 + \dots \right] = 1.$$

Neglecting higher-order terms in  $\omega$ , we obtain

$$\omega^4 - \omega_p^2 \omega^2 - \frac{3\omega_p^2}{5} c^2 k^2 = 0.$$

For the case when  $c^2k^2/\omega_p^2 \ll 1$ , the above expression reduces to

$$\omega^2 = \omega_p^2 + \frac{6}{5} c^2 k^2. \quad (15)$$

This expression shows that for QGP near equilibrium, the longitudinal color collective mode of oscillation represents a timelike behavior. Evidently, the thermal speed chosen (i.e.,  $v=c$ ) exceeds the phase velocity of the perturbation and thus the two speeds fail to resonate. The resulting collective mode therefore does not exhibit damping unlike the electron plasma waves where for sufficiently large momenta  $k$ , the perturbations undergo damping of the order of few times  $\omega_p$  [21]. This is the consequence of having treated quarks, and gluons as massless. For massive quarks, however, the plasma does show some weak Landau damping [8].

### 2. Strongly relativistic case (i.e., $v \lesssim c$ )

The thermodynamical properties of QGP in heavy-ion collisions will involve many nonequilibrium dynamical effects since the short-lived (5–10 fm/c) plasma will be subjected to various phenomena, e.g., hadronization. If the

damping rate is fast compared with the lifetime of the QGP, it is hard to probe the plasma state and interpret the signatures of the experiments. The excitation of color fluctuations will also give rise to energy loss, hadronization of the fast quarks and quark jets propagating through the plasma [8]. The space-time development of matter produced in ultrarelativistic heavy-ion collisions can be described by the hydrodynamical model, neglecting most of the details and by assuming the initial conditions to be in local thermal equilibrium, which is presumably maintained during the evolution. Therefore, a detailed transport theory is in order for describing the rapid time-dependent complex phenomena which could address the problems of finite size plasma effect, inhomogeneities,  $N$ -body phase space, particle resonance production, freezing out, and other collective effects prevalent in QGP of RHIC or LHC.

The simple semiclassical treatment that we have considered here (based on equilibrium distribution for the fermions and bosons of the plasma species) may not provide adequate information regarding QGP signals in the heavy ion collision experiments.

In QCD, there exists only one kind of gauge field which is massless under the strict  $SU(3)_c$  symmetry. However, using the gauge field with massive gauge field bosons [22], another version of QCD has been proposed to describe the strong interactions [23]. In this model, two sets of gluon fields are introduced, one set is massive and the other massless. Correspondingly three sets of glueballs arise having the same spin parity but different masses. Although all gluons are in a color octet, some gluons are colorless. If the colorless gluons are not restricted by color confinement, they may exist in a state of free particles. Such massive colorless gluons will exhibit properties similar to the glueballs especially in their decay modes. They are massive vector particles and can couple with quarks and massless gluons but not with leptons. Thus they may possibly appear in the strong decays or in the  $p\bar{p}$  collisions but not in  $e^+e^-$  collisions. Search for these massive gluons and glueballs becomes important both for the theory and the experiment [24,25]. The existence of the said massive gluons as free particle may shed light on the understanding of the nature of color confinement [26].

Similarly, in analogy with Glashow-Winberg-Salam model [27] based on  $SU(2)_L \times U(1)_Y$  gauge theory, a new gauge theory, quantum nuclear dynamics (QND) based on  $SU(2)_N \times U(1)_z$  has been proposed. In QND, two kinds of strong interactions are assumed to exist, one producing Coulomb-like potential for effectively massless gluons and the other generates Yukawa potential for strongly interacting massive gluons [28]. Here the massive gluons mediate the strong interaction just as the massive vector bosons mediate the weak interactions. Other issues of masses of quarks, chiral symmetry restoration, and phase structure have been discussed in detail by Corleto Detar [3].

In the present case, we treat the quark-gluon soup (of the early Universe) as a semiclassical system of particles (quark, antiquarks, and gluons including their massive components) analogous to the ordinary plasma of charged species. We also assume the plasma in global equilibrium with negligible baryon density and without involving phase transitions. As

mentioned before, in such an extreme temperature environment ( $T > 200$  MeV), although the rest mass energy is ignorable, the plasma particles of the early Universe (in QGP state) must possess some finite mass and hence thermal speed.

From Eq. (13) using  $v = p/m$ , where  $m$  is the average mass of the QGP particles, and changing the variables as before, by letting  $x = cp/T$ , we obtain

$$\begin{aligned} \Pi_{00} = & -\frac{4g^2T^2}{zc^2(2\pi)^2} \int_0^\infty \left( \frac{N_f \exp(x)}{[z^{-1}\exp(x)+1]^2} + \frac{z^2 N_f \exp(x)}{[z \exp(x)+1]^2} \right. \\ & + \left. \frac{N_c \exp(x)}{[z^{-1}\exp(x)-1]^2} \right) x^2 \left\{ \left[ -\frac{1}{3} \left( \frac{kTx}{m\omega c} \right)^2 - \frac{1}{5} \left( \frac{kTx}{m\omega c} \right)^4 \right. \right. \\ & \left. \left. - \frac{1}{7} \left( \frac{kTx}{m\omega c} \right)^6 + \dots \right] + i\pi \frac{\omega mc}{kTx} \Theta \left( 1 - \frac{\omega mc}{kTx} \right) \right\} dx. \end{aligned} \quad (16)$$

The imaginary part of  $\Pi_{00}$  is

$$\begin{aligned} I_1 = & i\pi \frac{\omega mc}{kT} \int_0^\infty \Theta \left( 1 - \frac{\omega mc}{kTx} \right) \left( \frac{N_f \exp(x)}{[z^{-1}\exp(x)+1]^2} \right. \\ & \left. + \frac{z^2 N_f \exp(x)}{[z \exp(x)+1]^2} + \frac{N_c \exp(x)}{[z^{-1}\exp(x)-1]^2} \right) x dx. \end{aligned}$$

Keeping in mind the property of the Heaviside unit-step function  $\Theta(1 - \omega mc/kTx)$ , we choose the lower limit of the integration to be  $\alpha = \omega mc/kT$ . Therefore the above integral yields the following expression:

$$\begin{aligned} I_1 = & i\pi z \frac{\omega mc}{kT} \left\{ N_f \alpha \left( -2 + \frac{z}{\exp(\alpha) + z} + \frac{1}{1 + z \exp(\alpha)} \right) \right. \\ & + N_f \left[ \ln \left( 1 + \frac{1}{z} \right) - \ln(1 + z) \right] + N_f \{ \ln[\exp(\alpha) + z] \\ & + \ln[z \exp(\alpha) + 1] \} + N_c \left( \frac{\alpha \exp(\alpha)}{\exp(\alpha) - z} \right. \\ & \left. - \ln[\exp(\alpha) - z] \right) \left. \right\}. \end{aligned}$$

The real part of  $\Pi_{00}$  can be integrated in terms of polylogarithmic series,  $\text{poly log}[n, z] = \sum_{l=1}^\infty z^l/l^n$ , as

$$\begin{aligned} I_2 = & -2z \left( N_f \sum_{l=1}^\infty \frac{(-z)^l}{l^2} + N_f \sum_{l=1}^\infty \frac{\left( -\frac{1}{z} \right)^l}{l^2} - N_c \sum_{l=1}^\infty \frac{z^l}{l^2} \right) \\ & \times \left( 2 + \sum_{s=0}^{\text{infy}} [1 + (-1)^s] \frac{\left( \frac{kT}{m\omega c} \right)^s}{s+1} \frac{(s+2)!}{l^s} \right). \end{aligned}$$

Now after substituting the values of  $I_1$  and  $I_2$ , the conductivity tensor  $\Pi_{00}$  becomes

$$\begin{aligned} \Pi_{00} = & -\frac{4g^2T^2}{c^2(2\pi)^2} \left[ \frac{1}{2} i\pi \frac{\omega m}{k} \left( N_f \frac{\omega mc}{kT} \left( -2 + \frac{z}{\exp\left(\frac{\omega mc}{kT}\right) + z} + \frac{1}{1+z \exp\left(\frac{\omega mc}{kT}\right)} \right) + N_f \left[ \ln\left(1 + \frac{1}{z}\right) - \ln(1+z) \right] \right. \right. \\ & \left. \left. + N_f \left\{ \ln \left[ \exp\left(\frac{\omega mc}{kT}\right) + z \right] + \ln \left[ z \exp\left(\frac{\omega mc}{kT}\right) + 1 \right] \right\} + N_c \left\{ \frac{\frac{\omega mc}{kT} \exp\left(\frac{\omega mc}{kT}\right)}{\exp\left(\frac{\omega mc}{kT}\right) - z} - \ln \left[ \exp\left(\frac{\omega mc}{kT}\right) - z \right] \right\} \right) \right. \\ & \left. - 2 \left( N_f \sum_{l=1}^{\infty} \frac{(-z)^l}{l^2} + N_f \sum_{l=1}^{\infty} \frac{\left(-\frac{1}{z}\right)^l}{l^2} - N_c \sum_{l=1}^{\infty} \frac{z^l}{l^2} \right) \left[ 1 - \frac{1}{4} \sum_{s=0}^{\infty} \frac{[1+(-1)^s]}{s+1} \left(\frac{kT}{\omega mc}\right)^s \frac{(s+2)!}{l^s} \right] \right]. \end{aligned}$$

The mutual dependence of phase transition, chemical potential  $\mu$ , and temperature  $T$  [critical temperature  $T_c$  with reference to the bag constant  $B(m, T)$ ] has been discussed in detail elsewhere [29,30]. Here we assume thermal and chemical equilibrium for relatively long-lived noninteracting quark-gluon plasma of the early Universe with global color neutrality and negligible average baryon density.

The gluons being field particles, such as photons and phonons, have zero chemical potential. We may assume the same for the quark and the antiquark with equal number density, like for an equal density electron-positron plasma [3]. Thus for this special case of overall color charge neutrality, when the fugacity number  $z = \exp(\mu/T)$  becomes unity, the expression for  $\Pi_{00}$  reduces to

$$\begin{aligned} \Pi_{00} = & -\frac{4g^2T}{c(2\pi)^2} \frac{1}{2} i\pi \frac{\omega mc}{kT} N_f \frac{\omega mc}{kT} \\ & \times \left( -2 + \frac{2}{\exp\left(\frac{\omega mc}{kT}\right) + 1} \right) + 2N_f \ln \left[ \exp\left(\frac{\omega mc}{kT}\right) + 1 \right] \\ & + N_c \left\{ \frac{\frac{\omega mc}{kT} \exp\left(\frac{\omega mc}{kT}\right)}{\exp\left(\frac{\omega mc}{kT}\right) - 1} - \ln \left[ \exp\left(\frac{\omega mc}{kT}\right) - 1 \right] \right\} \\ & + \frac{8g^2T}{c(2\pi)^2} \left( 2N_f \sum_{l=1}^{\infty} \frac{(-1)^l}{l^2} - N_c \sum_{l=1}^{\infty} \frac{(1)^l}{l^2} \right) \\ & \times \left( 1 - \frac{1}{4} \sum_{s=0}^{\infty} \frac{[1+(-1)^s]}{s+1} \left(\frac{kT}{\omega mc}\right)^s \frac{(s+2)!}{l^s} \right). \end{aligned}$$

The series of the logarithmic function over  $s$  depends upon  $(kT/m\omega c)$ . For  $(kT/m\omega c) = V_{th}^2/V_\phi c < 1$ , where thermal velocity  $v_{th} = \sqrt{T/m}$  and the phase velocity  $v_\phi = \omega/k$ , we can neglect higher powers (above 4), and obtain

$$\begin{aligned} \Pi_{00} = & -\frac{4g^2T^2}{c^2(2\pi)^2} \left[ \frac{1}{2} i\pi \frac{\omega mc}{kT} \left( N_f \frac{\omega mc}{kT} \left( -2 \right. \right. \right. \\ & \left. \left. + \frac{2}{\exp\left(\frac{\omega mc}{kT}\right) + 1} \right) N_f \frac{\omega mc}{kT} \left( -2 + \frac{2}{\exp\left(\frac{\omega mc}{kT}\right) + 1} \right) \right. \\ & \left. \left. + N_c \left\{ \frac{\frac{\omega mc}{kT} \exp\left(\frac{\omega mc}{kT}\right)}{\exp\left(\frac{\omega mc}{kT}\right) - 1} - \ln \left[ \exp\left(\frac{\omega mc}{kT}\right) - 1 \right] \right\} \right) \right. \\ & \left. - \frac{1}{15} \pi^4 \left(\frac{kT}{\omega mc}\right)^2 \left\{ \frac{1}{3} (4N_c + 7N_f) \right. \right. \right. \\ & \left. \left. - \frac{1}{7} \pi^6 \left(\frac{kT}{\omega mc}\right)^4 (16N_c + 31N_f) \right\} \right]. \end{aligned}$$

Since  $(\omega mc/kT) > 1$  and thus  $\exp(\omega mc/kT) \gg 1$ ,  $\Pi_{00}$  may be approximated to

$$\begin{aligned} \Pi_{00} = & \frac{4g^2T^2}{c^2(2\pi)^2} \left[ \frac{1}{45} \pi^4 \left(\frac{c^2k^2}{\omega^2}\right) \left(\frac{T}{mc^2}\right)^2 \frac{1}{3} (4N_c + 7N_f) \right. \\ & \left. + \frac{1}{105} \pi^6 \frac{c^4k^4}{\omega^4} \left(\frac{T}{mc^2}\right)^4 (16N_c + 31N_f) \right] \\ & - \frac{1}{2} i\pi \frac{4g^2T^2}{c^2(2\pi)^2} \left[ N_f \frac{\omega^2}{c^2k^2} \left(\frac{mc^2}{T}\right)^2 \exp\left(-\frac{\omega mc^2}{ckT}\right) \right]. \end{aligned} \quad (17)$$

### III. DISPERSION RELATION

We recall here that the longitudinal component of the dielectric permittivity tensor is given by

$$\epsilon_l = 1 - \frac{\Pi_{00}}{c^2 k^2},$$

and using the value of  $\Pi_{00}$  from Eq. (17),  $\epsilon_l$  becomes

$$\epsilon_l = 1 - \frac{A}{w^2} - \frac{Bc^2 k^2}{w^4} + iC \frac{w^2}{c^4 k^4} \exp\left[-\frac{w}{ck} \left(\frac{c^2}{v_{th}^2}\right)\right], \quad (18)$$

where  $A = \frac{1}{45} \pi^2 c^2 k_d^2 (T/mc^2)^2 (4N_c + 7N_f)$ ,  $B = (\pi^4/105)(c^2 k_d^2)(T/mc^2)^4 (16N_c + 31N_f)$ , and  $C = (1/2\pi)(c^2 k_d^2)(mc^2/T)^2 N_f$ , where the Debye wave number  $k_d = gT/c^2$ .

Defining  $\omega = \omega_r + i\omega_i$  and assuming  $\omega_i \ll \omega_r$ , we can write the dispersion relation as

$$\omega_r^4 - A\omega_r^2 - Bc^2 k^2 + i \left[ \omega_i (4\omega_r^3 - 2\omega_r A) + \frac{C}{c^4 k^4} \omega_r^6 \exp\left(\frac{w_r mc^2}{ck} \frac{mc^2}{T}\right) \right] = 0.$$

From the real part of the above expression, we obtain

$$\omega_r^2 = \frac{A}{2} + \frac{1}{2} \sqrt{A^2 + 4Bc^2 k^2}. \quad (19)$$

The second root with negative sign is neglected as it yields nonphysical values of  $\omega_r$ . The imaginary component  $\omega_i$  or the Landau damping rate is given by

$$\omega_i = - \frac{\frac{c^2 k_d^2}{2\pi} \left(\frac{mc^2}{T}\right)^4 \frac{\omega_r^6}{c^4 k^4} \exp\left(-\frac{w_r}{ck} \left(\frac{mc^2}{T}\right)\right) N_f}{4\omega_r^3 - 2\omega_r \left[ \frac{1}{45} \pi^2 c^2 k_d^2 \left(\frac{T}{mc^2}\right)^2 (4N_c + 7N_f) \right]}. \quad (20)$$

Let us now consider two special cases, i.e., long wave length when  $k < k_d$  and short wave length when the opposite holds true.

(i) For longer wave lengths  $k^2 \ll k_d^2$ , the real component becomes

$$\omega_r^2 \approx A + \frac{Bc^2 k^2}{A}.$$

Using the values of  $A$  and  $B$ , we have

$$\left(\frac{mc^2}{T}\right)^2 \frac{\omega_r^2}{c^2 k_d^2} = \frac{\pi^2}{45} (4N_c + 7N_f) + \frac{3}{7} \pi^2 \frac{k^2}{k_d^2} \left(\frac{16N_c + 31N_f}{4N_c + 7N_f}\right).$$

We note here that in the limit  $k \rightarrow 0$ , the oscillation frequency  $\omega_r$  behaves like an ordinary Langmuir oscillation.

The imaginary component for the long wave length case becomes

$$\frac{\omega_i}{\omega_r} = - \frac{\frac{N_f}{8\pi} \left(\frac{mc^2}{T}\right)^4 \frac{\omega_r^4}{c^4 k^4} \exp\left[-\frac{w_r}{ck} \left(\frac{mc^2}{T}\right)\right]}{\frac{\pi^2}{90} (4N_c + 7N_f) + \frac{3}{7} \pi^2 \frac{k^2}{k_d^2} \left(\frac{16N_c + 31N_f}{4N_c + 7N_f}\right)}.$$

(ii) For the short wave lengths case, i.e., when  $k_d^2 \ll k^2$ , we have

$$\omega_r^2 \approx \frac{A}{2} + ck\sqrt{B} \left(1 + \frac{A^2}{8Bc^2 k^2}\right)$$

or

$$\begin{aligned} \omega_r^2 &\approx \frac{\pi^2}{90} c^2 k_d^2 \left(\frac{T}{mc^2}\right)^2 (4N_c + 7N_f) + \frac{\pi^2}{\sqrt{105}} (ck)(ck_d) \\ &\times \left(\frac{T}{mc^2}\right)^2 (16N_c + 31N_f)^{1/2} \\ &\times \left(1 + \frac{1}{154} \frac{(c^2 k_d^2)}{c^2 k^2} \frac{(4N_c + 7N_f)^2}{(16N_c + 31N_f)}\right). \end{aligned}$$

The imaginary component  $\omega_i$ , i.e., the Landau damping becomes

$$\frac{\omega_i}{\omega_r} = - \frac{\frac{N_f}{8\pi} \left(\frac{k_d^2}{k^2}\right) \beta^4 \exp(-\beta)}{\frac{\pi^2}{\sqrt{105}} \left(\frac{k_d}{k}\right) (16N_c + 31N_f)^{1/2} \left[1 + \frac{1}{154} \left(\frac{k_d^2}{k^2}\right) \frac{(4N_c + 7N_f)^2}{(16N_c + 31N_f)}\right]},$$



where  $\beta = (\omega_r / ck)(mc^2/T)$ .

In order to calculate the color current polarization tensor for the electrostatic oscillations  $\Pi_{00}$  and linear Landau damping in QGP, we have considered two special cases of the plasma particles in the velocity range of strongly relativistic and extreme-relativistic cases. We observe that for the extreme-relativistic case (i.e.,  $v=c$ ) the Landau damping disappears since the phase velocity  $\omega/k$  in this case remains smaller than the thermal speed of the particle ( $c$ ).

On the other hand, for the strongly relativistic case, the two velocities resonate and thus the QGP exhibits Landau damping, for both the long- and short-wavelength regions. Further we see that the wavelength of the perturbations and Landau damping rate are sensitive to the choice of QGP parameters of interest. We have also noted that for longer

wavelength regimes, in the limit of  $k \rightarrow 0$ , the oscillation frequency shows a behavior similar to that of the ordinary plasma frequency  $\omega_p$ . On the other hand for the short wavelengths, there is an additional term that appears in the real part of the frequency (thus also in the Landau damping term) that depends on  $k_d$  and its higher orders.

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