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# Parallel propagating electromagnetic modes with the generalized $(r,q)$ distribution function

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In the present paper, it is argued that non-Maxwellian distribution functions are better suited to model space plasmas. A new model distribution function called the generalized $(r,q)$ distribution function which is the generalized form of the generalized Lorentzian (kappa) distribution function has been employed to carry out theoretical investigation for parallel propagating waves in general and for Alfvén waves in particular. New plasma dispersion functions have been derived and their properties investigated. The new linear dispersion relation for Alfvén waves is investigated in detail. © *2004 American Institute of Physics.* [DOI: 10.1063/1.1688329]

## I. INTRODUCTION

Recently it has become possible to construct distribution functions from data and it is seen that these deviate significantly from Maxwellian distribution functions due to the presence of high-energy tails or shoulders in the profile of the distribution function.[1–7] Until very recently, kinetic theory relied mainly on the use of Maxwellian distribution functions for analyzing the dynamics of charged particles and investigating the instabilities present in the plasmas. However, as is evident from current data analysis results, in real plasmas the particle distributions deviate significantly from Maxwellian distributions.[8–10] These deviations imply that when we use theoretical models using Maxwellian distribution function to explain or to predict different waves and instabilities, the ensuing results do not give good quantitative fits with observations.[11–13] Since it is now possible to construct realistic distribution function based on actual data, it is necessary to use such distribution functions in our theoretical models. This means that for different experiments or observations of different space and experimental plasmas, we will need to re-derive the dispersion relations, regions of stability, and instability, etc. and most importantly provide new formulations of the plasma dispersion function each time a new (real) distribution functions is used. The use of non-Maxwellian distribution functions can themselves cause an enhancement of the electromagnetic instabilities, e.g., the Whistler mode instability.[1,14,7] Many problems related to the heating of particles in different areas of space may now be more satisfactorily resolved.[15]

It is now well known that space plasmas, e.g., planetary magnetospheres and solar wind, frequently contain particle components that exhibit high or super thermal energy tails with approximate power-law distributions in velocity space. Such nonthermal distributions, with an overabundance of fast particles, can be better fitted by generalized Lorentzian (kappa) distribution functions than by Maxwellian distribution function. Summers *et al.* in a series of papers[2–4] extensively used generalized Lorentzian (kappa) distribution function and derived a new plasma dispersion function for integral values of kappa ($\kappa$) and investigated its properties. Later they modified the dispersion function so as to incorporate half-integral values as well.[5] Hellberg *et al.*[16] generalized the same plasma dispersion function given by Summers and Thorne[2] for both integer and noninteger values and expressed the dispersion function through hypergeometric functions. In the last few years several authors employed these kappa distribution functions to give explanations of many unresolved questions pertaining to Maxwellian distribution functions.[7,17]

The use of the family of kappa distributions to model the observed nonthermal features of electron and ion structures was frequently criticized due to lack of its formal derivation. A classical analysis addressing this problem was performed by Hasegawa *et al.*[18] who demonstrated how the kappa distributions emerge as a natural consequence of the presence of super thermal radiation fields in plasmas. Collier[19] considers the generation of kappa-like distributions using velocity space Levy flights. More recently a justification for the formation of power-law distributions in space plasma due to electron acceleration by whistler mode has been proposed by Ma and Summers[17,20,21] and a kinetic theory has been developed showing that kappa-like velocity space distributions present a particular thermodynamic equilibrium state, presumably valid for a turbulent system.[22,23] Also the family of kappa distributions emerges as a consequence of the entropy generalization in nonextensive statistics, thus providing the

 

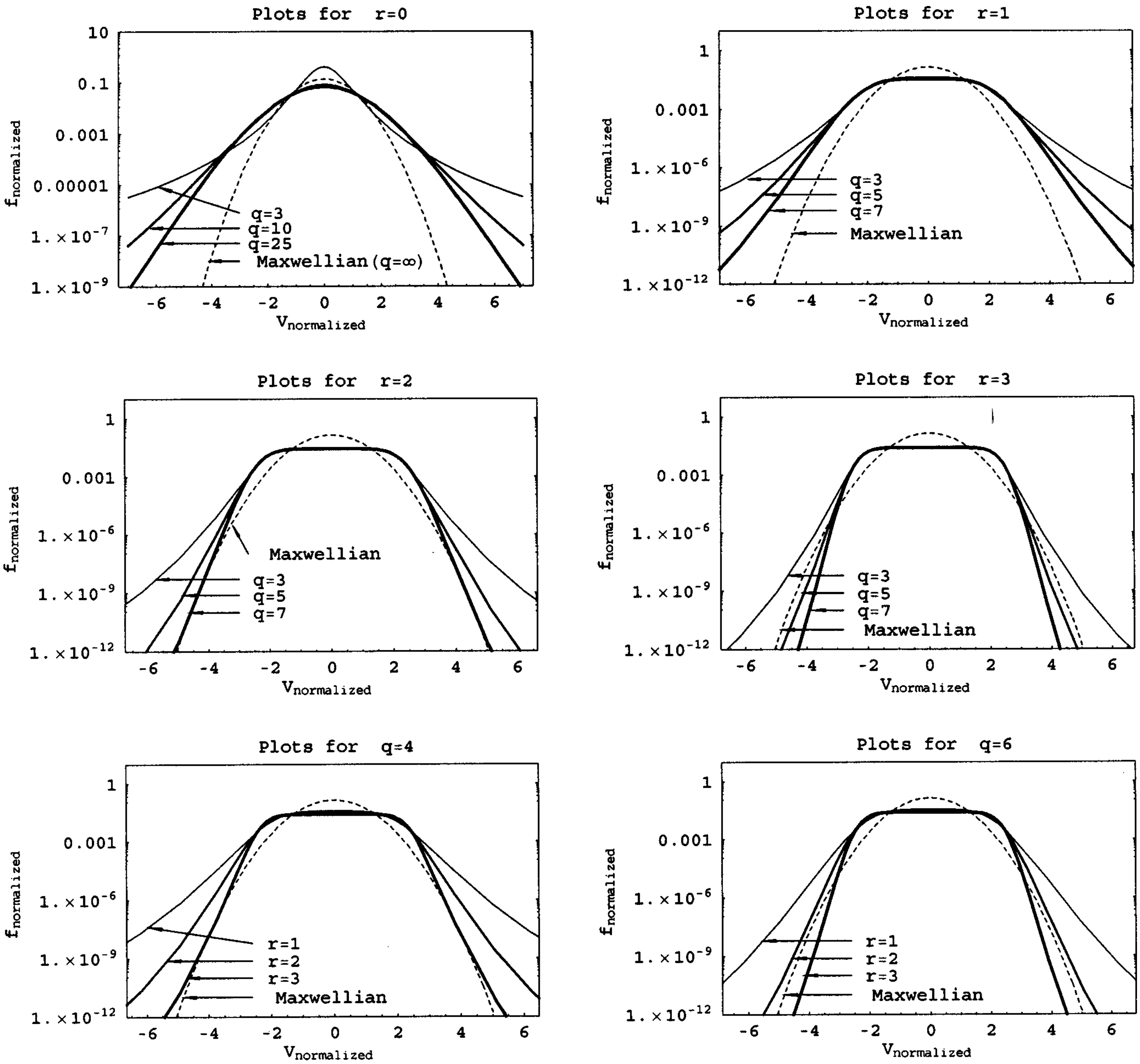


FIG. 1. Comparison of $(r,q)$ generalized distribution function for the different values of spectral indices $r$ and $q$ with the Maxwellian distribution function.

missing link for the derivation of kappa distributions from fundamental principles.

In this paper we adopt a new non-Maxwellian distribution function, which consists of two spectral indices $r$ and $q$, instead of one spectral index used in the kappa distribution function, and has the form

$$f_{(r,q)}=\frac{3(q-1)^{-3/2(1+r)}\Gamma(q)}{4\pi\Psi_{\perp}^{2}\Psi_{\parallel}\Gamma\left(q-\frac{3}{2(1+r)}\right)\Gamma\left(1+\frac{3}{2(1+r)}\right)} \times\left[1+\frac{1}{(q-1)}\left\{\left(\frac{v_{\parallel}}{\Psi_{\parallel}}\right)^{2}+\left(\frac{v_{\perp}}{\Psi_{\perp}}\right)^{2}\right\}^{r+1}\right]^{-q}, \qquad (1)$$

where

$$\Psi_{\parallel}=\sqrt{\frac{T_{\parallel}}{m}} \times\sqrt{\frac{3(q-1)^{-1/1+r}\Gamma\left(\frac{3}{2(1+r)}\right)\Gamma\left(q-\frac{3}{2(1+r)}\right)}{\Gamma\left(\frac{5}{2(1+r)}\right)\Gamma\left(q-\frac{5}{2(1+r)}\right)}}$$

and

$$\Psi_\perp = \sqrt{\frac{T_\perp}{m}} \times \sqrt{\frac{3(q-1)^{-1/(1+r)}\Gamma\left(\frac{3}{2(1+r)}\right)\Gamma\left(q-\frac{3}{2(1+r)}\right)}{\Gamma\left(\frac{5}{2(1+r)}\right)\Gamma\left(q-\frac{5}{2(1+r)}\right)}}$$

are the respective thermal speeds with respect to magnetic field, with $T_\parallel$ and $T_\perp$ the temperatures parallel and perpendicular to the ambient magnetic field. Here we note that $q>1$ and $q(1+r)>\frac{5}{2}$ are conditions which arise from the normalization and definition of temperature for the distribution function given above; $\Gamma$ is the gamma function and $f_{(r,q)}$ has been normalized so that $\int f_{(r,q)}\, d^3v = 1$.

This distribution is a generalized form of the kappa distribution function, which reduces back to kappa distribution function when we choose $r=0$ and $q=\kappa+1$, and to a Maxwellian when we take limit $q\to\infty$ and $r=0$. Figure 1 exhibits the behavior of this distribution fuction and it can be seen that it reduces to a Lorentzian (kappa) distribution function when $r=0$; and approaches a Maxwellian when $r=0$ and $q\to\infty$. We can see in Fig. 1 that if we fix the value of $q$ and increase the value of $r$ then the contribution of high-energy particles reduces but of the shoulders in the distribution function increases. Similarly, if we fix the value of $r$ and increase the value of $q$ then the result will be the same.

We adopt this $(r,q)$ distribution function assuming it to give better data fit results especially when there are shoulders in the profile of the distribution function along with the high-energy tail. We use the generalized $(r,q)$ distribution function to find the linear dispersion relation for left-hand and right-hand circularly polarized waves in a hot magnetized plasma. In the course of this derivation we obtain two new plasma dispersion functions and investigate some of their properties. In particular we give the asymptotic and power series expansions of the new dispersion functions for different values of spectral indices in order to give ready reference for future work. We also derive the dispersion relation for Alfvén waves; the plots for the right-hand and left-hand modes are also presented for the real part of the frequency.

## II. DISPERSION RELATION FOR ELECTROMAGNETIC (*R* MODE AND *L* MODE) WAVES PROPAGATING PARALLEL TO A MAGNETIC FIELD

We follow the general formalism of the kinetic theory to evaluate the linear dispersion relation for electromagnetic waves propagating in a hot magnetized plasma given in any standard textbook.[24–26] The special case of parallel propagating waves in hot magnetized plasmas is considered and linear dispersion relations for the right-hand and left-hand modes for our new distribution function are calculated.

For waves propagating exactly parallel to the magnetic field ($k_\perp=0$) the elements of dielectric tensor reduce to the following form:[4,24]

$$\begin{bmatrix} K_{xx}-1 \\ K_{zx} \\ K_{xy} \\ K_{yy}-1 \\ K_{zy} \end{bmatrix} = \sum_\alpha \frac{\omega_{p\alpha}^2}{\omega}\int v_\perp d^3v \begin{bmatrix} \frac{1}{4}\left(\frac{1}{\omega-k_\parallel v_\parallel+\Omega_\alpha}+\frac{1}{\omega-k_\parallel v_\parallel-\Omega_\alpha}\right) \\ 0 \\ \frac{i}{4}\left(\frac{-1}{\omega-k_\parallel v_\parallel+\Omega_\alpha}+\frac{1}{\omega-k_\parallel v_\parallel-\Omega_\alpha}\right) \\ \frac{1}{4}\left(\frac{1}{\omega-k_\parallel v_\parallel+\Omega_\alpha}+\frac{1}{\omega-k_\parallel v_\parallel-\Omega_\alpha}\right) \\ 0 \end{bmatrix} \hat{G} f_{0\alpha}, \tag{2}$$

$$\begin{bmatrix} K_{xz} \\ K_{yz} \\ K_{zz}-1 \end{bmatrix} = \sum_\alpha \frac{\omega_{p\alpha}^2}{\omega}\int d^3v \begin{bmatrix} 0 \\ 0 \\ \frac{v_\parallel(\partial f_{0\alpha}/\partial v_\parallel)}{\omega-k_\parallel v_\parallel} \end{bmatrix}, \tag{3}$$

$$K_{yx} = -K_{xy}, \tag{4}$$

where $K_{ij}$ are the elements of the dielectric tensor; $\Omega_\alpha$ is the cyclotron frequency of particles of species $\alpha$ ($i$: ions; $e$: electrons, etc.), and the operator $\hat{G}$ is given by

$$\hat{G} = \left(1-\frac{k_\parallel v_\parallel}{\omega}\right)\frac{\partial}{\partial v_\perp} + \frac{k_\parallel v_\parallel}{\omega}\frac{\partial}{\partial v_\parallel}. \tag{5}$$

Following the standard procedure[25] the right-hand and left-hand modes in the standard Stix notation[4,24,25] are given by

$$\tilde{R} = 1-\sum_\alpha \frac{\omega_{p\alpha}^2}{2\omega}\int_{-\infty}^{+\infty}\frac{v_\perp \hat{G} f_{0\alpha} d^3v}{k_\parallel v_\parallel-\omega-\Omega_\alpha}, \tag{6}$$

$$\tilde{L} = 1-\sum_\alpha \frac{\omega_{p\alpha}^2}{2\omega}\int_{-\infty}^{+\infty}\frac{v_\perp \hat{G} f_{0\alpha} d^3v}{k_\parallel v_\parallel-\omega+\Omega_\alpha}. \tag{7}$$

We find from Eqs. (2)–(4) that[25]

$$K_{xx} = \tilde{S} = \frac{(\tilde{R}+\tilde{L})}{2} = K_{yy}, \tag{8}$$

$$iK_{xy}=\tilde{D}=\frac{(\tilde{R}-\tilde{L})}{2}=-iK_{yx}. \tag{9}$$

The symbols $\tilde{S}$, $\tilde{D}$, $\tilde{R}$, and $\tilde{L}$ are the well-known Stix notations and refer to the sum, difference, right-, and left-hand plasma terms, respectively, in cold plasma theory. Thus the dispersion relation can be written concisely as

$$n_{\parallel}^2=\tilde{R}, \tag{10}$$

$$n_{\parallel}^2=\tilde{L}. \tag{11}$$

Equations (10) and (11) describe electromagnetic $R$ and $L$ mode waves propagating parallel to a given magnetic field in a hot plasma, respectively. Using now the distribution function given by Eq. (1), then Eqs. (6) and (7) take the form

$$R=1-\sum_{\alpha}\frac{\omega_{p\alpha}^2}{\omega^2}\left[\begin{array}{c}\frac{\omega}{k_{\parallel}\Psi_{\parallel\alpha}}\left\{Ac_1\left(\frac{\omega+\Omega_\alpha}{k_{\parallel}\Psi_{\parallel\alpha}}\right)+\left(\frac{\omega+\Omega_\alpha}{k_{\parallel}\Psi_{\parallel\alpha}}\right)^2 Z_1(\xi_\alpha)-Z_2(\xi_\alpha)\right\}\\ -(1-\Psi_{\perp\alpha}^2/\Psi_{\parallel\alpha}^2)\left(\frac{\omega+\Omega_\alpha}{k_{\parallel}\Psi_{\parallel\alpha}}\right)\left\{\begin{array}{c}A\frac{1}{\left(\frac{\omega+\Omega_\alpha}{k_{\parallel}\Psi_{\parallel\alpha}}\right)}c_2+A\left(\frac{\omega+\Omega_\alpha}{k_{\parallel}\Psi_{\parallel\alpha}}\right)c_1-\frac{AB}{\left(\frac{\omega+\Omega_\alpha}{k_{\parallel}\Psi_{\parallel\alpha}}\right)}c_3\\ +\left(\frac{\omega+\Omega_\alpha}{k_{\parallel}\Psi_{\parallel\alpha}}\right)^2 Z_1(\xi_\alpha)-Z_2(\xi_\alpha)\end{array}\right\}\end{array}\right], \tag{12}$$

$$L=1-\sum_{\alpha}\frac{\omega_{p\alpha}^2}{\omega^2}\left[\begin{array}{c}\frac{\omega}{k_{\parallel}\Psi_{\parallel\alpha}}\left\{c_1\left(\frac{\omega-\Omega_\alpha}{k_{\parallel}\Psi_{\parallel\alpha}}\right)+\left(\frac{\omega-\Omega_\alpha}{k_{\parallel}\Psi_{\parallel\alpha}}\right)^2 Z_1(\xi_\alpha)-Z_2(\xi_\alpha)\right\}\\ -(1-\Psi_{\perp\alpha}^2/\Psi_{\parallel\alpha}^2)\left(\frac{\omega-\Omega_\alpha}{k_{\parallel}\Psi_{\parallel\alpha}}\right)\left\{\begin{array}{c}A\frac{1}{\left(\frac{\omega-\Omega_\alpha}{k_{\parallel}\Psi_{\parallel\alpha}}\right)}c_2+A\left(\frac{\omega-\Omega_\alpha}{k_{\parallel}\Psi_{\parallel\alpha}}\right)c_1-\frac{AB}{\left(\frac{\omega-\Omega_\alpha}{k_{\parallel}\Psi_{\parallel\alpha}}\right)}c_3\\ +\left(\frac{\omega-\Omega_\alpha}{k_{\parallel}\Psi_{\parallel\alpha}}\right)^2 Z_1(\xi_\alpha)-Z_2(\xi_\alpha)\end{array}\right\}\end{array}\right]. \tag{13}$$

Equations (12) and (13) are the dispersion relation for right-hand and left-hand circularly polarized waves. In the above equations

$$A=\frac{3(q-1)^{-3/2(1+r)}\Gamma(q)}{4\Gamma\left(q-\frac{3}{2(1+r)}\right)\Gamma\left(1+\frac{3}{2(1+r)}\right)}, \tag{14}$$

$$B=\frac{q(1+r)(1-q)^q}{(q+qr-1)}, \tag{15}$$

$$c_1=\int_{-\infty}^{+\infty}\left\{1+\frac{1}{(q-1)}s^{2(1+r)}\right\}^{-q}ds, \tag{16}$$

$$c_2=\int_{-\infty}^{+\infty}s^2\left\{1+\frac{1}{(q-1)}s^{2(1+r)}\right\}^{-q}ds, \tag{17}$$

$$c_3=\int_{-\infty}^{+\infty}s^{2-2q-2qr}\,{}_2F_1\left(q+1,q-\frac{1}{1+r},q+\frac{r}{1+r},-(q-1)s^{-2(r+1)}\right)ds; \tag{18}$$

also

$$Z_1^{(r,q)}(\xi_\alpha)=\frac{3(q-1)^{-3/2(1+r)+q}\Gamma(q)}{4\Gamma\left(q-\frac{3}{2(1+r)}\right)\Gamma\left(1+\frac{3}{2(1+r)}\right)}\int_{-\infty}^{+\infty}\frac{\left\{1+\frac{1}{(q-1)}s^{2(1+r)}\right\}^{-q}}{(s-\xi_\alpha)}ds \tag{19}$$

and

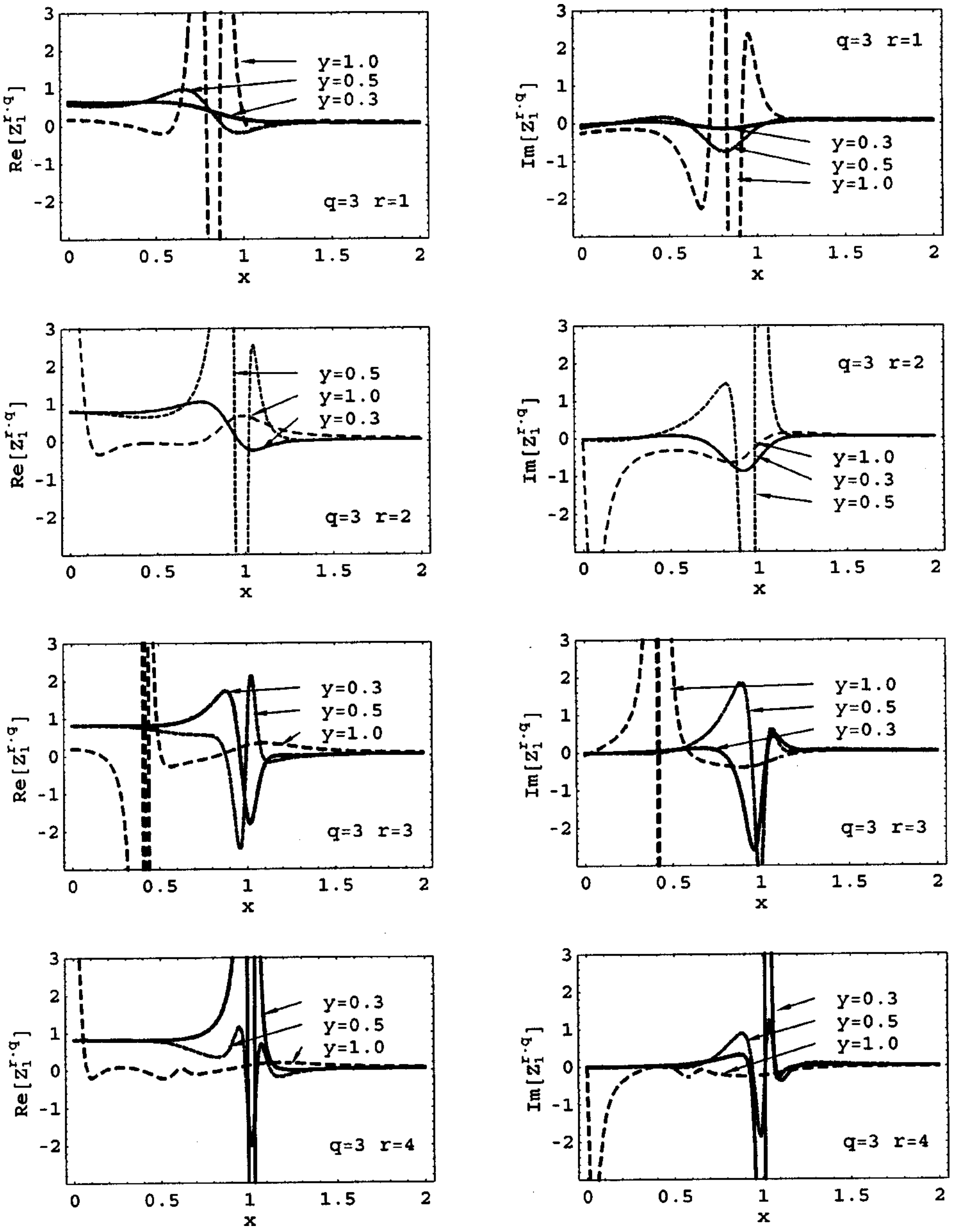


FIG. 2. The real and imaginary parts of $Z_1^{(r,q)}(x+i\,y)$ for different values of spectral indices $r$ and $q$ at different values of $y$.

$$Z_2^{(r,q)}(\xi_\alpha)=\frac{3(q-1)^{-3/2(1+r)}\Gamma(q)}{4\Gamma\left(q-\frac{3}{2(1+r)}\right)\Gamma\left(1+\frac{3}{2(1+r)}\right)}\frac{q(1+r)}{(q+qr-1)}\int_{-\infty}^{+\infty}\frac{s^{2-2q-2qr}}{(s-\xi_\alpha)}$$

$$\times{}_2F_1\left[q+1,q-\frac{1}{1+r},q+\frac{r}{1+r},-(q-1)s^{-2(r+1)}\right]ds \tag{20}$$

with

$$\xi_\alpha=\left(\frac{\omega\mp\Omega_\alpha}{k_\parallel\Psi_{\parallel\alpha}}\right)$$

are the new plasma dispersion functions.

In the limit $r=0$, $Z_2^{(r,q)}(\xi_\alpha)$ in Eqs. (12) and (13) reduces to $Z_1^{(r,q)}(\xi_\alpha)$ and if we further take the limit $q\rightarrow\kappa+1$ then the above dispersion function $Z_1^{(r,q)}(\xi_\alpha)$ reduces to the modified plasma dispersion function $Z_\kappa^*$ given by Summers and Thorne,[2] which in turn reduces to the well-known plasma dispersion function $Z(\xi_\alpha)$ of a Maxwellian plasma in the limit $r=0$ and $q\rightarrow\infty$. The imaginary and real parts of $Z_1^{(r,q)}(x+iy)$ and $Z_2^{(r,q)}(x+iy)$ for different values of $r$ and $q$ can be seen in Figs. 2 and 3, respectively. Under the limiting case $r=0$ and $q=\infty$, we can see how the new plasma dispersion functions $Z_1^{(r,q)}(x+iy)$ and $Z_2^{(r,q)}(x+iy)$ reduce to the well-known plasma dispersion function $Z(x+i\,y)$

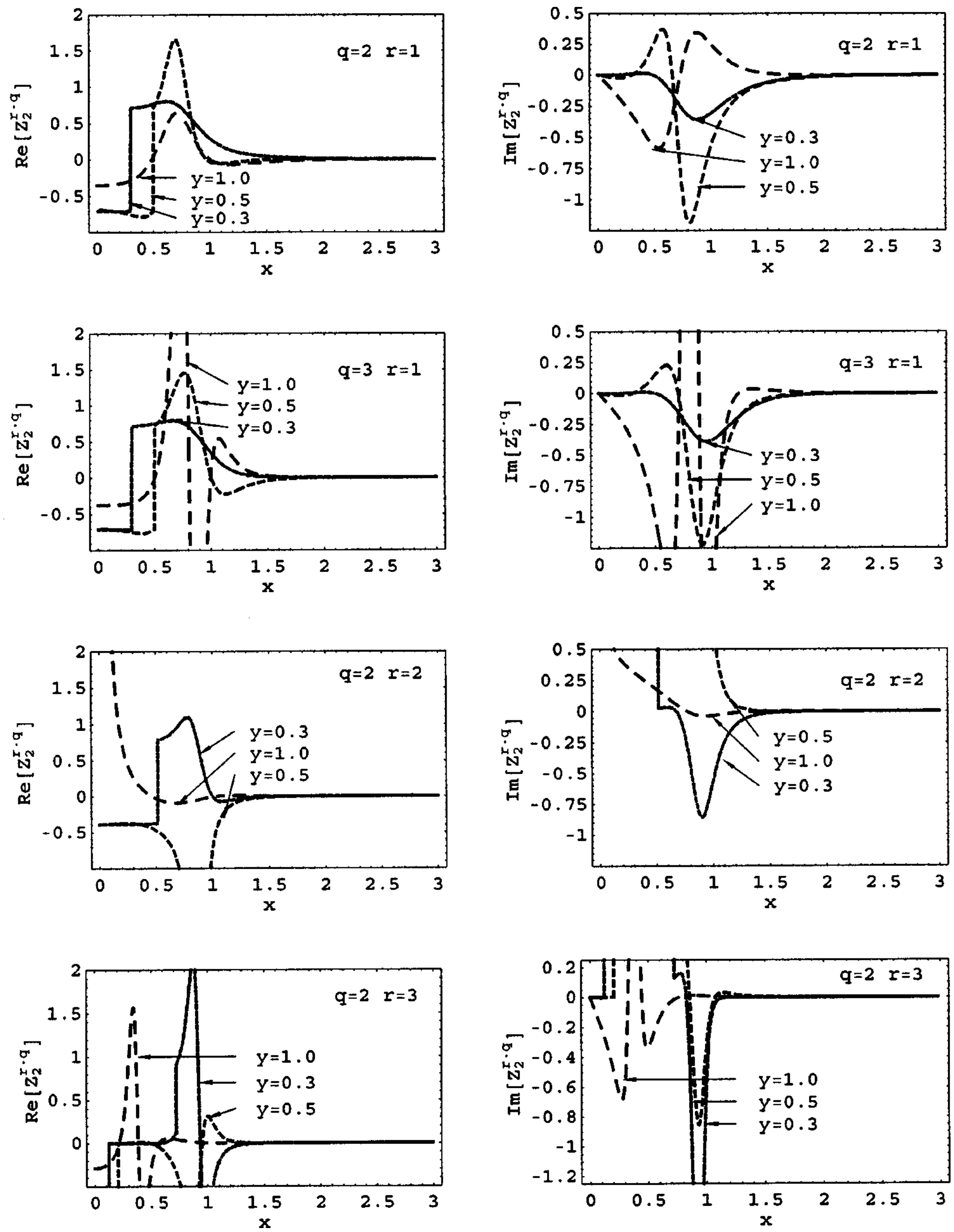


FIG. 3. The real and imaginary parts of $Z_2^{(r,q)}(x+i\ y)$ for different values of spectral indices $r$ and $q$ at different values of $y$.

(Fig. 4). The same can also be verified mathematically from Eqs. (12) and (13).

Likewise, expressions (12) and (13) for right-handed and left-handed circularly polarized waves reduce to the standard dispersion relations for the bi-Maxwellian distribution given by Eq. (38) of Summers and Thorne[2] (and in many standard textbooks).

Specific forms of the new plasma dispersion functions $Z_1^{(r,q)}(\xi_\alpha)$ and $Z_2^{(r,q)}(\xi_\alpha)$ in terms of asymptotic and power series expansions where $\xi = x + i\ y$ that are obtainable from Eqs. (19) and (20) can be seen in Appendixes A and B, respectively. These expressions provide a ready form for use in theoretical work.

## III. DISPERSION RELATION FOR ALFVÉN WAVES

In this section we derive the dispersion relation for Alfvén waves using the dispersion relation given by Eqs. (12) and (13) for parallel propagating waves. For this purpose we assume that the frequency is well below the ion cyclotron frequency, i.e., $\omega \ll \Omega_i$, such that

$$|\xi_i| \simeq \frac{|\Omega_i|}{k_\parallel \Psi_{\parallel i}} \simeq \frac{|\Omega_i|}{\omega} \frac{v_A}{\Psi_\parallel} \simeq \frac{|\Omega_i|}{\omega} \beta_i^{-1/2} \gg 1, \tag{21}$$

where $\beta_i \ll \Omega_i^2/\omega^2$. The above condition also holds for electrons. We now use the asymptotic expansions of the new plasma dispersion functions given in Eqs. (19) and (20) for both ions and electrons and using the assumption that $m_e/m_i \ll T_i/T_e$, and that the electrons contribute little to the damping owing to the smallness of their mass in comparison with those of ions,[27] the above Eqs. (12) and (13) take the form (here the $v_A^2/c^2$ term is neglected)

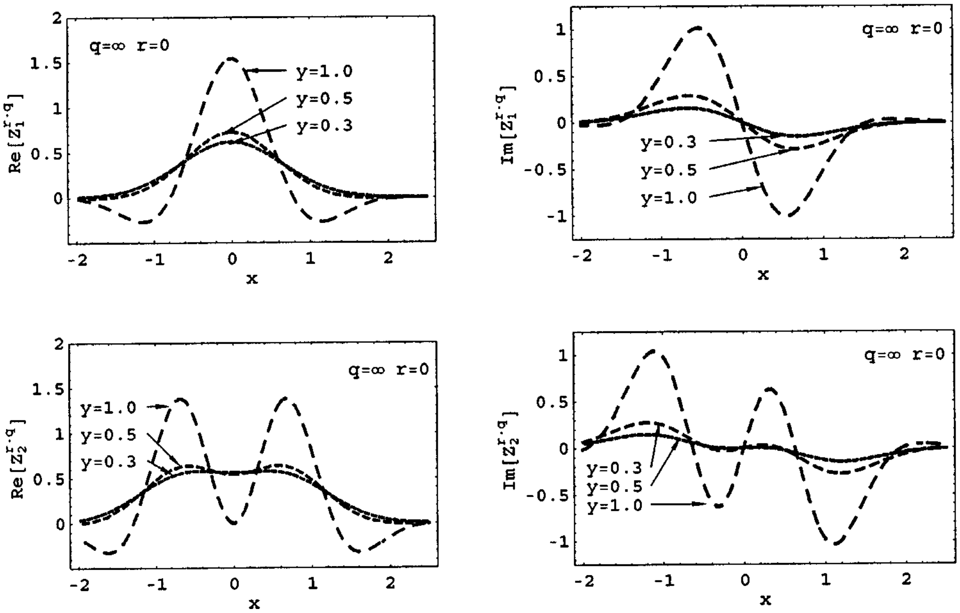


FIG. 4. The real and imaginary parts of $Z_1^{(r,q)}(x+i\,y)$ and $Z_2^{(r,q)}(x+i\,y)$ in the Maxwellian limit when $r=0$ and $q=\infty$ for different values of $y$.

$$[A(Bc_3-c_2)]\omega^2-k_\parallel^2 v_A^2 \mp AB(c_5)\frac{k_\parallel^2\Psi_{\parallel i}^2}{\Omega_i}\omega - iA\sqrt{\pi}\frac{\Omega_i^2}{k_\parallel\Psi_{\parallel i}}\omega\left[\begin{array}{c}\left(\frac{\Omega_i^2}{k_\parallel^2\Psi_{\parallel i}^2}\right)\left\{1+\frac{1}{q-1}\left(\frac{\Omega_i}{k_\parallel\Psi_{\parallel i}}\right)^{2(r+1)}\right\}^{-q}\\ -B\left(\frac{\Omega_i}{k_\parallel\Psi_{\parallel i}}\right)^{2-2q-2qr}{}_2\hat{F}_1^i\end{array}\right]+AB\omega k_\parallel^2\Psi_{\parallel i}^2 D(c_5)$$

$$+iA\sqrt{\pi}\Omega_i^2\omega\left[T_i\left(\frac{\Omega_i}{k_\parallel\Psi_{\parallel i}}\right)^3\left\{1+\frac{1}{q-1}\left(\frac{\Omega_i}{k_\parallel\Psi_{\parallel i}}\right)^{2(r+1)}\right\}^{-q}-B\left(\frac{\Omega_i}{k_\parallel\Psi_{\parallel i}}\right)^{3-2q-2qr}{}_2\hat{F}_1\right]=0, \qquad (22)$$

where

$$(c_5)=\frac{1}{\pi^{1/2}}\int_{-\infty}^{\infty}(s^{4-2q-2qr})\,{}_2F_1\left[q+1,q-\frac{1}{1+r},q+\frac{r}{1+r},-(q-1)s^{-2(r+1)}\right]ds$$

and

$${}_2\hat{F}_1={}_2F_1\left[q+1,q-\frac{1}{1+r},q+\frac{r}{1+r},-(q-1)\left(\frac{\Omega_i}{k_\parallel\Psi_{\parallel i}}\right)^{-2(r+1)}\right],$$

$$T_{i,e}=\left(1-\frac{\Psi_{\perp i,e}^2}{\Psi_{\parallel i,e}^2}\right),$$

$$D=\left(T_i+T_e\frac{T_{\parallel e}}{T_{\perp e}}\right),$$

where the values of $A$, $B$, $c_1$, $c_2$, and $c_3$ are given in Eqs. (14)–(18).

The real part of the above dispersion relation (22) is plotted in Fig. 5 which exhibits the variation of normalized frequency $f=\omega/v_A k_0$ against the normalized wave number $\alpha=k_\parallel/k_0$ where $k_0=\Omega_i v_A/\Psi_\parallel^2$ is the characteristic wave number[28] for the two modes $R$ and $L$ with different values of $r$ and $q$. The upper sign in Eq. (22) corresponds to the left-hand polarized slow Alfvén ($L$) wave and the lower sign corresponds to the right-hand polarized magnetoacoustic ($R$) or fast wave. In Fig. 5 the real part of the dispersion relation for the new distribution function given by Eq. (1) is plotted. We note that the solid lines correspond to the generalized $(r,q)$ distribution function and the dashed lines represent the dispersion relations for a Maxwellian distribution function. From the graphs we can see the deviation of the new dispersion relations from the Maxwellian as we change the values of $r$ and $q$. Thus in a high-$\beta$ plasma ($\beta>1$), even with

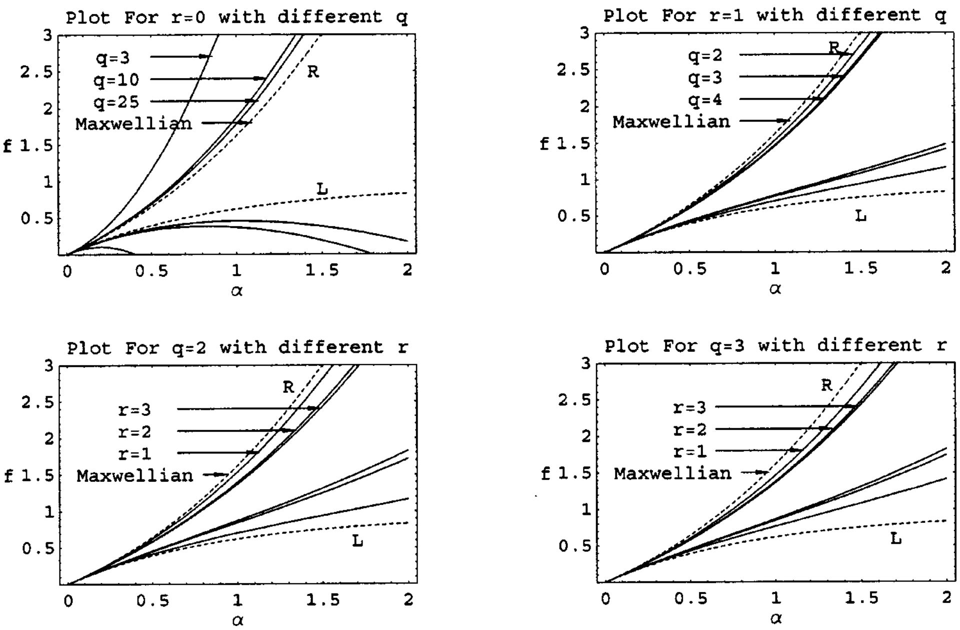


FIG. 5. The normalized frequency $f=\omega/V_A k_0$ plotted against the normalized wave number $\alpha=k_\parallel/k_0$ in a high $\beta_i$ plasma for the left-hand polarized Alfvén $(L)$ and right-hand polarized magnetoacoustic $(R)$ parallel propagating modes for the different values of $r$ and $q$ with the Maxwellian.

frequency well below the ion cyclotron frequency, the two oppositely circularly polarized parallel propagating waves have significantly different behaviors for the values of $r$ and $q$. If we fix the value of $q$ or $r$ and increase the value of the other parameter then it goes on deviating from Maxwellian. If we fix $r=0$ and increase the value of $q$ then it shifts towards Maxwellian and for $q\rightarrow\infty$ reduces to Maxwellian.

## IV. DISCUSSION AND CONCLUSION

In this paper we have considered the general formalism of the kinetic theory to derive the linear dispersion relations for electromagnetic waves propagating in a hot magnetized plasma by employing the generalized $(r,q)$ distribution function, which we feel will have better quantitative fits with real plasmas. The special case of parallel propagating waves in hot magnetized plasmas has been considered and the linear dispersion relations for the right-hand and left-hand modes are calculated for the new distribution function. Also the expressions for the slow and fast Alfvén waves for the new distribution function have been investigated. A graphical representation for the slow and fast modes have also been given to aid comparison with the Maxwellian distribution function. The newly constructed plasma dispersion function has been discussed and their power series and asymptotic expansions have also been given. These provide a handy form for use in theoretical work.

We hope that the work presented here can form a basis for obtaining new results for different space plasmas. This work also provides possibilities of reinterpreting old observations in space plasmas and to resolve some of the questions left unaddressed.

In conclusion, we feel that the work presented here is of a more general nature than the work where the generalized Lorentzian (kappa) distribution function has been used. More dispersion relations need to be evaluated and comparison with observations need to be made on the basis of the work presented here.

## APPENDIX A: THE ASYMPTOTIC EXPANSIONS OF THE DISPERSION FUNCTIONS $Z_1^{r,q}(\xi)$ AND $Z_2^{r,q}(\xi)$ FOR $\xi\rightarrow$LARGE

### 1. Asymptotic expansions for $Z_1^{r,q}$

$$Z_1^{1,3}=-\frac{1}{\xi}\left(1.485+\frac{0.5}{\xi^2}+\frac{0.424}{\xi^4}+---\right)+i\left(\frac{6.85}{\xi^{12}}-\frac{41.11}{\xi^{16}}+\frac{164.45}{\xi^{20}}----\right),$$

$$Z_1^{1,4}=-\frac{1}{\xi}\left(1.482+\frac{0.5}{\xi^2}+\frac{0.404}{\xi^4}+---\right)+i\left(\frac{68.24}{\xi^{16}}-\frac{818.9}{\xi^{20}}+\frac{6142.4}{\xi^{24}}----\right),$$

$$Z_1^{1,5}=-\frac{1}{\xi}\left(1.480+\frac{0.5}{\xi^2}+\frac{0.395}{\xi^4}+---\right)+i\left(\frac{855.8}{\xi^{20}}-\frac{17116.5}{\xi^{24}}+----\right),$$

$$Z_1^{2,3}=-\frac{1}{\xi}\left(1.6168+\frac{0.5}{\xi^2}+\frac{0.3266}{\xi^4}+---\right)+i\left(\frac{7.203}{\xi^{18}}-\frac{43.215}{\xi^{24}}+----\right),$$

$$Z_1^{2,4}=-\frac{1}{\xi}\left(1.60+\frac{0.5}{\xi^2}+\frac{0.324}{\xi^4}+---\right)+i\left(\frac{71.45}{\xi^{24}}-\frac{857.43}{\xi^{30}}+-----\right),$$

$$Z_1^{2,5}=-\frac{1}{\xi}\left(1.592+\frac{0.5}{\xi^2}+\frac{0.323}{\xi^4}+---\right)+i\left(\frac{894.03}{\xi^{30}}-\frac{17880.6}{\xi^{36}}+-----\right),$$

$$Z_1^{3,4}=-\frac{1}{\xi}\left(1.62+\frac{0.5}{\xi^2}+\frac{0.302}{\xi^4}+---\right)+i\left(\frac{71.02}{\xi^{32}}-\frac{852.17}{\xi^{40}}+-----\right),$$

$$Z_1^{3,5}=-\frac{1}{\xi}\left(1.61+\frac{0.5}{\xi^2}+\frac{0.408}{\xi^4}+---\right)+i\left(\frac{889.33}{\xi^{40}}-\frac{17786.}{\xi^{48}}+-----\right),$$

$$Z_1^{4,3}=-\frac{1}{\xi}\left(1.621+\frac{0.5}{\xi^2}+\frac{0.294}{\xi^4}+---\right)+i\left(\frac{7.03}{\xi^{30}}-\frac{42.18}{\xi^{40}}+-------\right),$$

$$Z_1^{4,4}=-\frac{1}{\xi}\left(1.637+\frac{0.5}{\xi^2}+\frac{0.293}{\xi^4}+---\right)+i\left(\frac{70.03}{\xi^{40}}-\frac{840.46}{\xi^{50}}+-------\right),$$

$$Z_1^{4,5}=-\frac{1}{\xi}\left(1.613+\frac{0.5}{\xi^2}+\frac{0.295}{\xi^4}+---\right)+i\left(\frac{878.06}{\xi^{50}}-\frac{17561.4}{\xi^{60}}+-------\right).$$

### 2. Asymptotic expansions for $Z_2^{r,q}$

$$Z_2^{1,3}=-\frac{1}{\xi}\left(1.5+\frac{0.707}{\xi^2}+\frac{0.84}{\xi^4}+---\right)+i\left(\frac{8.22}{\xi^{10}}-\frac{46.98}{\xi^{14}}+\frac{182.72}{\xi^{18}}----\right),$$

$$Z_2^{1,4}=-\frac{1}{\xi}\left(1.5+\frac{0.673}{\xi^2}+\frac{0.7}{\xi^4}+---\right)+i\left(\frac{77.99}{\xi^{14}}-\frac{909.99}{\xi^{18}}+\frac{6700.84}{\xi^{22}}----\right),$$

$$Z_2^{1,5}=-\frac{1}{\xi}\left(1.5+\frac{0.658}{\xi^2}+\frac{0.646}{\xi^4}+---\right)+i\left(\frac{950.92}{\xi^{14}}-\frac{18672.6}{\xi^{22}}+--------\right),$$

$$Z_2^{2,3}=-\frac{1}{\xi}\left(1.5+\frac{0.544}{\xi^2}+\frac{0.411}{\xi^4}+---\right)+i\left(\frac{8.10}{\xi^{16}}-\frac{47.14}{\xi^{22}}+-----\right),$$

$$Z_2^{2,4}=-\frac{1}{\xi}\left(1.5+\frac{0.538}{\xi^2}+\frac{0.387}{\xi^4}+---\right)+i\left(\frac{77.94}{\xi^{22}}-\frac{918.67}{\xi^{28}}+-----\right),$$

$$Z_2^{2,5}=-\frac{1}{\xi}\left(1.5+\frac{0.537}{\xi^2}+\frac{0.388}{\xi^4}+---\right)+i\left(\frac{957.892}{\xi^{28}}-\frac{18932.5}{\xi^{34}}+-----\right),$$

$$Z_2^{3,3}=-\frac{1}{\xi}\left(1.5+\frac{0.503}{\xi^2}+\frac{0.331}{\xi^4}+---\right)+i\left(\frac{7.79}{\xi^{22}}-\frac{45.73}{\xi^{30}}+-------\right),$$

$$Z_2^{3,4}=-\frac{1}{\xi}\left(1.5+\frac{0.504}{\xi^2}+\frac{0.328}{\xi^4}+---\right)+i\left(\frac{75.74}{\xi 30}-\frac{897.02}{\xi^{38}}+--------\right),$$

$$Z_2^{3,5}=-\frac{1}{\xi}\left(1.5+\frac{0.504}{\xi^2}+\frac{0.326}{\xi^4}+---\right)+i\left(\frac{936.13}{\xi^{38}}-\frac{18560.0}{\xi^{46}}+-------\right),$$

$$Z_2^{4,3}=-\frac{1}{\xi}\left(1.5+\frac{0.488}{\xi^2}+\frac{0.303}{\xi^4}+---\right)+i\left(\frac{7.533}{\xi^{28}}-\frac{44.41}{\xi^{38}}+-------\right),$$

$$Z_2^{4,4}=-\frac{1}{\xi}\left(1.5+\frac{0.490}{\xi^2}+\frac{0.303}{\xi^4}+---\right)+i\left(\frac{73.724}{\xi^{38}}-\frac{857.48}{\xi^{48}}+-------\right).$$

## APPENDIX B: THE SERIES EXPANSIONS OF THE DISPERSION FUNCTIONS $Z_1^{r,q}(\xi)$ AND $Z_2^{r,q}(\xi)$ FOR $\xi\rightarrow$SMALL

### 1. Power series expansions for $Z_1^{r,q}$

$$Z_1^{1,3}=\xi(-2.25-2.72\xi^2+2.925\xi^4--)+i(0.856-1.285\xi^4+1.285\xi^8--),$$

$$Z_1^{1,4}=\xi(-2.17-2.47\xi^2+2.46\xi^4--)+i(0.843-1.123\xi^4+0.936\xi^8--),$$

$$Z_1^{1,5}=\xi(-2.13-2.34\xi^2+2.231\xi^4--)+i(0.836-1.045\xi^4+0.783\xi^8--),$$

$$Z_1^{1,6}=\xi(-2.10-2.27\xi^2+2.10\xi^4--)+i(0.831-0.998\xi^4+0.698\xi^8--),$$

$$Z_1^{2,3}=\xi(-2.12-1.25\xi^2-2.748\xi^4--)+i(0.900-1.350\xi^6+1.350\xi^{12}--),$$

$$Z_1^{2,4}=\xi(-2.05-1.16\xi^2-2.454\xi^4--)+i(0.882-1.176\xi^6+0.98\xi^{12}--),$$

$$Z_1^{2,5}=\xi(-2.02-1.125\xi^2-2.309\xi^4--)+i(0.873-1.091\xi^6+0.818\xi^{12}--),$$

$$Z_1^{2,6}=\xi(-1.99-1.10\xi^2-2.223\xi^4--)+i(0.868-1.041\xi^6+0.728\xi^{12}--),$$

$$Z_1^{3,3}=\xi(-2.01-0.96\xi^2-1.05\xi^4--)+i(0.893-1.34\xi^8+1.34\xi^{16}--),$$

$$Z_1^{3,4}=\xi(-1.95-0.907\xi^2-0.97\xi^4--)+i(0.877-1.169\xi^8+0.974\xi^{16}--),$$

$$Z_1^{3,5}=\xi(-1.92-0.88\xi^2-0.925\xi^4--)+i(0.868-1.08\xi^8+0.814\xi^{16}--),$$

$$Z_1^{3,6}=\xi(-1.90-0.867\xi^2-0.90\xi^4--)+i(0.864-1.036\xi^8+0.725\xi^{16}--),$$

$$Z_1^{4,3}=\xi(-1.93-0.829\xi^2-0.73\xi^4--)+i(0.879-1.318\xi^{10}+1.318\xi^{20}--),$$

$$Z_1^{4,4}=\xi(-1.88-0.794\xi^2-0.686\xi^4--)+i(0.864-1.153\xi^{10}+0.96\xi^{20}--),$$

$$Z_1^{4,5}=\xi(-1.86-0.777\xi^2-0.663\xi^4--)+i(0.857-1.072\xi^{10}+0.804\xi^{20}--),$$

$$Z_1^{4,3}=\xi(-1.84-0.766\xi^2-0.649\xi^4--)+i(0.853-1.023\xi^{10}+0.717\xi^{20}--).$$

### 2. Power series expansions for $Z_2^{r,q}$

$$Z_2^{1,3}=\xi(-1.485-0.75\xi^2-1.63\xi^4--)+i(2.24-2.69\xi^6+3.229\xi^{10}--),$$

$$Z_2^{1,4}=\xi(-1.482-0.72\xi^2-1.48\xi^4--)+i(2.25-2.35\xi^6+2.35\xi^{10}--),$$

$$Z_2^{1,5}=\xi(-1.480-0.71\xi^2-1.41\xi^4--)+i(2.25-2.18\xi^6+1.97\xi^{10}--),$$

$$Z_2^{1,6}=\xi(-1.480-0.70\xi^2-1.36\xi^4--)+i(2.26-2.09\xi^6+1.76\xi^{10}--),$$

$$Z_2^{2,3}=\xi(-1.62-0.71\xi^2-0.75\xi^4--)+i(2.39-3.18\xi^8+3.637\xi^{14}--),$$

$$Z_2^{2,4}=\xi(-1.60-0.68\xi^2-0.70\xi^4--)+i(2.39-2.77\xi^8+2.64\xi^{14}--),$$

$$Z_2^{2,5}=\xi(-1.59-0.67\xi^2-0.675\xi^4--)+i(2.38-2.57\xi^8+2.20\xi^{14}--),$$

$$Z_2^{2,6}=\xi(-1.58-0.66\xi^2-0.66\xi^4--)+i(2.38-2.32\xi^8+1.88\xi^{14}--),$$

$$Z_2^{3,3}=\xi(-1.64-0.64\xi^2-0.497\xi^4--)+i(0.78-1.024\xi^{10}+0.954\xi^{18}--),$$

$$Z_2^{3,4}=\xi(-1.62-0.65\xi^2-0.544\xi^4--)+i(0.77-0.935\xi^{10}+0.866\xi^{18}--),$$

$$Z_2^{3,5}=\xi(-1.61-0.64\xi^2-0.529\xi^4--)+i(0.77-0.868\xi^{10}+0.723\xi^{18}--),$$

$$Z_2^{3,6}=\xi(-1.60-0.63\xi^2-0.520\xi^4--)+i(0.76-0.753\xi^{10}+0.646\xi^{18}--),$$

$$Z_2^{4,3}=\xi(-1.64-0.64\xi^2-0.497\xi^4--)+i(2.44-3.451\xi^{12}+--),$$

$$Z_2^{4,4}=\xi(-1.62-0.63\xi^2-0.476\xi^4--)+i(2.43-3.018\xi^{12}+--),$$

$$Z_2^{4,3}=\xi(-1.61-0.62\xi^2-0.466\xi^4--)+i(2.43-2.806\xi^{12}+--),$$

$$Z_2^{4,3}=\xi(-1.61-0.61\xi^2-0.459\xi^4--)+i(2.42-2.681\xi^{12}+--).$$

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