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2005 Phys. Scr. 2005 93

(http://iopscience.iop.org/1402-4896/2005/T116/019)

Physica Scripta. Vol. T116, 93–96, 2005

Collective Modes and Linear Landau Damping in a Quark-Gluon Plasma

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Received July 12, 2004

Abstract

The semiclassical kinetic approach of Boltzmann-Vlasov model along with the Yang-Mills equation in a covariant guage is used to study Quark-Gluon Plasma. The longitudinal and transverse color collective modes and Landau damping is investigated for extreme and for strongly relativistic cases. The relevant integrals for the polarization tensor are evaluated and the dispersion relations are obtained for both longitudinal and transverse modes. The regime of the QGP wherein the thermal speed of the plasma species equals the velocity of light, the linear Landau damping for the isotropic medium vanishes; while for a slight departure from the extreme relativistic case (finite mass of the plasma species), the damping term survives.

1. Introduction

The Relativistic Heavy Ion Collision (RHIC) and Large Hadron Collider (LHC) experiments at Brookhaven National Laboratory and at CERN, respectively, search signatures for a new state of matter called Quark-Gluon Plasma (QGP) [1–4]. The new state of matter is a macroscopic system of deconfined quarks and gluons interacting via the strong force, and is believed to have existed in the early universe a couple of microseconds after the Big Bang. A similar state of matter also exists in the core of compact stellar objects.

The study of the color-field fluctuations in the QGP, both with reference to the on-going experiments on heavy ion collisions (LHC & RHIC) and the matter in the early universe, constitute an important ingredient to our understanding of its color conduction properties and of the relevance of the color degree of freedom to the hadron formation out of the plasma. Field theoretical techniques have been employed to calculate the non-abelian plasma properties and in particular its excitation spectrum and that the dispersion relations for the longitudinal and transverse modes have been obtained in the finite temperature QCD within a one loop approximation [5–12]. Further it was recognized [13–20] that almost all the results obtained from the hard thermal loop approximation (with certain limits) can also be described in terms of kinetic transport equations and that the linear Landau damping was absent due to the massless gluons [21]. Markov and Markova [22] developed a theory of nonlinear damping and showed how nonlinear effects play a role in the Landau damping phenomenon. The advantage of the kinetic approach is that the physical picture is more transparent and its classical character more pronounced.

In this paper we re-examine the linear Landau damping for color collective modes of oscillations (longitudinal and transverse) for extreme ($v = c$) and strongly-relativistic ($v \lesssim c$) velocities using Vlasov and Yang-Mills equations in a covariant gauge. We observe that for both of these modes the Landau damping survives only if the thermal speed of the plasma species is somewhat smaller than the speed of light.

In section 2, we develop the linearized Vlasov kinetic equation for the perturbed distributions for the quark, anti-quark and gluon plasma species and derive a relation for the dielectric response function in terms of the polarization tensor using the Yang-Mills equation. The integrals of the polarization tensor are evaluated for electrostatic perturbations for both the extreme- and strongly-relativistic cases. In section 3, the plasma dispersion relations are developed.

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2. Linearized Vlasov Theory of Quark-Gluon Plasma

In the ultrarelativistic high temperature collisionless quark-gluon plasma, the plasma species quarks, anti-quarks and gluons are supposed to be in thermal equilibrium and behave like free gas particles obeying Fermi-Dirac and Bose-Einstein statistics respectively. In order to consider the problem of linear Landau damping for such a phase, we need to solve the Boltzmann-Vlasov kinetic and Maxwell's equations. These equations in relativistic notation can be expressed [2, 3, 21–23] as

$$p^\mu D_\mu f_{q,\bar{q}} \pm \tfrac{1}{2} g p^\mu \left\{ F_{\mu\nu}, \frac{\partial f_{q,\bar{q}}}{\partial p_\nu} \right\} = 0, \tag{1}$$

$$p^\mu \tilde{D}_\mu f_g + \tfrac{1}{2} g p^\mu \left\{ \tilde{F}_{\mu\nu}, \frac{\partial f_g}{\partial p_\nu} \right\} = 0. \tag{1a}$$

The force term $F_{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu) - \mathrm{i}g[A_\mu, A_\nu]$, satisfies Maxwell's equation in relativistic notation

$$D_\mu F^{\mu\nu}(x) - \zeta^{-1}\partial^\nu \partial^\mu A_\mu(x) = -J^\nu(x)$$

where ζ is a gauge parameter and g is the coupling parameter. A_μ is the color field potential and D_μ and $\tilde{D}_\mu$ are the covariant derivatives which act as

$$D_\mu = \partial_\mu - \mathrm{i}g[A_\mu(x), .],$$
$$\tilde{D}_\mu = \partial_\mu - \mathrm{i}g[\tilde{A}_\mu(x), .].$$

Here [,] and {, } denote the commutator and the anticommutator respectively. The generators of the color symmetry group are denoted by t^a and T^a for the fundamental and adjoint representation respectively. Thus for the fundamental representation, the color field is expressed as $A_\mu = A^a_\mu t^a$ and the field tensor as $F_{\mu\nu} = F^a_{\mu\nu} t^a$, and similarly for the adjoint representation we have, $\tilde{A}_\mu = A^a_\mu T^a$ and $\tilde{F}_{\mu\nu} = F^a_{\mu\nu} T^a$ with

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu.$$

Here J^ν is the color current density given by

$$J^\nu = g t^a \int \mathrm{d}^4 p p^\nu [Tr t^a (f_q - f_{\bar{q}}) + Tr(T^a f_g)]. \tag{2}$$

We also note that $Tr(t^a t^b) = \delta^{ab}, Tr(T^a T^b) = N_c \delta^{ab}$ and $[t^a, t^b] = \mathrm{i} f^{abc} t^c$, where the structure constants $f^{abc} = \mathrm{i}(T^a)^{bc}$. Here μ, ν, are the Minkowski indices which vary from 0 to 3; and $a, b, c \ldots$ are the color indices which run from 1, to $N^2 - 1$ of $SU(N)$ gauge group with N_f flavors of quarks.

Linearizing the above equations and taking Fourier Transforms, we obtain the turbulent part of the distribution function $f^{(1)}$ in terms of the regular (background) distribution functions $f^{(0)}$ as [22, 23]

$$f_{q,\bar{q}}^{(1)}(k, p) = \mp g \frac{\chi^{\nu\lambda}(k, p)}{pk + \mathrm{i}\epsilon p_0} \frac{\partial f_{q,\bar{q}}^{(0)}}{\partial p^\lambda} A_\nu^{(1)}(k),$$

$$f_g^{(1)}(k, p) = -g \frac{\chi^{\nu\lambda}(k, p)}{pk + \mathrm{i}\epsilon p_0} \frac{\partial f_g^{(0)}}{\partial p^\lambda} \tilde{A}_\nu^{(1)}(k),$$

where $\chi^{\nu\lambda}(k, p) = ((pk)\, g^{\nu\lambda} - p^\nu k^\lambda)$ and k is the wave number of the perturbation.

Now using the above perturbed distribution functions along with Maxwell's equation, we can calculate the color current density $J_\mu^{(1)}$ [23] as

$$J_\mu^{(1)} = g t^a \int \mathrm{d}^4 p p^\mu \frac{-g\chi^{\nu\lambda}(k, p)}{pk + \mathrm{i}\epsilon p_0} Tr \left[t^a \left(\frac{\partial f_q^{(0)}}{\partial p^\lambda} A_\nu^{(1)}(k) - \frac{\partial f_{\bar{q}}^{(0)}}{\partial p^\lambda} A_\nu^{(1)}(k) \right) + T^a \left(\frac{\partial f_g^{(0)}}{\partial p^\lambda} \tilde{A}_\nu^{(1)}(k) \right) \right]. \quad (2a)$$

In a more standard form, the perturbed current density $J_\mu^{(1)}$ can be expressed as

$$J_\mu^{(1)}(k) = \Pi^{\mu\nu}(k)\, A_\nu^{(1)}(k)$$

where

$$\Pi^{\mu\nu}(k) = g^2 \int \mathrm{d}^4 p \frac{p^\mu}{pk + \mathrm{i}\epsilon p_0} \left(p^\nu k \cdot \frac{\partial}{\partial p} - p \cdot k \frac{\partial}{\partial p^\nu} \right) N_{eq} \quad (3)$$

is the polarization tensor and $N_{eq} = \frac{1}{2}(f_q^{(0)} + f_{\bar{q}}^{(0)}) + N_c f_g^{(0)}$ is the quark-gluons equilibrium number density. The above equation for the current density is the tensor analog of the generalized Ohm's Law.

The permittivity tensor $\in^{\mu\nu}$ can be defined in terms of a polarization tensor $\Pi^{\mu\nu}$ as

$$\in^{\mu\nu} = \delta^{\mu\nu} - \frac{1}{\omega^2} \Pi^{\mu\nu}. \quad (4)$$

The tensor structure of the dielectric response function $\in^{\mu\nu}$ can be separated into longitudinal and transverse components as

$$\in_L = 1 - \frac{1}{c^2 k^2} \Pi_{00} \quad (5)$$

and

$$\in_T = 1 - \frac{1}{2\omega^2} \Pi_{ii} + \frac{1}{2c^2 k^2} \Pi_{00} \quad (6)$$

respectively.

Since the quarks and anti-quarks are fermions, and the gluons are bosons, their equilibrium distribution functions $f_{q\bar{q}}$ and f_g are given by

$$f_{q\bar{q}}^{(0)} = \frac{1}{z^{\mp 1} \exp(cp/T) + 1}, \qquad f_g^{(0)} = \frac{1}{z^{-1} \exp(cp/T) - 1} \quad (7)$$

where cp/T is the kinetic energy of the plasma particles for a very high temperature relativistic case, normalized over temperature (in eV). We note that in the high temperature limit ($T > 150\,\mathrm{MeV}$), the rest mass energy is ignored in the relativistic energy equation. The fugacity number $z = \exp(\mu/T)$ depends on the chemical potential μ of the particles in that phase.

The conductivity tensor Π_{00} [2, 3], in spherical momentum space is written as [23]

$$\Pi_{00} = -\frac{2g^2 c}{zT(2\pi)^2} \times \int_0^\infty \left(\frac{N_f \exp(cp/T)}{(z^{-1}\exp(cp/T) + 1)^2} + \frac{z^2 N_f \exp(cp/T)}{(z\exp(cp/T) + 1)^2} + \frac{N_c \exp(cp/T)}{(z^{-1}\exp(cp/T) - 1)^2} \right) p^2 \times \left(\int_0^\pi \frac{kv\cos(\theta)}{kv\cos(\theta) - \omega - \mathrm{i}\epsilon} \sin(\theta)\,\mathrm{d}\theta \right) \mathrm{d}p. \quad (8)$$

The wave number k and frequency ω correspond to the propagation of oscillations in the QGP and v is the thermal speed of the plasma particles. Similarly the conductivity tensor Π_{ii} [2, 3], in spherical momentum space, in terms of Π_{00}, can also be expressed as

$$\Pi_{ii} = \frac{2g^2 c}{zT(2\pi)^2} \times \int_0^\infty \int_0^\pi \left\{ \frac{1}{2} \frac{N_f \exp(cp/T)}{(z^{-1}\exp(cp/T) + 1)^2} + \frac{1}{2} \frac{z^2 N_f \exp(cp/T)}{(z\exp(cp/T) + 1)^2} + \frac{N_c \exp(cp/T)}{(z^{-1}\exp(cp/T) - 1)^2} \right\} \times p^2 \sin(\theta)\,\mathrm{d}\theta\,\mathrm{d}p + \Pi_{00}. \quad (9)$$

Using the well known Plemelj formula and performing the integration over θ in the expression for Π_{00}, we obtain

$$\Pi_{00} = -\frac{4g^2 c}{zT^2(2\pi)^2} \int_0^\infty \left\{ \frac{1}{2} \frac{N_f \exp(cp/T)}{(z^{-1}\exp(cp/T) + 1)^2} + \frac{1}{2} \frac{z^2 N_f \exp(cp/T)}{(z\exp(cp/T) + 1)^2} + \frac{N_c \exp(cp/T)}{(z^{-1}\exp(cp/T) - 1)^2} \right\} \times \left\{ 1 + \frac{\omega}{2kv} \left[\log\left| \frac{\omega - kv}{\omega + kv} \right| - \mathrm{i}\pi\Theta\left(1 - \frac{\omega}{kv}\right) \right] \right\} p^2\,\mathrm{d}p. \quad (10)$$

The mutual dependence of phase transition, chemical potential μ and temperature T (critical temperature T_c with reference to the bag constant $B(m, T)$) has been discussed in detail in the literature [24, 25]. Here we assume thermal and chemical equilibrium for the relatively long-lived non-interacting quark-gluon plasma of the early universe with global color neutrality and negligible average baryon density [23]. Thus for this special case the fugacity number $z = \exp(\mu/T)$ becomes unity.

Now, we consider two special cases i.e., the extreme or ultra- relativistic case (where the plasma particles behave like radiation in extreme temperature environment) and the case where the plasma species have some finite mass and hence thermal speed.

2.1. *Extreme or Ultra-relativistic Case (i.e. $v = c$)*

In this case, the particle thermal speed equals the velocity of light in vacuum i.e., $v = c$ and the integration becomes easy to perform. As a result the longitudinal and transverse components of the permittivity tensor become

$$\epsilon_L = 1 + \frac{3\omega_p^2}{c^2 k^2} \left(1 - \frac{\omega}{2ck} \left[\log\left| \frac{\omega + kc}{\omega - kc} \right| - \mathrm{i}\pi\Theta\left(1 - \frac{\omega}{kc}\right) \right] \right)$$

and

$$\epsilon_T = 1 - \frac{3\omega_p^2}{2c^2k^2}\left(1 - \frac{\omega}{2ck}\left(1 - \frac{k^2c^2}{\omega^2}\right)\left[\log\left|\frac{\omega + kc}{\omega - kc}\right| - i\pi\Theta\left(1 - \frac{\omega}{kc}\right)\right]\right)$$

where $3\omega_p^2 = \frac{g^2T^2}{c^2}(\frac{1}{6}(N_f + 2N_c))$ is the analog of the plasma frequency [2, 3]. The imaginary parts depend on the Heaviside unit-step function $\Theta(1 - \frac{\omega}{kc})$ which leads to the conclusion that this term survives only if its argument is positive i.e., the phase velocity $\omega/k < c$ (being the particle thermal speed). Therefore, Landau damping vanishes for the extreme-relativistic case.

Solving for ω^2, the real parts of the dispersion relations yield

$$\omega_{Lr}^2 = \omega_p^2 + \tfrac{3}{5}c^2k^2$$

and

$$\omega_{Tr}^2 = \omega_p^2 + \tfrac{6}{5}c^2k^2.$$

These expressions show that for QGP near equilibrium, the color collective modes of oscillations are time-like ($\omega^2 > c^2k^2$) i.e., the phase velocity of the wave is larger than the velocity of light. The resulting collective mode therefore does not exhibit damping unlike electron plasma waves. This is a consequence of having treated quarks and gluons as massless. For massive quarks however, the plasma does show some weak Landau damping [13].

2.2. *Strongly-relativistic Case (i.e. $v \lesssim c$)*

In this case, we treat the quark-gluon soup (of the early universe) as a semiclassical system of particles (quark, anti-quarks and gluons including their massive components) analogous to an ordinary plasma, in chemical equilibrium, with some finite average mass of the plasma species and hence thermal speed less than the speed of light [23].

We proceed to determine ϵ_L and ϵ_T for the strongly relativistic case i.e. $v \lesssim c$. This case is catered by considering $v = p/m$, where m is the average mass of the plasma particles (in QGP soup). After lengthy algebric manipulations, we obtain

$$\Pi_{00} = \frac{4g^2T^2}{c^2(2\pi)^2} \times \left(\tfrac{1}{90}\pi^4\left(\frac{c^2k^2}{\omega^2}\right)\left(\frac{T}{mc^2}\right)^2(8N_c + 7N_f) + \tfrac{1}{210}\pi^6\frac{c^4k^4}{\omega^4}\left(\frac{T}{mc^2}\right)^4(32N_c + 31N_f)\right) - \tfrac{1}{2}i\pi\frac{4g^2T^2}{c^2(2\pi)^2}\left(N_f\frac{\omega^2}{c^2k^2}\left(\frac{mc^2}{T}\right)^2\exp\left(-\frac{\omega}{ck}\frac{mc^2}{T}\right)\right) \quad (10a)$$

and

$$\Pi_{ii} = \frac{g^2T^2}{6c^2}(N_f + 2N_c) + \frac{4g^2T^2}{c^2(2\pi)^2}\left(\tfrac{1}{90}\pi^4\left(\frac{c^2k^2}{\omega^2}\right)\left(\frac{T}{mc^2}\right)^2 \times(8N_c + 7N_f) + \tfrac{1}{210}\pi^6\frac{c^4k^4}{w^4}\left(\frac{T}{mc^2}\right)^4(32N_c + 31N_f)\right) - \tfrac{1}{2}i\pi\frac{4g^2T^2}{c^2(2\pi)^2}\left(N_f\frac{\omega^2}{c^2k^2}\left(\frac{mc^2}{T}\right)^2\exp\left(-\frac{\omega}{ck}\frac{mc^2}{T}\right)\right). \quad (11)$$

3. Dispersion Relations

Using the expressions for the dielectric response functions, we obtain the following expressions for the permittivity tensor ϵ_l and ϵ_T

$$\epsilon_l = 1 - \frac{A}{\omega^2} - \frac{Bc^2k^2}{\omega^4} + iC\frac{\omega^2}{c^4k^4}\exp\left(-\frac{\omega}{ck}\frac{mc^2}{T}\right),$$

$$\varepsilon_T = 1 - \frac{\omega_p^2}{2\omega^2} + \frac{A}{2\omega^2} - A\frac{c^2k^2}{2\omega^4} + c^2k^2\frac{B}{2\omega^4} - \frac{i}{4\pi}\left(\frac{1}{c^2k^2} - \frac{1}{\omega^2}\right) \times\frac{c^2k_d^2\omega^2}{c^2k^2}N_f\left(\frac{mc^2}{T}\right)^2\left(\exp\left(-\frac{\omega}{ck}\frac{mc^2}{T}\right)\right)$$

where

$$\omega_p^2 = \tfrac{1}{6}c^2k_d^2(N_f + 2N_c),$$

$$A = \tfrac{1}{180}\pi^2c^2k_d^2\left(\frac{T}{mc^2}\right)^2(8N_c + 7N_f),$$

$$B = \frac{\pi^4}{420}(c^2k_d^2)\left(\frac{T}{mc^2}\right)^4(32N_c + 31N_f),$$

$$C = \frac{1}{4\pi}(c^2k_d^2)\left(\frac{mc^2}{T}\right)^2N_f.$$

The Debye wave number $k_d = gT/c^2$.

Defining $\omega = \omega_r + i\omega_i$ and assuming $\omega_i \ll \omega_r$, the real part of the dispersion relation for the longitudinal component becomes [23]

$$\omega_r^2 = \frac{A}{2} + \tfrac{1}{2}\sqrt{A^2 + 4Bc^2k^2}$$

and the imaginary part ω_i or the Landau damping rate is given by

$$\omega_i = -\frac{\frac{1}{2}C\frac{\omega_r^6}{c^4k^4}\exp\left(-\frac{w_r}{ck}\left(\frac{mc^2}{T}\right)\right)N_f}{4\omega_r^3 - \omega_rA}. \quad (12)$$

Similarly for the transverse component, the real part of the dispersion relation and the Landau damping rate become

$$\omega_r^2 = \left(\frac{3\omega_p^2}{2} - \frac{A}{2} + c^2k^2\right) + \sqrt{\left(\frac{3\omega_p^2}{2} - \frac{A}{2} + c^2k^2\right)^2 + 2(A - B)c^2k^2},$$

$$\omega_i = -\frac{C\frac{\omega_r^6}{c^2k^2}\left(\frac{1}{\omega_r^2} - \frac{1}{c^2k^2}\right)\left(\exp\left(-\frac{\omega_r}{ck}\frac{mc^2}{T}\right)\right)}{4\omega_r^3 - 2\omega_r\left(\frac{3\omega_p^2}{2} - \frac{A}{2} + c^2k^2\right)}.$$

We note that for both longitudinal and transverse oscillations, both the real part ω_r and the imaginary part ω_i depend strongly upon the coupling constant $g(T)$ and temperature T through the parameters A, B, C and ω_p. The Landau damping term vanishes for extreme thermal velocities (i.e. $v = c$) of the plasma species as has been reported in earlier literature. However, a slight departure from this extreme relativistic case introduces Landau damping. The sensitivity of the damping term can also be attributed to the choice of the mass of the plasma species.

We therefore conclude that in the extreme temperature environment of the massless plasma species, the Landau damping disappears while for the case of a relatively massive plasma the damping term survives. We may therefore expect that the wave-particle interaction in QGP may result in different signatures coming out from the different thermal regimes of the QGP.

Acknowledgment

G. M. is grateful to the 'Office of the External Activities', AS-ICTP, Trieste, Italy and PAEC for providing partial financial support.

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