AIP Physics of Plasmas

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Citation: Physics of Plasmas (1994-present) **13**, 072103 (2006); doi: 10.1063/1.2219740
View online: http://dx.doi.org/10.1063/1.2219740
View Table of Contents: http://scitation.aip.org/content/aip/journal/pop/13/7?ver=pdfcov
Published by the AIP Publishing

A specific property of electromagnetic waves interacting with dust-laden plasma

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(Received 8 May 2006; accepted 13 June 2006; published online 12 July 2006)

The propagation pattern of electromagnetic waves (EMWs) in dusty plasmas is quite different from that in electron-ion plasmas. For instance, here the ponderomotive force acts on dust grains as a negative pressure, and a nonlinear Schrödinger equation with an additional nonlinear term is obtained. Based on this equation, the modulation instability is examined and it is shown that the growth rate becomes maximum when that additional term compensates the diffraction term. The main part of this work is devoted to the localization of the grains by the EMW. Considering both subsonic and supersonic regimes, it has been shown that under certain conditions the grains are localized and the ions circumnavigate the grains, whereas the electrons escape from the region of localization. Further, the localization of grains by the EMW is found to be shape-dependent of the pulse. Comparing pancake and light bullet shaped pulses in the supersonic regime, and it is shown that only the light bullet shape leads to the compression of grains. Finally, investigating nonstationary solution, it is shown that for some parameters, the nonlinear wave breaking and the formation of a shock wave can take place.
[DOI: 10.1063/1.2219740]

I. INTRODUCTION

Nonlinear wave propagation in plasmas has become one of the most important subjects of plasma studies, and much work has been devoted to the nonlinear interactions of high-frequency electromagnetic (EM) waves in an electron-ion plasma.[1–15] However, when an electron-ion plasma contains extremely massive, micrometer-sized charged dust grains, there appears the possibility of new normal modes.[16–25] The latter include the dust-acoustic (DA) waves that have extremely low phase velocities (in comparison with the electron and ion thermal velocities) and which appear as a normal mode of a three-component dusty plasma comprising of electrons, ions, and dust grains. In a dusty plasma, both the electrons and the ions can at times be considered to be Boltzmann-distributed, whereas the charged dust particles are always inertial. Thus, the pressures of the electrons and ions provide the restoring force, whereas the inertia comes from the dust mass. Experimental confirmations about the existence of DA waves have also been made in several laboratory experiments.[28–30] In addition, a number of studies have discussed the properties of complex (dusty) plasmas.[26,27]

Many studies have been conducted to examine nonlinear coupling of high-frequency EM waves (EMWs) and DA or dust-ion acoustic waves in both unmagnetized and magnetized dusty plasmas, and it has been found that the presence of extremely massive charged dust grains modifies the strength of the coupling coefficient, because the number densities of the ions and electrons are not equal. It is well known that the slow modulation of a monochromatic EM plane wave can be described by the nonlinear Schrödinger (NLS) equation. For a medium with a positive coefficient of cubic nonlinearity, the instability that arises in the transverse direction is known as self-focusing, while that in the longitudinal direction is referred to as the modulational instability. Several theoretical attempts have been made to investigate modulational instability and to search for nonlinear structures in dusty plasmas. Recently, the stimulated scattering of a light wave in a dusty plasma was also considered by Shukla and Stenflo, in which the dust charge fluctuation effects were considered.[31]

In this paper, we consider the nonlinear propagation of high-frequency, long-wavelength transverse (EM) waves in a collisionless dusty plasma. A NLS-like equation with an additional term is derived and its modulational instability is investigated, leading to the excitation of DA waves. It is observed that when the extra nonlinear term compensates the diffraction term, the growth rate becomes maximum. In this paper we assume that the size of the dust grains is much smaller than the Debye radii and the wavelengths of the EM waves. Our basic emphasis is on the focusing of EM waves

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and we show how, in the area of localization of wave energy, the density of grains increases and the ions, following the grains, start clustering around them, while electrons are pushed away from that region. Further, this localization of EM waves is dependent on the shape of the EM pulse. Considering the example of pancake and bullet shaped pulses, we find that only the latter leads to compression of grains in the supersonic regime of the focusing region. Here we might speculate on the possible existence of dust atoms that Tsintsadze *et al.*[32] have recently proposed, by deriving a Thomas-Fermi type equation for dust grains in plasmas, where the negatively charged dust grain acts as the positive nucleus and the positively charged trapped ions circumnavigate around it.

The article is organized in the following fashion. In Sec. II, we present the basic formulation for the motion of dust grains and discuss the condition for compression of the dust grains. In Sec. III, we derive a NLS equation governing the dynamics of modulated DA waves. The discussion of the compression phenomenon and the possibility of crystallization of the dust atoms is presented in Sec. IV. Section V deals with the nonstationary solution of the NLSE. Finally, conclusions are given in Sec. VI.

II. MATHEMATICAL FORMALISM

We consider a three-component fully ionized plasma composed of electrons (with mass m and charge e), ions (mass m_i and charge $q_i=Z_ie$), and heavy charged dust particulates with mass m_d and charge $q_d=-Z_de$ in thermodynamic equilibrium. Our aim here is to consider some phenomena that can arise during the propagation of electromagnetic waves in such plasmas.

We shall consider some specific properties of such interaction in a collisionless dusty plasma, by assuming the oscillation time t_0 of EM waves ($t_0\sim 2\pi/\omega_0$, where ω_0 is some characteristic frequency associated with the EM wave) to be much less than that of all other particulates. Here we are interested in the motion of dust grains, so we consider that the time with which velocity and density of grains changes to be much larger than that of electrons and ions; i.e.,

$$v_d\left(\frac{\partial v_d}{\partial t}\right)^{-1}, \quad n_d\left(\frac{\partial n_d}{\partial t}\right)^{-1} \gg t_e\frac{1}{\omega_{pe}}, \quad t_i\frac{1}{\omega_{pi}},$$

where ω_{pe} and ω_{pi} are the Langmuir frequencies for electrons and ions, respectively. It is for this reason that electrons and ions are taken to be inertialess. We also note as the electrons are lighter than the ions, they are most effected by the electromagnetic field via the ponderomotive force. For the spatial scale, we assume inequalities $a\ll r_{\mathrm{D}\alpha}\ll\lambda$, where a, $r_{\mathrm{D}\alpha}$, and λ are, respectively, the dust grain radius, Debye radius of the electrons or ions, and wavelength of the EM wave 9. It is important to mention here that we have assumed that the occurrence time for the nonlinear processes discussed is much smaller than that required for further significant change in dust charge, and therefore we take the dust charge to be constant.

The Boltzmann distribution for the electrons and ions is expressed as

$$n_e = n_{oe}\exp\left(\frac{e\varphi-\dfrac{e^2|\mathbf{A}|^2}{2m_oc^2}}{T_e}\right), \tag{1}$$

$$n_i = n_{oi}\exp\left(\frac{-Z_ie\varphi}{T_i}\right). \tag{2}$$

We note that the effect of the ponderomotive force is taken into account only for the electrons for the reason given above. The ponderomotive force here has been expressed through the vector potential $\mathbf{A}$, which has the form

$$\mathbf{A}=\mathbf{A}_o(\mathbf{r},t)\exp i(\mathbf{k}_o\cdot\mathbf{r}-\omega_ot), \tag{3}$$

where $\mathbf{A}_0(\mathbf{r},t)$ is the amplitude of vector potential of the EM wave, which is slowly varying in space and time.

The continuity and momentum equations of dust grains are

$$\frac{\partial n_d}{\partial t}+\nabla\cdot(n_d\mathbf{v}_d)=0, \tag{4}$$

$$\frac{\partial \mathbf{v}_d}{\partial t}+(\mathbf{v}_d\cdot\nabla)\mathbf{v}_d=\frac{\nabla(Z_{\mathrm{D}}e\varphi)}{m_d}. \tag{5}$$

In Eq. (5), we have neglected the pressure term, by considering that for the temperatures the following inequality, i.e., $T_d\ll T_i,T_e$, holds. The electrons and ion number densities are given by $n_e=n_{0e}+\delta n_e$, $n_i=n_{0i}+\delta n_i$, respectively, where δn_e and δn_i are the respective perturbations. The quasi-neutrality condition can be expressed as

$$Z_i\delta n_i=Z_d\delta n_d+\delta n_e. \tag{6}$$

We further suppose that $|e\varphi-e^2|\mathbf{A}|^2/2m_oc^2|\ll T_e$ and $Z_ie|\varphi|\ll T_i$, and use in Eq. (1) and Eq. (2) along with the condition of quasi-neutrality, to express the potential energy $e\varphi$ through the density of dust grains and ponderomotive potential. We thus obtain

$$e\varphi=-\alpha T_eZ_d\frac{\delta n_d}{n_{od}}+\frac{n_{oe}}{n_{od}}\alpha\frac{e^2|\mathbf{A}|^2}{2m_oc^2}7, \tag{7}$$

where $\alpha=n_{od}T_i/(n_{oe}T_i+Z_iT_en_{oi})$. Substituting Eq. (5) into Eq. (3), we get

$$\frac{\partial \mathbf{v}_d}{\partial t}+(\mathbf{v}_d\cdot\nabla)\mathbf{v}_d=-u_d^2\nabla\left(\frac{\delta n_d}{n_{od}}\right)+\beta\nabla U_{\mathrm{pond}}, \tag{8}$$

where $\underline{\mathbf{u}_d}$ is the dust acoustic velocity, given by $u_d=Z_d\sqrt{T_e\alpha/m_d}$, $\beta=(Z_d\alpha/m_d)(n_{0e}/n_{0d})$, and the ponderomotive potential is $U_{\mathrm{pond}}=e^2|\mathbf{A}|^2/2m_oc^2$.

It is important to emphasize here that the structure of Eq. (8) is quite different from the case of a two component plasma, where the ponderomotive force is added to the equation of motion of the ions as a usual pressure term with a minus sign. In our case β is positive and we can say that the ponderomotive force acts on dust grains as a negative pressure

$$P_{\mathrm{pond}}=-\beta m_dn_dU_{\mathrm{pond}}. \tag{9}$$

We now assume that the ponderomotive force due to EM waves is not strong enough to effect the nonlinearity of the dust grains; i.e., we can neglect the second term of left-hand side (l.h.s.) of Eq. (8) and linearize the continuity equation (4). Combining the two equations, we obtain

$$\left(\frac{\partial^2}{\partial t^2} - \mathbf{u}_d^2\nabla^2\right)\frac{\delta n_d}{n_{0d}} = -\beta\nabla^2 U_{\text{pond}}. \tag{10}$$

Equation (10) shows that the right-hand side (r.h.s.) contains a negative ponderomotive pressure, unlike Zakharov's equation,[10] where a positive ponderomotive force always appears.

III. SCHRÖDINGER EQUATION AND MODULATION INSTABILITY

In spite of the large number of publications devoted to the derivation of the NLS equation and the investigation of modulational and filamenational instabilities, envelop solitons, self-focusing, etc., in dusty plasmas, we will show that in a three-component dusty plasma, the structure of the nonlinear Schrödinger equation is physically different from that in a two-component plasma.

In order to construct the nonlinear Schrödinger equation and then to investigate the excitation of dust acoustic waves, we start with Maxwell's equations, assuming the amplitude of the electromagnetic waves to be nonrelativistic. Thus, we can obtain the following equation for the vector potential:

$$\nabla^2\mathbf{A} - \frac{\partial^2\mathbf{A}}{\partial t^2} = \frac{n_e}{n_{oe}}\mathbf{A}. \tag{11}$$

Here we have introduced the following dimensionless variables: $\mathbf{r}\to(\omega_{pe}/c)\mathbf{r}$ and $t\to\omega_{pe}t$, where $\omega_{pe}=\sqrt{4\pi e^2 n_0/m_e}$ is the Langmuir frequency. Substituting expression (6) into Eq. (11) and expressing the density deviation of the equilibrium density $(n_e - n_{0e}\simeq\delta n_e)$ and the density variation of dust grains and ponderomotive potential, we obtain a new type of nonlinear Schrödinger equation:

$$2i\omega_o\left(\frac{\partial}{\partial t} + \mathbf{v}_g\cdot\frac{\partial}{\partial\mathbf{r}}\right)\mathbf{A}_o + \nabla^2\mathbf{A}_o + \left(Z_{\mathrm{D}}\alpha\frac{\delta n_d}{n_{od}} + \frac{Z_i n_{0i}}{n_{0d}}\alpha\frac{U_{\text{pond}}}{T_i}\right)\mathbf{A}_o = 0, \tag{12}$$

where ω_0 is dimensionless frequency, i.e., ω_0/ω_p, and we have made use of the linear dispersion relation for an EM wave

$$\omega_o^2 = \omega_p^2 + k_0^2c^2. \tag{13}$$

We note that in Eq. (12), $\mathbf{v}_g$ is the dimensionless group velocity given by $(1/c)(\partial\omega_o/\partial\mathbf{k}_o)=\mathbf{k}_0c/\omega_0$.

It is important to note that the Schrödinger equation [Eq. (12)] looks quite different than that obtained in the usual electron-ion plasma, since the last term of Eq. (12) is new, and we will show this term introduces new physics, which contributes to the development of a strong modulation instability. Using Madelung's representation of the complex amplitude of the electromagnetic wave

$$A_0 = a(\mathbf{r},t)e^{i\psi(\mathbf{r},t)}, \tag{14}$$

where the amplitude a and the phase ψ are real, and substituting this into the nonlinear Schrödinger equation, we obtain from the imaginary part of Eq. (12) after multiplying through by a the following equation:

$$\frac{\partial}{\partial t}a^2 + (\mathbf{v}_g\cdot\nabla)a^2 + \frac{1}{\omega}\nabla(a^2\nabla\psi) = 0, \tag{15}$$

and from the real part we get

$$\frac{\partial\psi}{\partial t} + (\mathbf{v}_g\cdot\nabla)\psi + \frac{1}{2\omega}(\nabla\psi)^2 - \frac{1}{2a\omega}\nabla^2 a - \frac{\alpha}{2\omega}\times\left(Z_d\frac{\delta n_d}{n_{0d}} + \frac{Z_i n_{0i}}{n_{0d}}\frac{U_{\text{pond}}}{T_i}\right) = 0. \tag{16}$$

We will now show how the ponderomotive potential in Eq. (16) changes the dispersion relation of the linear perturbation. To this end, we linearize Eqs. (15) and (16) with respect to the perturbations, which are represented as $a=a_0+\delta a$, $\psi=\psi_0+\delta\psi$, where a_0, ψ_0 denote the equilibrium values and δa, $\delta\psi$ are small perturbations. Upon solving Eq. (16), we observe that ψ_0 is time dependent and is expressed as

$$\psi_0(t) = \frac{\alpha}{2\omega}\frac{Z_i n_{0i}}{n_{0d}}\frac{e^2a_0^2}{2mc^2T_i}t. \tag{17}$$

After linearization of Eqs. (10), (15), and (16), we seek plane wave solutions proportional to $\exp[i(\mathbf{q}\cdot\mathbf{r}-\Omega t)]$. Finally, we obtain the following dispersion relation:

$$\left\{(\Omega-\mathbf{q}\cdot\mathbf{v}_g)^2 - \frac{q^2}{4\omega^2}\left(q^2 - \alpha\frac{Z_i n_{0i}}{n_{0d}}\frac{e^2a_0^2}{m_oc^2T_i}\right)\right\}[\Omega^2 - q^2\mathbf{u}_d^2] = \frac{\alpha\beta q^4}{4\omega^2}\frac{e^2a_0^2}{m_0c^2}. \tag{18}$$

The third term on the l.h.s. of Eq. (18) is an additional term, which appears only for a three-component dusty plasma. This additional term can increase the growth rate, when the diffraction term becomes of the same order as this term, or conversely, stabilizes the instability, when the first and second terms on the l.h.s. are smaller than the term on the r.h.s. of Eq. (18) and that $\Omega^2 > q^2u_d^2$. We first consider the case when the third term on the l.h.s. of Eq. (18) compensates the diffraction term $q^4/4\omega^2$. By taking $\omega_{pe}\gg k_0c$ (in a dense plasma, the group velocity becomes small and can be of the same order as $\mathbf{u}_d$), we obtain from the dispersion relation (18) having coincidental roots $\Omega=\mathbf{q}\cdot\mathbf{v}_g+\gamma$ and $\Omega=\mathbf{q}\cdot\mathbf{u}_d+\gamma$, the following expression for the growth rate:

$$\mathrm{Im}\,\gamma = \frac{\sqrt{3}}{4}qc\left[\frac{n_{0e}}{Z_dZ_in_{0i}}\left(\frac{T_i}{m_0c^2}\right)\left(\frac{ea_0}{T_e}\right)^2\right]^{1/3}. \tag{19}$$

Now we assume the plasma to be tenuous $(\omega_{pe}\ll k_0c)$ and consider propagation of a short-wavelength EM wave. Assuming that the diffraction term is much smaller than the new additional term of ponderomotive pressure in Eq. (15), and that $\Omega\approx qc\gg qu_d$, we can get the imaginary part of Ω as

$$\mathrm{Im}\,\Omega = qc\left(\frac{\omega_p}{2\omega_0}\right)\left(\frac{e^2a_0^2}{m_0c^2T_e}\right)^{1/2}\left[1-\left(\frac{T_i}{T_e}\right)\left(\frac{T_i}{m_dc^2}\right)\right]^{1/2}, \tag{20}$$

where we have made the simplifying assumption that $Z_in_i \approx Z_dn_{0d} \approx n_{0e}$.

IV. COMPRESSION PHENOMENON OF DUST GRAINS BY EMWs

In the present section we investigate the phenomenon of clustering of dust grains in the region where the EM wave is localized. If the nonlinear terms (last two terms) in the Schrödinger equation [Eq. (12)] are positive then the EM wave can be focused at the focal point. We will now show the existence of compression of dust grains in the localized area of electromagnetic wave. First we suppose that the dust grains are inertialess, so that Eq. (8) reduces to

$$\frac{\delta n_d}{n_{0d}} = \frac{\beta}{u_d^2}U_{\mathrm{pond}} > 0, \tag{21}$$

which means that when the energy density of the EM wave increases, then at that point, the density of the dust gains also increases. Further, while the ions are attracted towards the grains, the electrons are expelled from that region. Thus, in the localized region of the EM wave, dust grains and ions are found in abundance, suggesting the possibility of crystallization of dust atoms.[32] We may also note that Eq. (10) describes subsonic motion when the inequality $\partial^2/\partial t^2 \ll u_d^2\boldsymbol{\nabla}^2$ is satisfied.

In the opposite case, i.e., when the regime is supersonic ($\partial^2/\partial t^2 \gg \mathbf{u}_d^2\boldsymbol{\nabla}^2$), then Eq. (10) becomes

$$\frac{\partial^2}{\partial t^2}\left(\frac{\delta n_d}{n_{od}}\right) = -\beta\boldsymbol{\nabla}^2U_{\mathrm{pond}}. \tag{22}$$

If we assume here that the EM wave propagates along the z axis and introduce $\mathbf{r}_\perp$ and $\tau = t - z/\mathbf{v}_g$, then integrating Eq. (22) twice, we obtain

$$\frac{\delta n_d}{n_{od}} = -\frac{\beta}{v_g^2}\left(U_{\mathrm{pond}} + v_g^2\Delta_\perp\int_{\tau_\perp}^{\tau} d\tau^\perp \int_{\tau_\perp}^{\tau_\perp} d\tau^\parallel U_{\mathrm{pond}} + C\tau + C_1\right). \tag{23}$$

If we further assume that $U_{\mathrm{pond}}(\mathbf{r}_\perp,\tau)=0$, then we obtain the condition that $C=0=C_1$.

Let us now consider the example of a radiation pulse (EM wave) with a unit step function having a profile of the form

$$U_{\mathrm{pond}} = U_0 e^{-\mathbf{r}_\perp^2/2r_o^2}\{\Theta(\tau-\tau_1) - \Theta(\tau-\tau_2)\}, \tag{24}$$

where $\tau_2-\tau_1$ is the pulse width, r_o characterizes the pulse length, and $\Theta(x)$ is the unit step function, which has the property that when $x>0$, $\Theta(x)=1$, and if $x \geqslant 0$, then $\Theta(x)=0$. Substituting Eq. (24) into Eq. (23) and integrating, we obtain the following expression for the fluctuating density of the dust grains:

$$\frac{\delta n_d}{n_{od}} = -\frac{\beta}{2v_g^2}U_{\mathrm{pond}}\left\{1 - \frac{v_g^2(\tau-\tau)^2}{r_o^2}\left(1-\frac{r_\perp^2}{2r_o^2}\right)\right\}. \tag{25}$$

If we consider in the above expression $r_\perp=0$ and $\tau=\tau_2$, then in this case $U_{\mathrm{pond}}=U_0=\mathrm{const}$ and

$$\frac{\delta n_d}{n_{od}} = -\frac{\beta U_0}{2v_g^2}\left(1-\frac{v_g^2\tau_o^2}{r_o^2}\right), \tag{26}$$

where $\tau_0=\tau_2-\tau_1$. The above equation shows that if the initial shape of the EM wave pulse has the form of a light bullet, i.e., $v_g\tau_0 > r_0$, then the density of dust grains increases in the area where focusing of the EM wave takes place. However, when the shape of pulse has a pancake form, i.e., $r_0 > v_g\tau_0$, then the opposite occurs; i.e., $\delta n_d < 0$. Thus, we have shown that there exists a regime of interaction of the EM wave where the dust grains can gather together in the focusing region of the EM wave.

V. ENVELOPE OF AN EM WAVE

We have shown in Sec. III that the modulated amplitude of EM waves leads to the excitation of dust acoustic waves, whose amplitude grows exponentially. After a certain time the wave stops growing due to the appearance of the nonlinear terms, which did not exist in the linear analysis.

Now we will take into account nonlinear terms but retain only the quadratic nonlinearities, which is a satisfactory approximation for nonrelativistic amplitude of the EM waves. In this case the nonlinearities enter only through the ponderomotive force, which redistributes the particles and changes the density of the plasma (the hydrodynamic nonlinearities, i.e., the convective derivative term $\{(\mathbf{v}_d.\boldsymbol{\nabla})\mathbf{v}_d\}$, remain irrelevant, at least as long as the wave does not steepen too much).

Introducing the notation $F=(1/2m_0c^2T_e)^{1/2}eA_0(\mathbf{r},t)$, $\delta n = (Z_d\alpha/n_{0d})\delta n_d$ and rewriting the equations (10) and (12), we obtain the following coupled equations:

$$2i\omega_o\left(\frac{\partial}{\partial t} + \mathbf{v}_g\cdot\boldsymbol{\nabla}\right)F + \boldsymbol{\nabla}^2F + (\delta n + |F|^2)F = 0, \tag{27}$$

$$\left(\frac{\partial^2}{\partial t^2} - u_d^2\boldsymbol{\nabla}^2\right)\delta n = -u_d^2\left(\frac{n_{0e}}{n_{od}}\right)\alpha\boldsymbol{\nabla}^2|F|^2. \tag{28}$$

First, we shall consider the one-dimensional steady state problem. To this end, we introduce the new variable $\xi = x - v_gt$, and write $F=F_0(\xi,t)\exp[-i(\omega-\Delta\omega)t]$. The function $F_0(\xi,t)$ is real and $\Delta\omega$ is a negative correction to the frequency, which in dimensionless form is

$$\Delta\omega = \frac{\omega_p^2 + k_0^2c^2 - \omega_0^2}{2\omega_0}. \tag{29}$$

In this case, from Eq. (27) and (28) we get the following coupled equations:

$$\frac{d^2F_0}{d\xi^2} - 2\omega_0\Delta\omega F_0 + (\delta n + F_0^2)F_0 = 0, \tag{30}$$

$$\delta n = \frac{1}{1-M^2}\left(\frac{n_{0e}}{n_{0d}}\right)\alpha F_0^2, \tag{31}$$

where $M=v_g/u_d$ is the Mach number of the DA wave. Substituting Eq. (31) into Eq. (30), we obtain the NLS equation with a cubic nonlinearity, the solution of which is well known. However, our case is physically different compared to the case of an electron-ion plasma, and to obtain a soliton solution, it is necessary that the density perturbation of the dust grains must be positive. In our case, $M<1$, which corresponds to the subsonic regime, and therefore the variation of the density of dust grains is positive, which means that the dust gains would cluster in the maximum regime (crests) of solitons.

We now consider the quasi-nonstationary regime by introducing new variables τ and ξ and also assume that $v_g \approx u_d$. In this case Eq. (28) will reduce to

$$\frac{\partial \delta n}{\partial \tau} = \frac{\alpha u_d}{2}\left(\frac{n_{0e}}{n_{0d}}\right)\frac{\partial}{\partial \xi}F_0^2. \tag{32}$$

We may rewrite Eq. (30) and Eq. (32) as

$$\frac{\partial^2 \chi}{\partial z^2} - \chi + (\delta N + f_c^2 \chi^2)\chi = 0, \tag{33}$$

$$\frac{\partial \delta N}{\partial \tau} = \frac{\partial}{\partial z}\chi^2, \tag{34}$$

where $\chi=(1/\sqrt{2\omega\Delta\omega})(F_0/f_c)$, $f_c^2=(2/\alpha u_d\sqrt{2\omega\Delta\omega})(n_{0e}/n_{0d})$, $z=\sqrt{2\omega\Delta\omega}\xi$, and $\delta N=(Z_d\alpha/2\omega\Delta\omega)(\delta n_d/n_{0d})$.

We now examine the general solution of Eq. (33) and (34) by following Refs. 33 and 34 for arbitrary initial distribution $\chi(z,0)$. Substituting δN from Eq. (33) into Eq. (34), we obtain the nonlinear equation for the amplitude of the EM field

$$\frac{\partial}{\partial \tau}\left(\frac{1}{\chi}\frac{\partial^2 \chi}{\partial z^2}\right) + f_c^2\frac{\partial \chi^2}{\partial \tau} = -\frac{\partial}{\partial z}\chi^2. \tag{35}$$

Since $\chi(z,\tau)$ is a slowly varying function of time τ, we can neglect the second term on the l.h.s. in comparison with the r.h.s., and further by multiplying both sides of Eq. (35) by χ^2, we rewrite the first term in Eq. (35) in the form

$$\frac{\partial}{\partial z}\left[\chi^2\frac{\partial}{\partial z}\left(\frac{1}{\chi}\frac{\partial \chi}{\partial \tau}\right)\right].$$

Integrating once and using boundary conditions $|z|\to\infty$ and $\chi(\pm\infty)=0$, we obtain

$$\frac{\partial}{\partial z}\left(\frac{1}{\chi}\frac{\partial \chi}{\partial \tau}\right) = -\frac{\chi^2}{2}. \tag{36}$$

We now introduce the function

$$\Phi(z,\tau) = \int_{-\infty}^{z} dz' \chi^2(z',\tau), \tag{37}$$

so that Eq. (36) on integration yields

$$\frac{\partial \Phi}{\partial \tau} = -\frac{\Phi^2}{2} + C\Phi + C_1, \tag{38}$$

where $C_1=0$, because $\Phi(-\infty,\tau)=\partial\Phi(-\infty,\tau)/\partial\tau=0$.

For $z\to\infty$, we obtain from Eq. (38) the following:

$$\Phi(+\infty,\tau) = \int_{-\infty}^{\infty} dz' \chi^2(z',\tau) = \Phi_o, \tag{39}$$

where Φ_o is a constant. In this case, from Eq. (36), we have $C=\Phi_o/2$; thus, Eq. (36) can be written as

$$\frac{\partial \Phi}{\partial t} = -\frac{\Phi}{2}(\Phi - \Phi_o). \tag{40}$$

The above equation has two different solutions corresponding to $\Phi>\Phi_o$ or $\Phi<\Phi_o$.

We first consider the case $\Phi_o>\Phi$; then the solution of Eq. (38) is

$$\Phi = \frac{\Phi_o}{2}\left\{1 + \tanh\frac{\Phi_o}{4}[\tau + \tau_o(z)]\right\}. \tag{41}$$

From Eq. (41) at $\tau=0$, we obtain for $\tau_o(z)$

$$\tau_o(z) = -\frac{2}{\Phi_o}\ln\left(\frac{\Phi_o}{\Phi(z,0)} - 1\right), \tag{42}$$

for which we have $\tau_0(+\infty)=\infty$ when $\Phi=\Phi_o$ and $\tau_0(-\infty)=-\infty$ when $\Phi(-\infty)=0$.

The expression for the energy density of an EM wave can be obtained by differentiating the function $\Phi(z,\tau)$ with respect to z to obtain

$$\chi^2(z,\tau) = \frac{\partial \Phi}{\partial z} = \frac{\Phi_o^2}{8}\frac{\partial \tau_o(z)}{\partial z}\left\{\sec h^2\left(\frac{\Phi_o}{4}\right)[\tau + \tau_o(z)]\right\}. \tag{43}$$

Now we analyze Eq. (43) for the special case when the initial function is

$$\chi(z,\tau=0) = \frac{\chi_o}{\cosh z}. \tag{44}$$

In this case, $\tau_o(z)=(4/\Phi_o)z$ and $\Phi_o/2=\chi_o^2$.

Finally, we have

$$\chi^2 = \chi_o^2 \sec h^2\left(z + \frac{\chi_o^2}{2}\tau\right). \tag{45}$$

For the density of dust grains in this case, we obtain

$$Z_d\frac{\delta n_d}{n_{0d}} = \frac{2}{\alpha}\left(\frac{\omega_p^2 + k_0^2c^2 - \omega_o^2}{\omega_p^2}\right)\frac{1}{\cosh^2\left(\frac{\chi_o^2}{2}\tau + z\right)}. \tag{46}$$

We now investigate the second case, when $\Phi>\Phi_o$. In this case the term Φ_o dominates the r.h.s. in Eq. (38) and leads to $\Phi(z,\tau)\to\infty$ in finite time. The solution of Eq. (40) is

$$\Phi(z,\tau) = \frac{\Phi_o}{2}\left\{1 + \coth\left(\frac{\Phi_o}{4}\right)[\tau + \tau_o(z)]\right\}. \tag{47}$$

In this, it follows that at $\tau+\tau_o(z)\to 0$, when $\Phi(z,\tau)\to\infty$.

At $\tau=0$, we obtain from Eq. (47) that

$$\tau_o(z) = \frac{2}{\Phi_o} \ln\left(1 - \frac{\Phi_o}{\Phi(z,0)}\right). \tag{48}$$

By differentiating the function $\Phi(z,\tau)$ with respect to z, we find

$$\frac{\partial \Phi}{\partial z} = \chi^2 = -\frac{\Phi_o^2}{8} \frac{\partial \tau_o}{\partial z} \left\{ \frac{1}{\sinh^2\left[\left(\frac{\Phi_o}{4}\right)[\tau + \tau_o(z)]\right]} \right\}. \tag{49}$$

Expressions (47) and (49) describe the nonlinear breaking of the wave front and for time $\tau = -\tau_o(z)$, a shock wave must appear.

VI. CONCLUSIONS

In the present work, we have investigated some aspects of the nonlinear wave propagation in a dusty plasma that to the best of our knowledge have not been considered before. We have shown in Sec. II that we are able to obtain an equation that resembles Zakharov's equation, but with an important difference that our equation has a negative ponderomotive pressure term. This negative ponderomotive pressure term introduces new physics having important consequences, which were discussed in later sections. In Sec. III, we obtained a NLS type equation for the amplitude of EM waves in a dusty plasma with an additional term that leads to a strong modulational instability. The growth rates of this strong modulational instability have been obtained for the cases of a dense and a tenuous plasma, respectively.

In Sec. IV, we have shown that in our case the EM wave can be focused and that the dust grains cluster in the focus region. We have also shown that when the EM wave pulse has the form of a light bullet, the dust grain density increases in the focusing region of the EM wave, while the opposite happens when the EM wave pulse has a pancake shape.

As shown in Sec. III, the nonlinear interaction is governed by a Schrödinger-like equation for the EM wave envelopes and a driven DA wave equation. The coupled nonlinear equations admit both stationary and nonstationary solutions. In Sec. V, the stationary solutions are characterized as EM wave crest that propagate with a velocity close to the dust sonic speed and lead to the localization of dust grains, which in turn may lead to crystallization. On the other hand, a nonstationary density response to DA wave admits shock-like structures.

We believe that the results obtained here are important for the physics associated with the dusty plasmas and will help in the better understanding of nonlinear phenomena in such complex plasmas.

ACKNOWLEDGMENT

Miss Zahida Ehsan acknowledges the financial support provided by the Salam Chair in Physics, Government College University Lahore, Pakistan.

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