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Dust charge fluctuation instability in a dusty plasma with equilibrium density and magnetic field inhomogeneities

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Dust charge fluctuation instability in a dusty plasma in the presence of equilibrium density and external/ambient static magnetic field inhomogeneities has been examined in detail. The plasma ions acquire a uniform drift speed due to the equilibrium magnetic field gradient. For strongly magnetized electrons and ions, the dust charge fluctuation effect is contributed dominantly by ion dynamics. This results in an instability when the ion drift speed exceeds the perpendicular phase velocity of the waves under consideration.
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In the last two decades, physics of dusty plasmas with or without an external static magnetic field has been investigated in considerable detail.[1,2] In laboratory or space situations, the effects of a homogeneous and isotropic external/ambient static magnetic field have been considered. However, real laboratory and astrophysical dusty plasmas can occur logically in the presence of a nonuniform density and magnetic field. Recently, Shukla[3–5] has presented the effects of nonuniformities in density and magnetic field on linear and nonlinear instability of electromagnetic waves. Shukla *et al.*[6] have also studied the instability of the Shukla mode in a dusty plasma containing equilibrium density and magnetic field inhomogeneities.

In addition, a well known phenomenon has been discovered that the low-frequency electrostatic dusty plasma modes suffer a new mechanism of damping known as "Tromsø damping"[7–9] due to dust charge fluctuation effects, where conventional Landau damping is negligible. In all these studies, plasmas are considered unmagnetized and homogeneous.

In this Brief Communication, we have investigated the dust charge fluctuation effects on damping/growth of the low-frequency electrostatic waves in a dusty plasma in the presence of both equilibrium density and static magnetic field gradients. For the case of strongly magnetized electrons and ions, the conditions for the plasma instability due to the equilibrium density and static magnetic field gradients have been derived.

We consider a nonuniform dusty plasma in the presence of a nonuniform static magnetic field $[\mathbf{B}_0(x) \parallel \hat{\mathbf{z}}]$. The inhomogeneities are taken in one dimension; that is, the $\hat{\mathbf{x}}$ direction. The quasineutrality condition is assumed to be satisfied,

$$en_{e0}(x) = en_{i0}(x) + q_{d0}(x)n_{d0}(x), \tag{1}$$

where $n_{j0}(x)$ with $j=e,i,d$ is the equilibrium nonuniform density of the jth species, $q_{d0}(x)=-Z_{d0}(x)e$ with Z_{d0} as the number of electrons residing on a typical negatively charged dust grain, and e is the charge of a free electron.

Due to density nonuniformity, the electrons and ions will acquire diamagnetic drift frequencies. However, because of the magnetic field inhomogeneity, ions will have an additional drift velocity $V_0\hat{\mathbf{y}}$, and the relatively massive dust particles are considered unmagnetized but may be mobile or immobile depending on the frequency regimes of the waves under consideration. To calculate the ion drift $\mathbf{V}_0$ due to the magnetic field gradient $\partial B_0(x)/\partial x$, we write the equations of motion for electrons and ions as

$$\frac{\partial \mathbf{v}_e}{\partial t} = -\frac{e}{m_e}\left(\mathbf{E} + \frac{\mathbf{v}_e \times \mathbf{B}(x)}{c}\right), \tag{2}$$

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_i \cdot \boldsymbol{\nabla}\right)\mathbf{v}_i = \frac{e}{m_i}\left(\mathbf{E} + \frac{\mathbf{v}_i \times \mathbf{B}(x)}{c}\right), \tag{3}$$

where $\mathbf{B}(x)$ contains the external magnetic field $B_0(x)\hat{\mathbf{z}}$ and the wave magnetic field if any, m_j is the mass of the jth species $(j=e,i)$, and c is the velocity of light in a vacuum.

For a low-frequency wave and neglecting electron inertia, we obtain from Eq. (2)

$$\mathbf{E} = -\frac{\mathbf{v}_e \times \mathbf{B}(x)}{c}. \tag{4}$$

From Ampere's law, one can write

$$n_i\mathbf{v}_i - n_e\mathbf{v}_e = \frac{c}{4\pi e}\boldsymbol{\nabla} \times \mathbf{B}. \tag{5}$$

Using Eqs. (4) and (5) and the quasineutrality condition $n_e - n_i = q_d n_d/e$, Eq. (3) can be written as

$$m_i D_i \mathbf{v}_i = \frac{(\boldsymbol{\nabla} \times \mathbf{B}) \times \mathbf{B}}{4\pi n_e} + \frac{q_d n_d}{c n_e}\mathbf{v}_i \times \mathbf{B}, \tag{6}$$

where $D_i = \partial/\partial t + \mathbf{v}_i \cdot \boldsymbol{\nabla}$. At equilibrium, Eq. (6) yields

$$0 = \frac{(\mathbf{B}_0 \cdot \boldsymbol{\nabla})\mathbf{B}_0 - \boldsymbol{\nabla} B_0^2/2}{4\pi n_{e0}} + \frac{q_{d0}n_{d0}}{c n_{e0}}(\mathbf{V}_0 \times \mathbf{B}_0),$$

$$= \frac{B_0^2(\partial B_0/\partial x)}{4\pi n_{e0}}\hat{\mathbf{y}} + \frac{q_{d0}n_{d0}}{c n_{e0}}(-B_0^2\mathbf{V}_0 + B_0^2 V_{0z}\hat{\mathbf{z}}). \tag{7}$$

Here, $\mathbf{V}_0$ is the equilibrium ion drift velocity and the subscript "0" denotes equilibrium quantities.

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Thus, we obtain the ion drift velocity due to the gradient in the equilibrium magnetic field $B_0(x)\hat{\mathbf{z}}$ as

$$\mathbf{V}_0 = -\frac{c[\partial B_0(x)/\partial x]}{4\pi q_{d0} n_{d0}}\hat{\mathbf{y}}. \tag{8}$$

We now consider the specific case of strongly magnetized plasmas. We study the dust charge fluctuation instability in a dusty plasma where

$$\omega \ll \omega_{cj},$$

$$k_{\parallel} v_{tj} \ll \omega, \quad j = e,i. \tag{9}$$

The dust component is obviously considered cold and unmagnetized, but may be considered mobile depending upon the frequency regime of the low-frequency electrostatic mode under consideration.

Following our earlier work,[10,11] we obtain the dielectric function of the dusty plasma corresponding to the low-frequency electrostatic wave from Poisson's equation as

$$\epsilon(\omega,\mathbf{k}) = 1 + \chi_e + \chi_i + \chi_d + \frac{4\pi i n_{d0}\beta}{k^2\omega}, \tag{10}$$

where

$$\chi_e = \frac{\omega_e^*}{\omega k^2 \lambda_{De}^2} + \frac{\omega - \omega_e^*}{\omega}\left(\frac{k_\perp^2}{k^2}\frac{\omega_{pe}^2}{\omega_{ce}^2} - \frac{k_\parallel^2}{k^2}\frac{\omega_{pe}^2}{\omega^2}\right), \tag{11}$$

$$\chi_i = \frac{\omega_i^*}{\omega' k^2 \lambda_{Di}^2} + \frac{\omega' - \omega_i^*}{\omega'}\left(\frac{k_\perp^2}{k^2}\frac{\omega_{pi}^2}{\omega_{ci}^2} - \frac{k_\parallel^2}{k^2}\frac{\omega_{pi}^2}{\omega'^2}\right), \tag{12}$$

$$\chi_d = -\frac{\omega_{pd}^2}{\omega^2}, \tag{13}$$

$$\beta = \frac{a_0^2}{\sqrt{2\pi}}\left[\frac{\omega_{pe}}{\lambda_{De}} e^{e\Phi_G/T_e} Y_e(\omega) + \frac{\omega_{pi}}{\lambda_{Di}}\left(1 - \frac{e\Phi_G}{T_i}\right) Y_i(\omega')\right], \tag{14}$$

$$Y_e \approx \sqrt{\frac{\pi}{2}}\frac{k_\perp v_{te}}{\omega_{ce}}\frac{\omega - \omega_e^*}{\omega_{ce}}, \tag{15}$$

$$Y_i \approx \sqrt{\frac{\pi}{2}}\frac{k_\perp v_{ti}}{\omega_{ci}}\frac{\omega' - \omega_i^*}{\omega_{ci}}, \tag{16}$$

with Doppler shifted frequency $\omega' = \omega - k_y V_0$ for uniform ion drift V_0 due to equilibrium magnetic field gradient. In Eqs. (11)–(16), $\omega_j^* = \mp k_y v_{tj}^2/2\omega_{cj} L_{nj}$, $v_{tj} = (2T_j/m_j)^{1/2}$, $\omega_{cj} = |q_j| B_0/m_j c$, $\lambda_{Dj} = v_{tj}/\sqrt{2}\omega_{pj}$, $L_{nj} = -n_{j0}/(\partial n_{j0}/\partial x)$, where T_j is the temperature in energy units of the jth species. $\Phi_G = q_{d0}/a_0$ is the grain surface potential, and q_{d0} is the equilibrium charge of a spherical dust grain of radius a_0.

Neglecting the imaginary part in Eq. (10), the dispersion relations of the wave modes are obtained from $\epsilon_r(\omega,\mathbf{k}) = 0$, where ϵ_r is the real part of ϵ. Now, neglecting the inhomogeneities in the electron and ion densities and for $\omega \ll \omega_{pd}$, we obtain from Eq. (10) the usual dust-lower-hybrid wave frequency[12]

$$\omega^2 = \omega_{\mathrm{dlh}}^2\left[1 + \frac{k_\parallel^2}{k^2}\frac{\omega_{pe}^2}{\omega_{pd}^2}\right], \tag{17}$$

where $\omega_{\mathrm{dlh}}^2 = \omega_{pd}^2 \omega_{ci}^2/\omega_{pi}^2$, and $k_\perp^2 \gg k_\parallel^2$ is assumed to take into account the maximum effect of the magnetic field.

For $\omega > \omega_{pd}$ for immobile grains and considering inhomogeneous dusty plasma with homogeneous external/ambient magnetic field B_0, we obtain the low-frequency electrostatic drift waves

$$\omega = \omega_i^*\left(1 + \frac{T_e}{T_i}\frac{m_e}{m_i}\frac{1}{\delta}\frac{L_{ni}}{L_{ne}}\right), \tag{18}$$

where non-neutrality parameter $\delta = n_{io}/n_{eo}$. In the dusty plasma with equilibrium density and magnetic field gradients for $V_0 \gg \omega/k_y$ and comparing Eqs. (15) and (16), we note that the ion contribution to $\beta \equiv \beta_e + \beta_i$ in Eq. (14) is greater than that for the electrons ($\beta_i \gg \beta_e$).

Thus, the imaginary part of the drift wave, i.e., $\omega \equiv \omega_r + i\gamma$ for $k_y V_0 \gg \omega$, ω_i^*, turns out to be

$$\gamma = -\frac{4\pi n_{d0}\beta_i}{k^2(\omega_{pi}^2/\omega_{ci}^2)},$$

$$\simeq +2\pi n_{d0} a_0^2 V_0. \tag{19}$$

For the dust-lower-hybrid wave with $k_y V_0 > \omega$, ω_i^*, the growth rate of the resulting instability turns out to be

$$\gamma = +\pi n_{d0} a_0^2 V_0\left(1 - \frac{e\Phi_G}{T_i}\right). \tag{20}$$

In deriving Eq. (20), we have calculated the damping rate of the wave using the formula $\gamma = -\epsilon_i/(\partial\epsilon_r/\partial\omega_r)$ and Eq. (10) with $\beta \approx \beta_i$.

Hence, both the normal modes, the dust-lower-hybrid wave or the drift wave in the dusty plasma exhibit the dust-charge-fluctuation instability when the uniform drift velocity of ions (V_0) produced by the equilibrium magnetic field inhomogeneity exceeds the perpendicular phase velocity of the waves. The dust-charge-fluctuation instability growth rate depends on the dusty plasma parameters n_{d0}, a_0, and V_0.

In conclusion, we have investigated the dust charge fluctuation effects for different conditions of a dusty plasma in the presence of equilibrium density and external/ambient static magnetic field inhomogeneities. The plasma ions acquire a uniform drift velocity (V_0) due to the uniform gradient of the static magnetic field [cf. Eq. (8)]. Usually, in unmagnetized dusty plasmas with finite temperature, the electrostatic waves are damped due to the induced dust charge fluctuations caused by the oscillating electric field of the waves.[7–9] Uniform external magnetic field is seen to reduce this dust charge fluctuation damping[11] in a homogeneous dusty plasma.

For strongly magnetized electrons and ions having a finite temperature, we find that the contribution of ion dynamics to the dust charge fluctuation effect is more effective than that of electrons. Thus, the low-frequency waves involving dust dynamics and propagating nearly perpendicular to the magnetic field grow when the equilibrium magnetic field gradient-induced drift velocity exceeds the perpendicular phase velocity of the waves ($V_0 > \omega/k_\perp, \omega_i^*/k_\perp$).

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[1] F. Verheest, *Waves in Dusty Space Plasmas* (Kluwer, Dordrecht, 2000), Chap. 7.
[2] P. K. Shukla and A. A. Mamun, *Introduction to Dusty Plasma Physics* (Institute of Physics, Bristol, 2002), Chap. 4.
[3] P. K. Shukla, Phys. Lett. A **316**, 238 (2003).
[4] P. K. Shukla, Phys. Lett. A **252**, 340 (1999).
[5] P. K. Shukla, Phys. Scr. **T107**, 256 (2004).
[6] P. K. Shukla, R. Bharuthram, and R. Schlickeiser, Phys. Plasmas **11**, 1732 (2004).
[7] R. K. Varma, P. K. Shukla, and V. Krishan, Phys. Rev. E **47**, 3612 (1993).
[8] F. Milandso, T. K. Aslaksen, and O. Havnes, Planet. Space Sci. **41**, 321 (1993).
[9] M. R. Jana, A. Sen, and P. K. Kaw, Phys. Rev. E **48**, 3930 (1993).
[10] M. Salimullah, A. M. Rizwan, M. Nambu, H. Nitta, and P. K. Shukla, Phys. Rev. E **70**, 026404 (2004).
[11] M. Salimullah, I. Sandberg, and P. K. Shukla, Phys. Rev. E **68**, 027403 (2003).
[12] M. Salimullah and G. E. Morfill, Phys. Rev. E **59**, R2558 (1999).