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The stability of a charged surface of a magnetoradiative dusty plasma is investigated. It is shown that for a particular condition, the surface can become unstable through two different mechanisms: one is when the surface is charged, while the other arises due to dissipation instabilities caused by the radiation energy flux. In the linear approximation, a general dispersion relation is derived, taking into account magnetoradiative effect, surface charge, and gravity. © *2007 American Institute of Physics*. [DOI: 10.1063/1.2749496]

I. INTRODUCTION

In recent years, dusty plasmas have attracted considerable attention, primarily in connection with their possible role in various astrophysical phenomena[1] as well as in different plasma processing technologies.[2–6] There has been considerable activity in the investigation of linear and nonlinear volume waves in dusty plasmas.[7–9] However, the investigation of surface waves, on the interface between a dusty plasma and a vacuum or on a longitudinal interface between two different dusty plasmas, has received considerably less attention.

The propagation of high frequency surface waves in electron-ion collisionless (or weakly collisional) plasmas, have been fairly intensively investigated[10] along with investigations in the magnetohydrodynamic limit[11,12] when the surface was uncharged. In the past decade or so, surface waves in dusty plasmas began to be investigated with different interfaces for the dusty plasmas (see, e.g., Cramer *et al.*[13]) investigated the propagation of surface waves at the interface of two dusty plasmas, and in Refs. 13–16, surface waves at the interface of a dusty plasma and a metallic or a dielectric interface were considered. This was, of course, due to the practical interest that such interfaces hold in various laboratory situations. Such studies were further enhanced with the inclusion of photoemission currents.

Quite recently,[17] the question of stability of a charged plane surface of an electron-positron-ion plasma was considered and it was shown that the charged surface of such a plasma, which interfaces with a vacuum is unstable to small surface perturbations. This instability is due to the surface charge density fluctuations in the low frequency regime.

When we consider light particles as inertialess, it is possible to construct one fluid magnetohydrodynamics for a dusty plasma, which means that the plasma is electrically neutral at each point. However, this assumption is not strictly fulfilled on the surface between the plasma and the vacuum.

In this paper we will consider the stability of a charged surface of a dusty plasma that is subjected to thermal radiations, as well as magnetic and gravitational fields. In particular, we will examine how the radiation effects the surface of such a dusty plasma and we specially take into account the effects of radiative transfer, assuming the dusty plasma to be nonrelativistic.

II. GOVERNING EQUATIONS

It is known[18] that if the temperature is not too high and the plasma density is not too low, then the radiant energy density and the radiation pressure are negligibly small in comparison with the energy and pressure of the plasma. However, in this case the effect of the radiations on energy balance and the motion of the plasma will be essential, because the radiant energy lost by the heated plasma and the radiant heat transferred in the plasma can become comparable or can exceed the plasma thermal conduction. This statement is based on the fact that photons usually have a much longer mean free path than that of the charged particles.

At very high temperatures or in a tenuous plasma, the radiation energy and pressure cannot be neglected and must be added to the internal energy and pressure of the plasma, and the radiated heat transfer term should be included in the hydrodynamic (magnetohydrodynamic) equations.

The hydrodynamic equations for nonrelativistic temperatures were derived in Ref. 19. Radiation electrodynamic and hydrodynamics for a relativistically hot plasma were later considered in Ref. 20. As already noted in equations describing dusty plasma, we shall include the energy density and radiation pressure as well as the radiation heat conduction, and assume that the density of the plasma remains constant. We shall consider nonrelativistic temperatures; in this case the energy equation including radiation has the form[20]

$$\frac{\partial}{\partial t}\left(\frac{3}{2}T+\frac{U_r}{n}\right)+(\mathbf{u}\cdot\nabla)\left[\frac{3}{2}T+\frac{P_g}{n}+\frac{4}{3}\frac{U_r}{n}\right]+\nabla\cdot\frac{\mathbf{S}}{n}=0, \tag{1}$$

where $U_r=\alpha_r T^4$ is the radiation energy density and $\alpha_r=(\pi^2/15)[k_B^4/(\hbar c)^3]$.

The heat flux transported by radiation heat conduction[21,22] is given by

$$\mathbf{S}=-\frac{lc}{3}\nabla U_r=-K_0\nabla T, \tag{2}$$

where $K_0=(lc/3)\alpha_r T^3$ is the coefficient of radiation thermal conductivity, l being the Rosseland radiation mean free path, where it is possible to consider l to be proportional to some power of the temperature (assuming the density of the medium to be constant), i.e., $l=AT^\eta$, $\eta=1,2,3,\ldots$.

In a fully ionized gas, where the radiation and absorption of light proceeds entirely via bremsstrahlung, i.e., $\eta=\frac{7}{2}$; thus, $K_0T^{13/2}$, and therefore Eq. (1) reduces in equilibrium to

$$\frac{d^2T^\tau}{dz^2}=0,$$

where $\tau=\eta+4$. Here we take the z axis to be normal to the surface of the dusty plasma. Integrating this equation twice with the boundary condition $T=0$ at $z=z_s$, we obtain for the temperature distribution

$$T_0=\theta_0|z_s-z|^{1/\tau}, \tag{3}$$

where z_s is the surface coordinate and θ_0 is the constant temperature inside the plasma. As is obvious from Eq. (3), the temperature vanishes at $z=z_s$, which implies the existence of a surface with a sharp boundary. In order to investigate the stability of the surface waves along an interface between a dusty plasma and vacuum, we shall assume that electrons and ions are inertialess and their respective equations of motion are

$$-en_e\mathbf{E}-\frac{e}{c}n_e(\mathbf{v}_e\times\mathbf{B})-\nabla(P_{ge}+P_r)=0 \tag{4}$$

and

$$+Z_ien_i\mathbf{E}+\frac{e}{c}n_i(\mathbf{v}_i\times\mathbf{B})-\nabla(P_{gi}+P_r)=0, \tag{5}$$

where $P_{ge}(P_{gi})$ are electron (ion) pressure and P_r is radiation pressure. The equation of motion of negatively charged dust grains is

$$m_dn_d\frac{d\mathbf{v}_d}{dt}=-Z_den_d\mathbf{E}-\frac{Z_den_d}{c}(\mathbf{v}_d\times\mathbf{B})-\nabla(P_{gd}+P_r)+m_dn_d\mathbf{g}. \tag{6}$$

We also use the following Maxwell's equations:

$$\nabla\cdot\mathbf{E}=4\pi\rho_e=4\pi e[Z_in_i-n_e-Z_dn_d], \tag{7}$$

$$\nabla\cdot\mathbf{B}=0, \tag{8}$$

$$\nabla\times\mathbf{B}=\frac{4\pi}{c}\mathbf{J}+\frac{1}{c}\frac{\partial\mathbf{E}}{\partial t}. \tag{9}$$

Adding Eqs. (4)–(6), we obtain

$$m_dn_d\frac{d\mathbf{v}_d}{dt}=e[Z_in_i-n_e-Z_dn_d]\mathbf{E}+\frac{1}{c}[\mathbf{J}\times\mathbf{B}]-\nabla(P_g^t+P_r^t)+m_dn_d\mathbf{g}, \tag{10}$$

where

$$\mathbf{J}=-e[-Z_in_i\mathbf{v}_i+n_e\mathbf{v}_e+Z_dn_d\mathbf{v}_d], \tag{11}$$

and

$$P_g^t=P_{ge}+P_{gi}+P_{gd},\quad P_r^t=3P_r. \tag{12}$$

In these equations, P_r denotes the radiation pressure, which can be expressed via the radiation energy

$$P_r=\frac{U_r}{3}=\frac{\alpha_rT^4}{3}. \tag{13}$$

It is important to emphasize that at the surface of the plasma the electric field does not have a tangential component, and therefore $\mathbf{E}=-\nabla\Phi$ must be normal to the surface of the plasma at every point.[23] Thus,

$$e(Z_in_i-n_e-Z_dn_d)\mathbf{E}=\frac{1}{4\pi}\mathbf{E}(\nabla\cdot E)=\frac{1}{8\pi}\nabla_nE^2, \tag{14}$$

where $\hat{n}$ is normal to the surface and $E=-\partial\Phi/\partial n$. Substituting Eqs. (14) and (9) into Eq. (10), we obtain

$$\rho_d\frac{d\mathbf{v}_d}{dt}=\frac{1}{8\pi}\nabla_nE^2-\nabla\left(P_g^t+P_r^t+\frac{B^2}{8\pi}\right)+\frac{1}{4\pi}(\mathbf{B}\cdot\nabla)\mathbf{B}+\rho_d\mathbf{g}-\frac{1}{4\pi c}\left(\frac{\partial\mathbf{E}}{\partial t}\times\mathbf{B}\right), \tag{15}$$

where $\rho_d=m_dn_d$ and the first term on the right-hand side of Eq. (15) is the negative pressure gradient, which acts on the charged surface of plasma. For the magnetic field, we use the equation

$$\frac{\partial\mathbf{B}}{\partial t}=\nabla\times(\mathbf{V}\times\mathbf{B}). \tag{16}$$

Assuming the plasma to be incompressible,

$$\nabla\cdot\mathbf{v}=0; \tag{17}$$

thus, $\mathbf{v}$ can be expressed through the gradient of a scalar potential Ψ and we have

$$\nabla^2\Psi=0.$$

Further, we suppose that besides the number density the temperature inside the plasma is also constant, but on the surface (transition area) the equilibrium temperature becomes a function of the coordinate z, which is directed along the normal to the surface. Equations (1), (7), and (15)–(17) are now in closed form. We note here that all quantities related to radiation are expressed in terms of temperature T. We further note that when the plasma is in equilibrium in the presence of magnetic, gravitational, and radiation fields and the free charged surface lies in the x,y plane. Thus, any small perturbation on the surface will propagate as a wave. We choose the direction of constant magnetic field along the y axis. In equilibrium, when the surface is at rest, the electric field given by Eq. (7) (using the fact that $E=0$ on the inner area) takes the form

$$E_z = 4\pi \int \rho_e dz = 4\pi\sigma, \tag{18}$$

where σ is the surface charged density; thus, for the potential Φ,

$$\Phi = -4\pi\sigma z. \tag{19}$$

Now we consider the propagation of waves in the linear approximation. If we perturb the surface the potential, Eq. (19) can be written as

$$\Phi = -4\pi\sigma[z + \zeta(x,y,t)] + \delta\Phi, \tag{20}$$

where ζ is the displacement of the surface and $\delta\Phi$ is the perturbed potential and may be expressed as $\delta\Phi \sim \exp[i(k_x x + k_y y - \omega t) - kz]$, where $k^2 = (k_x^2 + k_y^2)$ and $k > 0$, which vanishes for $z \to \infty$. Here, $\zeta(x,y,t)$ is the z coordinate of a point on the surface. The surface oscillations can be obtained by taking ζ, and using Eq. (20), we obtain a relation between $\delta\Phi$ and ζ given by $\delta\Phi = 4\pi\sigma\zeta(x,y,t)$. Using Eqs. (18) and (19), we obtain

$$\frac{E_z^2}{8\pi} \simeq 2\pi\sigma^2 + k\sigma\delta\Phi|_{z=0} = 2\pi\sigma^2 + 4\pi\sigma^2 k\zeta(x,y,t). \tag{21}$$

Now we can write the relation for the normal component of velocity v_z on the surface as

$$\left.\frac{\partial\Psi}{\partial z}\right|_{z=0} = \frac{\partial\zeta}{\partial t}. \tag{22}$$

Using Laplace's formula for the pressure difference, we obtain

$$P - P_0 = -\alpha\left(\frac{\partial^2\zeta}{\partial x^2} + \frac{\partial^2\zeta}{\partial y^2}\right), \tag{23}$$

where α is the coefficient of surface tension. Linearizing Eq. (15) and substituting Eqs. (21) and (23) in its z component, taking into account that the last term in Eq. (15) is

$$-\frac{B_0}{4\pi c}\frac{\partial E_x}{\partial t} = -\frac{B_0^2}{4\pi c^2}\frac{\partial v_z}{\partial t},$$

where $E_x = (B_0/c)v_z$, we obtain the condition at the surface as

$$\left[\left(1 + \frac{V_A^2}{c^2}\right)\frac{\partial\Psi}{\partial t} - \frac{k\sigma}{\rho_d}\delta\Phi + \left(\frac{\partial P_r}{\partial T}\right)\frac{\delta T}{\rho_d} - \frac{B_0}{4\pi\rho_d}\frac{\partial}{\partial y}\right.$$
$$\left.\left.\times \int B_z dz - g\zeta - \alpha\left(\frac{\partial^2\zeta}{\partial x^2} + \frac{\partial^2\zeta}{\partial y^2}\right)\right]\right|_{z=0} = 0, \tag{24}$$

where $V_{\mathrm{A}} = (1/\sqrt{4\pi\rho_d})B_0$ is the Alfvén velocity of the dust grains, which is much smaller than the speed of light and have in further calculations V_{A}^2/c^2 is neglected. From Eq. (1), after linearization we obtain

$$\left[\frac{\partial}{\partial t}\left(\frac{3}{2} + \frac{1}{n}\frac{\partial U_r}{\partial T}\right)\delta T + \frac{\partial\Psi}{\partial z}\frac{\partial}{\partial z}\left(\frac{3}{2}T_0 + \frac{p_g}{n} + \frac{4}{3}\frac{U_r}{n}\right)\right.$$
$$\left.\left. - \frac{4}{3}\alpha_r cA(2\nabla_z T_0^{\tau-1}\nabla_z\delta T + \Delta_z T_0^{\tau-1}\delta T)\right]\right|_{z=0} = 0. \tag{25}$$

On the surface of the plasma, the boundary condition can be expressed as

$$\left.\frac{\partial\delta\Phi}{\partial t}\right|_{z=0} = 4\pi\sigma\frac{\partial\zeta}{\partial t} = 4\pi\sigma\left(\frac{\partial\Psi}{\partial z}\right)_{z=0} \tag{26}$$

and for the z component of magnetic field, we obtain upon linearization of Eq. (16), and using here that $\mathbf{v}$ has been expressed through the gradient of a scalar potential Ψ

$$\frac{\partial B_z}{\partial t} = B_0\frac{\partial^2\Psi}{\partial y\partial z}. \tag{27}$$

If now we differentiate Eq. (24) with respect to t, and taking into account Eqs. (22), (26), and (27), we obtain a dynamical boundary condition given by

$$\left[\frac{\partial^2\Psi}{\partial t^2} - \frac{4\pi\sigma^2 k}{\rho_d}\left(\frac{\partial\Psi}{\partial z}\right) + \frac{1}{\rho_d}\left(\frac{\partial P_r}{\partial T}\right)\frac{\partial\delta T}{\partial t}\right.$$
$$\left.\left. - V_{\mathrm{A}}^2\frac{\partial^2\Psi}{\partial y^2} - g\frac{\partial\Psi}{\partial z} - \frac{\alpha}{\rho_d}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\frac{\partial\Psi}{\partial z}\right]\right|_{z=0} = 0. \tag{28}$$

In order to obtain the dispersion relation, we shall look for solutions of Eqs. (25) and (28), by taking $\delta\Psi, \delta T \sim \exp[i(k_x x + k_y y - \omega t) + kz]$. Substituting this solution into Eqs. (25) and (28), we obtain the dispersion relation for the surface waves given by

$$\omega^2 - k^2(V_{\mathrm{A}}^2\cos^2\theta - V_E^2) - k(g + g_r) - g\frac{a^2k^3}{2}$$
$$+ ig_r k\frac{[(|D|/\omega)]}{1 + (D^2/\omega^2)} = 0. \tag{29}$$

Equation (29) describes the surface magnetoradiative capillary gravity waves, where $V_E^2 = (1/4\pi\rho_d)E_0^2$ is electric Alfvén velocity, $a^2 = 2\alpha/\rho_d g$ is the capillary constant, and

$$g_r = \tfrac{4}{3}\alpha_r\frac{T^3}{\rho_d}\frac{\left[\frac{3}{2} + (\partial p_g/n\partial T_0) + \frac{4}{3}(1/n)(\partial U_r/\partial T_0)\right](\partial T_0/\partial z)}{[1 + (D^2/\omega^2)]},$$

where D is given by

$$D = \frac{4cA\alpha_r}{3\left[\frac{3}{2} + (1/n)(\partial U_r/\partial T)\right]}\left(2k\frac{\partial}{\partial z}T_0^{\tau-1} + \frac{\partial^2 T_0^{\tau-1}}{\partial z^2}\right).$$

Now we investigate this dispersion relation in some detail. First, we consider the case when radiation is absent. In this case, Eq. (29) reduces to

$$\omega^2 = k^2(V_A^2 \cos^2\theta - V_E^2) + kg\left(1 + \frac{k^2a^2}{2}\right). \tag{30}$$

If on the surface the charge density is also neglected, then $V_E = E_0/\sqrt{4\pi\rho_d} = 4\pi\sigma/\sqrt{4\pi\rho_d} = 0$, then the dispersion relation describes the propagation of surface magnetocapillary gravity waves.

On the other hand, if $\sigma \neq 0$, the surface becomes unstable, as is evident from Eq. (30). For an unstable solution, it is necessary that $\omega^2 < 0$ for some values of k, i.e., $V_E^2 > V_A^2 \cos^2\theta + g/k(1+k^2a^2/2)$. If we further assume that the magnetic field, the surface charge and the surface tension all are absent and that the imaginary part in Eq. (29) is also neglected, then

$$\omega^2 = k(g + g_r). \tag{31}$$

This dispersion relation describes the propagation of gravity-radiated waves on the surface. From Eq. (31) follows an interesting result. First, as we can see, the radiation does not change the spatial dispersion relation and appears as a gravitational effect. Second, on the surface, pure radiation waves can propagate when $g < g_r$, which leads to

$$\omega^2 = kg_r \tag{32}$$

and the group velocity becomes a function of wavelength, i.e., $V_g = \sqrt{g_r/8\pi\lambda}$, where $\lambda = 2\pi/k$.

Now we assume that the imaginary term in Eq. (29) is much smaller than the real terms, i.e., we suppose $\mathrm{Re}\,\omega \gg \mathrm{Im}\,\omega$, then

$$\mathrm{Im}\,\omega = -\frac{kg_r|D|}{2\,\mathrm{Re}\,\omega^2}, \tag{33}$$

which describes the damping of the surface waves. We emphasize here that in the presence of surface charge, there exist values of the wave vector

$$k_{1,2} = \frac{1}{ga^2}(V_E^2 - V_A^2\cos^2\theta) \pm \frac{1}{ga^2}\sqrt{(V_E^2 - V_A^2\cos^2\theta)^2 - (ga^2)^2(g+g_r)}$$

for which Eq. (29) reduces to (for $|D|^2 \ll \omega^2$)

$$\omega^3 + ikg_r|D| = 0. \tag{34}$$

The solution of Eq. (34) has one negative imaginary root and two complex roots; one of the two complex roots describes damping of surface waves and the other, which has a positive imaginary part, leads to the instability of surface waves, with a growth rate given by

$$\mathrm{Im}\,\omega = \frac{\sqrt{3}}{2}(k_{1,2}g_r|D|)^{1/3}. \tag{35}$$

Equation (35) corresponds to a dissipative instability due to the radiation energy flux.

III. CONCLUSIONS

In this communication, we have considered that when the plane surface of a dusty plasma is charged it can be unstable to small perturbations for some particular conditions and this instability arises as a result of the negative pressure. The charged surface describes the propagation of (when under the influence of radiation, magnetic, and gravitational fields) the magnetic-radiative-capillary-gravity waves. Several limiting cases were discussed and we have shown that thermal energy flux leads to dissipation of surface waves, but the surface charge and thermal energy flux together can cause the dissipation instability.

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