Physics Letters A 372 (2008) 6757–6760

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

A quantum hydrodynamic model for multicomponent quantum magnetoplasma with Jeans term

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ARTICLE INFO

Article history:
Received 18 July 2008
Received in revised form 29 August 2008
Accepted 17 September 2008
Available online 25 September 2008
Communicated by F. Porcelli

PACS:
52.27.Lw
52.35.Fp
52.30.Ex
52.25.Xz

ABSTRACT

The effect of Jeans term in a multicomponent self-gravitating quantum magnetoplasma is investigated employing the quantum hydrodynamic (QHD) model. The effects of quantum Bohm potential and statistical terms as well as the ambient magnetic field are also investigated on both dust and ion dynamics driven waves in this Letter. We state the conditions that can drive the system unstable in the presence of Jeans term. The limiting cases are also presented. The present work may have relevance in the dense astrophysical environments where the self-gravitating effects are expected to play a pivotal role.

1. Introduction

Dusty plasmas have engendered a lot of interest in the past two decades because they have been observed in a variety of physical situations of interest. This includes planetary rings, Earth's ionosphere, cometary tails, laboratory experiments, in the fabrication of semiconductor devices, etc. [1–4]. The inclusion of dust dynamics or even the presence of stationary dust grains in the electron-ion plasma can lead to the emergence of new modes in the system. The dust grains are massive by comparison with the electron and ions and, therefore the temporal and spatial scales associated with the dust species can be very different.

When an electron–dust–ion (e–d–i) plasma is cooled to extremely low temperatures, the de-Broglie wavelength of the charge carriers becomes comparable to the dimension of the system under consideration. In such a situation, e–d–i plasma behaves like a Fermi gas and quantum mechanical effects are expected to play a significant role in the behavior of charged particles [5–10]. The thermal de-Broglie wavelength for jth species is $\lambda_{Bj} = \frac{h}{2\pi m_j v_{Tj}} = \frac{a_j}{\lambda_{Dj}}$, where $a_j (= \frac{h}{4q_j\sqrt{\pi m_j n_{j0}}})$ characterizes the Bohr radius per unit number density n_{j0}, λ_{Dj} is the Debye length, and q_j is the charge. For classical regimes, we assume $\lambda_{Bj} \ll \lambda_{Dj}$ and, therefore, consider them point like. On the contrary, for quantum regimes $\lambda_{Bj} \gg \lambda_{Dj}$ and the quantum effects, therefore, could no longer be ignored [8,9].

Manfredi [8] wrote a review article on the Schrödinger–Poisson and the Wigner–Poisson models in a collisionless quantum plasma. The quantum hydrodynamic model (QHD) is an extension of the classical fluid model in a plasma. It comprises of a set of equations that describe the transport of momentum and energy of the charged species. The departure from the classical model lies in the fact that an additional term, the so-called "Bohm potential", is introduced in the equation of motion of the charged particles. In the limit that the quantum effects go to zero, the classical fluid equation of motion is retrieved in accordance with the correspondence principle.

The initial attempts to study the propagation of waves in quantum dusty plasmas were made by Marklund et al. [11] and Shukla and Ali [12]. Since then, numerous papers have been written on the effect of quantum corrections on the propagation characteristics of linear and nonlinear waves in homogeneous and inhomogeneous dusty plasmas. Masood et al. [10] studied the linear and nonlinear properties of the dust–ion acoustic wave in a quantum plasma using the quantum hydrodynamic (QHD) model and found that the quantum electron Bohm potential significantly altered the dispersion characteristics of dust ion acoustic wave in the linear regime whereas it was observed to shrink the width of the soliton in the nonlinear regime. Khan et al. [13] studied the obliquely propagating dust ion acoustic wave with transverse perturbation in a quantum magnetoplasma and derived Zakharov–Kuznetsov (ZK) equation in the small amplitude limit. The authors found that the quantum corrections, angle of propagation, as well as the dust con-

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0375-9601/$ – see front matter
doi:10.1016/j.physleta.2008.09.034

centration modified the dust ion acoustic wave in both the linear and nonlinear regimes.

As mentioned earlier, the dust grains are massive by comparison with electrons and ions. This fact brings the gravitational force in the picture that needs to be taken into account along with the electromagnetic force particularly in dense astrophysical environments. The importance of the gravitational-like instabilities in a dusty plasma has been recognized as one of the processes responsible for the production of stars, planets and smaller bodies like comets and asteroids [14]. Recently, Shukla and Stenflo [15] investigated the Jeans instability [16] in multi-component unmagnetized quantum plasmas and found that the electron Bohm potential effects stabilized Jeans instability. The authors however ignored the quantum statistical effects due to pressure and focused only on the tunneling effect produced by the quantum diffraction term.

In the present work, we investigate the effect of the Jeans term in a self-gravitating multi-component quantum magnetoplasma and also incorporate the quantum statistical effects. In Section 2, we write down the basic set of equations for the system under consideration. In Section 3, we derive an expression for the dust driven wave in a quantum magnetoplasma and state the conditions that can drive the system unstable in the presence of Jeans term. We also discuss the limiting cases. In Section 4, we study the effects of Jeans term on the ion driven wave, state the conditions that can drive the system unstable and also present the limiting cases. In Section 5, we recapitulate the main findings of the Letter and conclude.

2. Basic set of equations

Consider a three component quantum dusty plasma comprising of electrons, ions and dust in the presence of an ambient magnetic field $\mathbf{B}_0$. Let the ambient magnetic field in the z-direction and assume that the propagation is in the x and z directions, respectively. We assume cold dust and ignore the quantum diffraction effect of dust (meaning that we treat dust classically). The governing equations, therefore read as follows:

The dust equation of motion is given by

$$m_d n_d \frac{d\mathbf{v}_d}{\partial t} = \varepsilon e z_d n_d \nabla\phi + \frac{\varepsilon e z_d n_d}{c}(\mathbf{v}_d \times \mathbf{B}) - m_d n_d \nabla\psi, \tag{1}$$

where ε is ± 1 for positive and negative dust respectively.

The ion momentum equation is

$$m_i n_i \frac{d\mathbf{v}_i}{\partial t} = -e n_i \nabla\phi + \frac{e n_i}{c}(\mathbf{v}_i \times \mathbf{B}) - \nabla p_i + \frac{\hbar^2}{4m_i}\nabla\left[\frac{\nabla^2\sqrt{n_i}}{\sqrt{n_i}}\right]. \tag{2}$$

The electron momentum equation is

$$m_e n_e \frac{d\mathbf{v}_e}{\partial t} = e n_e \nabla\phi - \frac{e n_e}{c}(\mathbf{v}_e \times \mathbf{B}) - \nabla p_e + \frac{\hbar^2}{4m_e}\nabla\left[\frac{\nabla^2\sqrt{n_e}}{\sqrt{n_e}}\right]. \tag{3}$$

The continuity equation for the jth species is

$$\frac{\partial n_l}{\partial t} + \nabla \,.\, (n_l \mathbf{v}_l) = 0. \tag{4}$$

Using the Poisson equation, the electrostatic and gravitational potentials (represented by ϕ and ψ respectively) are given by

$$\nabla^2\phi = 4\pi e(n_e + \varepsilon z_d n_d - n_i), \tag{5}$$

$$\nabla^2\psi = 4\pi G m_d n_d, \tag{6}$$

where n_e, n_i, and n_d are the electron, ion, and dust number densities and m_e, m_i, and m_d are the electron, ion, and dust masses respectively. $\hbar$ is the Planck's constant divided by 2π, $\mathbf{v}_d$, $\mathbf{v}_i$, and $\mathbf{v}_e$ are the dust, ion, and electron velocities and $\mathbf{B}$ represents the ambient magnetic field. c is the velocity of light. l represents ion, electron, and dust species and G is the gravitational constant. p_e and p_i represent quantum statistical contributions of electrons and ions which follow the pressure law [10] $p = \frac{1}{3}\frac{m_l v_{Fl}^2}{n_{l0}^2} n_l^3$.

3. Dust dynamics driven wave

3.1. Dispersion relation

For a dust dynamics driven wave, we ignore the inertia of electrons and ions, and assume that on a dust time scale the electrons and ions are aligned along the field. Simplifying Eqs. (1), (2), and (3) by ignoring the inertia of the lighter species (ions and electrons) by comparison with the dust mass, linearizing them and employing plane wave analysis (i.e., all perturbed quantities assume to vary as $e^{(i\mathbf{k}.\mathbf{r}-i\omega t)}$ and $\mathbf{k} = k_x\hat{x} + k_z\hat{z}$), we obtain

$$-i\omega\mathbf{v}_{d1} = \frac{i\mathbf{k}\varepsilon e z_d}{m_d}\phi_1 + \mathbf{v}_{d1} \times \omega_{cd} - i\mathbf{k}\psi_1, \tag{7}$$

$$n_{i1} = -\frac{e\phi n_{i0}}{m_i v_{Fi}^2(1+\gamma_i)}, \tag{8}$$

$$n_{e1} = \frac{e\phi n_{e0}}{m_e v_{Fe}^2(1+\gamma_e)}, \tag{9}$$

where $v_{Fj} = \sqrt{2K_B T_{Fj}/m_j}$ are the Fermi velocities of the jth species and $\gamma_j = \frac{\hbar^2 k^2}{4m_j^2 v_{Fj}^2}$ for $j = i, e$. The continuity equation after applying the plane wave equation for the electrostatic wave reads as follows

$$\frac{n_{d1}}{n_{d0}} = \frac{\mathbf{k}\cdot\mathbf{v}_{d1}}{\omega}. \tag{10}$$

Substituting the above expression in Eq. (7), we obtain the following expression for dust perturbed density

$$n_{d1} = \frac{-\frac{\varepsilon e z_d n_{d0}}{m_d\omega^2(1-\omega_{cd}^2/\omega^2)}}{1+\frac{\omega_{jd}^2}{\omega^2}\left[\frac{k_\perp^2}{k^2}\frac{1}{1-\omega_{cd}^2/\omega^2}+\frac{k_\parallel^2}{k^2}\right]}k_\perp^2 - \frac{\frac{\varepsilon e z_d n_{d0}}{m_d\omega^2}}{1+\frac{\omega_{jd}^2}{\omega^2}\left[\frac{k_\perp^2}{k^2}\frac{1}{1-\omega_{cd}^2/\omega^2}+\frac{k_\parallel^2}{k^2}\right]}k_\parallel^2. \tag{11}$$

Substituting Eqs. (8), (9), and (11) in Eq. (5), we get the following dispersion relation for a dust dynamics driven wave in a self gravitating quantum dusty magnetoplasma

$$1 + \frac{\omega_{pe}^2}{k^2}\frac{1}{f_e} + \frac{\omega_{pi}^2}{k^2}\frac{1}{f_i} - \frac{\omega_{pd}^2}{\omega^2}\left[\frac{\left(\frac{k_\perp^2}{k^2}\frac{1}{1-\omega_{cd}^2/\omega^2}+\frac{k_\parallel^2}{k^2}\right)}{1+\frac{\omega_{jd}^2}{\omega^2}\left(\frac{k_\perp^2}{k^2}\frac{1}{1-\omega_{cd}^2/\omega^2}+\frac{k_\parallel^2}{k^2}\right)}\right] = 0, \tag{12}$$

where

$$f_e = v_{Fe}^2(1+\gamma_e) \tag{13}$$

and

$$f_e = v_{Fi}^2(1+\gamma_i). \tag{14}$$

Note that the quantum contribution here is different from Shukla and Stenflo [15] as the quantum statistical effects of both electrons and ions are incorporated here. In conventional susceptibility form, the above equation can be expressed as

$$1 + \chi_e + \chi_i + \chi_d = 0, \tag{15}$$

where

$$\chi_e = \frac{\omega_{pe}^2}{k^2}\frac{1}{f_e},$$

$$\chi_i = \frac{\omega_{pi}^2}{k^2}\frac{1}{f_i} \quad \text{and} \quad \chi_d = -\frac{\omega_{pd}^2}{\omega^2}\left[\frac{\left(\frac{k_\perp^2}{k^2}\frac{1}{1-\omega_{cd}^2/\omega^2}+\frac{k_\parallel^2}{k^2}\right)}{1+\frac{\omega_{jd}^2}{\omega^2}\left(\frac{k_\perp^2}{k^2}\frac{1}{1-\omega_{cd}^2/\omega^2}+\frac{k_\parallel^2}{k^2}\right)}\right],$$

$\omega_{cd} = \frac{ez_dB}{m_dc}$ is the dust Larmor frequency and $\omega_{jd} = 4\pi Gm_dn_{d0}$ is the dust Jeans frequency.

Eq. (12) can be rewritten as

$$\omega = \sqrt{\frac{-A_1 \pm \sqrt{A_1^2 - 4A_2}}{2\alpha}}, \tag{16}$$

where $A_1 = -\alpha\omega_{cd}^2 + \alpha\omega_{jd}^2 - \omega_{pd}^2$, $A_2 = (\omega_{pd}^2 - \alpha\omega_{jd}^2)\alpha\omega_{cd}^2\frac{k_\parallel^2}{k^2}$, and $\alpha = 1 + \frac{\omega_{pe}^2}{k^2}\frac{1}{f_e} + \frac{\omega_{pi}^2}{k^2}\frac{1}{f_i}$.

If $\omega_{pd}^2 < \alpha\omega_{jd}^2$, then the system is Jeans stable. It is clear from the stability/instability condition that both the quantum statistical and Bohm potential terms stabilize the system. Conversely, the system can become Jeans unstable provided $A_1^2 < 4A_2$.

Limiting cases

Case 1 ($\omega_{cd} = 0, \omega_{jd} \neq 0$)

This corresponds to the case when the ambient magnetic field is absent, however, quantum statistical and Bohm potential as well as the Jeans term is present. Eq. (12) in this case reads as follows

$$1 + \frac{\omega_{pe}^2}{k^2}\frac{1}{f_e} + \frac{\omega_{pi}^2}{k^2}\frac{1}{f_i} - \frac{\omega_{pd}^2}{\omega^2 + \omega_{jd}^2} = 0. \tag{17}$$

The above equation can be rewritten as

$$\omega = \sqrt{-\omega_{jd}^2 + \frac{\omega_{pd}^2}{\alpha}} \tag{18}$$

which leads to the condition $\omega_{pd}^2 > \alpha\omega_{jd}^2$ for the system to be Jeans stable.

Case 2 ($\omega_{cd} = 0, \omega_{jd} = 0$)

This corresponds to the case when both the ambient magnetic field and Jeans term are absent, however, quantum statistical and Bohm potential are present. Eq. (12) in this case reads as follows

$$1 + \frac{\omega_{pe}^2}{k^2}\frac{1}{f_e} + \frac{\omega_{pi}^2}{k^2}\frac{1}{f_i} - \frac{\omega_{pd}^2}{\omega^2} = 0. \tag{19}$$

Case 3 ($\omega_{cd} = 0, \omega_{jd} = 0$, and $\gamma_j = 0$)

This corresponds to the case when only the quantum statistical terms are taken into account and everything else is ignored. Eq. (12) in this case reads as follows

$$1 + \frac{\omega_{pe}^2}{k^2v_{Fe}^2} + \frac{\omega_{pi}^2}{k^2v_{Fi}^2} - \frac{\omega_{pd}^2}{\omega^2} = 0. \tag{20}$$

As mentioned earlier that we include here the quantum statistical pressure term along with the quantum Bohm potential term. We have calculated both the contributions using the following parameters in a dense dusty plasma environment: $n_{eo} = 10^{24}$ cm^{-3}, $n_{do} = 10^{21}$ cm^{-3}, and $zd = 100$. It is found that barring a very restricted wave number range, the quantum statistical pressure dominates the quantum Bohm potential term by three or four orders of magnitude. The wavelengths that make the quantum Bohm potential term dominate the quantum statistical pressure term correspond to the wave numbers of the order of the inter-particle distance (given by $d = 1/n^{1/3}$) indicating the severe limitations of such an assumption. Thus, in general both the quantum statistical and Bohm potential terms should be incorporated to study the quantum behavior of the system.

4. Ion dynamics driven wave

In this case, we assume that the dust grains are immobile. Thus, the contribution of the dust comes only through the equilibrium quasi-neutrality. In the presence of electrostatic and self-gravitational fields, the dynamics of the ions and electrons is governed by

$$m_in_{i0}\frac{\partial \mathbf{v}_i}{\partial t} = -en_{i0}\nabla\phi + \frac{en_{i0}}{c}(\mathbf{v}_i \times \mathbf{B}_0) - m_in_{i0}\nabla\psi, \tag{21}$$

$$\frac{\partial n_{i1}}{\partial t} + \nabla . (n_{i0}\mathbf{v}_{i1}) = 0, \tag{22}$$

$$0 = en_{e0}\nabla\phi - v_{Fe}^2\nabla n_{e1} + \frac{\hbar^2}{4m_e}\nabla\nabla^2 n_{e1}. \tag{23}$$

Simplifying Eqs. (21), (22), and (23), and employing the plane wave analysis, we obtain the following dispersion relation for an ion dynamics driven wave in a self-gravitating quantum dusty magnetoplasma

$$1 + \frac{\omega_{pe}^2}{k^2}\frac{1}{f_e} - \frac{\omega_{pi}^2}{\omega^2}\left[\frac{\left(\frac{k_\perp^2}{k^2}\frac{1}{1-\omega_{ci}^2/\omega^2}+\frac{k_\parallel^2}{k^2}\right)}{1+\frac{\omega_{ji}^2}{\omega^2}\left(\frac{k_\perp^2}{k^2}\frac{1}{1-\omega_{ci}^2/\omega^2}+k_\parallel^2/k^2\right)}\right] = 0, \tag{24}$$

where $\omega_{ci} = \frac{eB}{m_ic}$ is the ion Larmor frequency and $\omega_{ji} = 4\pi Gm_dn_{d0}$ is the ion Jeans frequency.

The above equation can be rewritten as

$$\omega = \sqrt{\frac{-A_3 \pm \sqrt{A_3^2 - 4A_4}}{2\alpha}} \tag{25}$$

where $A_3 = -\alpha_1\omega_{ci}^2 + \alpha_1\omega_{ji}^2 - \omega_{pi}^2$, $A_4 = (\omega_{pi}^2 - \alpha_1\omega_{ji}^2)\alpha_1\omega_{ci}^2\frac{k_\parallel^2}{k^2}$, and $\alpha_1 = 1 + \frac{\omega_{pe}^2}{k^2}\frac{1}{f_e}$.

If $\omega_{pi}^2 < \alpha_1\omega_{ji}^2$, then the system is Jeans stable. It is again clear from the stability/instability condition that like the dust driven wave, both the quantum statistical and Bohm potential terms also stabilize the ion driven wave. Conversely, the system can become Jeans unstable provided $A_3^2 < 4A_4$.

Limiting cases

Case 1 ($\omega_{ci} = 0, \omega_{ji} \neq 0$)

This corresponds to the case when the ambient magnetic field is absent, however, quantum statistical and Bohm potential of electrons as well as the ion Jeans term is present. Eq. (24) in this case reads as follows

$$1 + \frac{\omega_{pe}^2}{k^2}\frac{1}{f_e} - \frac{\omega_{pi}^2}{\omega^2 + \omega_{ji}^2} = 0. \tag{26}$$

The above equation can be rewritten as

$$\omega = \sqrt{-\omega_{ji}^2 + \frac{\omega_{pi}^2}{\alpha}} \tag{27}$$

which leads to the condition $\omega_{pi}^2 > \alpha\omega_{ji}^2$ for the system to be Jeans stable.

Case 2 ($\omega_{ci} = 0, \omega_{ji} = 0$)

This corresponds to the case when both the ambient magnetic field and Jeans term are absent, however, quantum statistical and Bohm potential are present. Eq. (24) in this case reads as follows

$$1+\frac{\omega_{pe}^2}{k^2}\frac{1}{f_e}-\frac{\omega_{pi}^2}{\omega^2}=0. \tag{28}$$

Case 3 ($\omega_{ci}=0$, $\omega_{ji}=0$, *and* $\gamma_e=0$)

This corresponds to the case when only the quantum statistical terms are taken into account and everything else is ignored. Eq. (24) in this case reads as follows

$$1+\frac{\omega_{pe}^2}{k^2 v_{Fe}^2}-\frac{\omega_{pi}^2}{\omega^2}=0. \tag{29}$$

5. Conclusion

The effect of Jeans term in a multi-component self-gravitating quantum magnetoplasma comprising of electrons, ions, and dust is investigated employing the quantum hydrodynamic (QHD) model. It is found that the quantum Bohm potential and statistical effects stabilize both the dust as well as the ion dynamics driven wave. The limiting cases of both the quantum dust and ion waves are also presented. It is also found that in general the quantum statistical term dominates the quantum Bohm potential term and both the effects should be incorporated to study the quantum behavior of the system. The present investigation may have relevance in the dense astrophysical environments where the self-gravitation effects are expected to dominate.

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