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Jeans instability in a magneto-radiative dusty plasma

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Abstract. The importance of thermal radiation on the Jeans instability is discussed for a magnetized dusty plasma with gravitational effects. The one-fluid MHD equations are developed by assuming that the entropy of each subsystem of plasma is conserved, when the temperature of the plasma species is non-relativistic. The dispersion relation in this case shows that thermal radiation helps to stabilize the Jeans instability. It is shown that the plasma is stable in a certain range of wavelengths. The magnetic field stabilizes the Jeans instability when the wave propagates across the magnetic field. However, for oblique propagation it is seen that the magnetic field does not stabilize the Jeans instability.

In recent years a great deal of attention has been paid to the phenomena of collective processes in dusty plasmas (see, for example, Verheest (2000); Shukla and Mamun (2002); Mendis (2002); Tsytovich et al. (2002); Morfill et al. (2003); Cramer and Verheest (2005); Tsintsadze et al. (1996, 2006a,b); Mamun and Shukla (2001)), because dusty plasmas are now known to be rather common in space—ranging from interplanetary, interstellar to intergalactic media and such plasmas can play a role in the formation of stars, galaxies, planetary systems, quasar accretion, planetary rings, tails of comets, etc. In cosmic conditions, self-gravitation of the medium is important in understanding the formation of dust clouds and equilibrium structures, as well as for the study of collective processes.

The properties of the Jeans aperiodic instabilities in fully (Tsintsadze et al. 2000) and partially ionized dusty magnetoplasmas (Verheest et al. 2000) have been considered, by assuming that the dusty plasma constituents are electrons, ions and extremely massive charged dust grains. Recently, a model was put forward, by Tsintsadze et al. (2000), where all species of dusty plasma were taken to be hot, a charge neutrality was maintained in the equilibrium state and all species were under the influence of both gravitational and electrostatic forces. It was shown that the propagation of spiral electrostatic gravitational waves in a rotating dusty plasma leads to the oscillatory Jeans instability. Jeans instability has also been investigated in papers by Shukla and Stenflo (2006a,b) where a self-gravitating dusty plasma was considered and later the same problem was considered in a quantum dusty plasma.

In this paper we consider Jeans instability in a dusty magnetized gravitating plasma in which thermal radiations are also present. For such a plasma, a set of MHD equations is constructed. We have derived a dispersion relation which is subsequently analyzed for different cases and then the main conclusion of our results is given.

The theory of thermal radiation in one-component systems was developed quite some time back and is referred to in, for example, Chandrasekhar (1957), Landau and Lifshitz (1998) and Zeldovich and Raiser (1967). We note here, however, that originally Planck considered radiation as a collection of particles (photons) in a vacuum where it became known as black-body radiation and the radiation frequency could be expressed through the standard dispersion relation $\omega = ck$. It was shown later (Tsintsadze 1995, Tsintsadze et al. 1996) that the behavior of photons in a plasma is radically different from the behavior in a vacuum. As plasma particles perform oscillatory motion in the field of electromagnetic waves, the radiation field is affected. The oscillation of electrons in an isotropic homogenous plasma causes the index of refraction to depend on the radiation frequency which is not close to unity in a dense plasma, and is given by

$$\omega = (\omega_{\mathrm{pe}}^2 + k^2c^2)^{1/2},$$

where

$$\omega_{\mathrm{pe}} = \left(\frac{4\pi n_0 e^2}{m_{\mathrm{e}}}\right)^{1/2}$$

is the Langmuir frequency, and n_{e} and m_{e} are the density and the rest mass of electrons, respectively. Plancks' theory is violated when $l_{0\alpha} = \hbar\omega_{\mathrm{p}\alpha}/k_{\mathrm{B}}T_\alpha \geqslant 1$ (where k_{B} is the Boltzmann constant and T_α is the temperature of the different species). As an example, in the early prestellar period of the evolution of the universe the parameter $l_{0\alpha} \approx 1$. However, it is obvious that the parameter $l_{0\alpha}$ is different in different astrophysical objects. If we apply this ratio to dusty plasmas in the interstellar medium, the ratio becomes much less than unity. Therefore, for such dusty clouds we can use Planck's theory, and in this case the energy density can be written for the thermal radiation as

$$u_{\mathrm{r}\alpha} = \frac{k_{\mathrm{B}}^4\pi^2}{45(\hbar c)^3}T_\alpha^4 = 7.57\times 10^{-15}T_\alpha^4 \text{ erg cm}^{-3}$$

and the radiation pressure as

$$p_{\mathrm{r}\alpha} = \frac{u_{\mathrm{r}\alpha}}{3}.$$

Thus, the equilibrium radiation can be considered from a thermodynamic point of view as a perfect gas with a specific heat energy $\gamma = 4/3$.

Tsintsadze (1995) derived the electromagnetohydrodynamic equations for a plasma and photon gas, where the latter was in thermal equilibrium with the plasma. In the equation of motion there were two pressure terms—the first being the usual plasma pressure $p_\alpha = k_{\mathrm{B}}n_\alpha T_\alpha$ and the second being the radiation pressure of each plasma species. In general, we follow here the formalism developed in the aforementioned reference. In order to establish the importance of radiation in determining the local properties of a plasma, we consider the ratio of the plasma internal energy density $\varepsilon_{\mathrm{in}}$ to the radiation energy density u_{r}, for all plasma species, when the temperature of plasma particles is non-relativistic,

$$\eta_\alpha = \frac{\varepsilon_{\mathrm{in}}^\alpha}{u_{\mathrm{r}\alpha}}$$

where $\varepsilon_{\mathrm{in}}^\alpha = \frac{3}{2} n_\alpha k_{\mathrm{B}} T_\alpha$. In general, this ratio can have any value for different astrophysical objects; however, radiation is important when $\eta_\alpha < 1$, which, as will be shown later, holds for the case under consideration.

Based on the condition of quasineutrality and the fact that the temperature of the dust grains is normally less than the temperature of the electrons and ions, it is possible to neglect the gas dynamics and radiation pressures of the dust grains. Now we derive magnetohydrodynamic equations for the dusty plasmas, under the effect of thermal radiation and a gravitating field. For this we write the set of fluid equations for each species separately (Tsintsadze et al. 2007):

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \mathbf{v}_\alpha) = 0, \tag{1}$$

$$m_\alpha n_\alpha \frac{d\mathbf{v}_{\alpha j}}{dt} = e_\alpha n_\alpha \left[\mathbf{E} + \frac{1}{c}(\mathbf{v}_\alpha \times \mathbf{B}) \right] - \nabla p_\alpha^{\mathrm{t}} - \mathbf{R}_\alpha - m_\alpha n_\alpha \nabla \Psi, \tag{2}$$

$$\nabla \cdot \mathbf{B} = \mathbf{0}. \tag{3}$$

The quasineutrality condition is

$$\sum_\alpha e_\alpha n_\alpha = 0$$

and Poisson's equation for the gravitational potential is

$$\nabla^2 \Psi = 4\pi G m_{\mathrm{d}} n_{\mathrm{d}}, \tag{4}$$

where G is the gravitational constant and n_α, $\mathbf{v}_\alpha$, p_α^{t} and $\mathbf{R}_\alpha$ are the number density, velocity, total pressure (which is the sum of the usual gas dynamics pressure p_α and the radiation pressure $p_{\mathrm{r}\alpha}$) and the frictional force of each plasma species, respectively. $\mathbf{R}_\alpha$ is given as

$$\mathbf{R}_\alpha = m_\alpha n_\alpha \sum v_{\alpha\beta} (\mathbf{v}_\alpha - \mathbf{v}_\beta), \tag{5}$$

where $v_{\alpha\beta}$ is the elastic collisional frequency of particle α with β. To obtain the one-fluid MHD equations we suppose that collisions between particles are frequent. In this case it follows from (5) that the mean velocities of the different species of the plasma must be almost equal ($\mathbf{v}_{\mathrm{i}} \approx \mathbf{v}_{\mathrm{e}} \approx \mathbf{v}_{\mathrm{d}}$) (see Ginsburg 1970), i.e. in this case the frictional forces can be balanced by each other, which also implies that $\partial/\partial t \ll v_{\alpha\beta}$. Furthermore, by assuming the electrons and ions to be inertialess, summing up (2) and taking into account the conservation of momentum, i.e. $\sum_\alpha R_\alpha = 0$, we obtain an ideal MHD set of equations,

$$\frac{\partial n_{\mathrm{d}}}{\partial t} + \nabla \cdot (n_{\mathrm{d}} \mathbf{v}_{\mathrm{d}}) = 0, \tag{6}$$

$$\rho_{\mathrm{d}} \frac{d\mathbf{v}_{\mathrm{d}}}{dt} = +\frac{1}{c}(\mathbf{J} \times \mathbf{B}) - \nabla p^{\mathrm{t}} - \rho_{\mathrm{d}} \nabla \Psi, \tag{7}$$

where $\rho_{\rm d} = m_{\rm d} n_{\rm d}$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}), \tag{8}$$

$$\nabla^2 \Psi = 4\pi G m_{\rm d} n_{\rm d}, \tag{9}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{10}$$

$$\mathbf{J} = \frac{c}{4\pi}(\nabla \times \mathbf{B}), \tag{11}$$

$$p^{\rm t} = p_{\rm e} + \frac{\alpha_{\rm r}}{3} T_{\rm e}^4 + p_{\rm i} + \frac{\alpha_{\rm r}}{3} T_{\rm i}^4, \tag{12}$$

where $\alpha_r = \pi^2 k_{\rm B}^4 / 15(\hbar c)^3$ is the radiation constant. As assumed, (7) and (12) take into account only the electron and ion pressures. However, in order to obtain a closed set of equations we shall express $p^{\rm t}$ in terms of the dust grain density. As shown in Avinash and Shukla (1994), the condition $\mathbf{v}_{\rm i} \backsimeq \mathbf{v}_{\rm e} \backsimeq \mathbf{v}_{\rm d}$ leads to a very important relationship between densities of different species,

$$\frac{n_{\rm e}}{n_{0\rm e}} = \frac{n_{\rm i}}{n_{0\rm i}} = \frac{n_{\rm d}}{n_{0\rm d}}. \tag{13}$$

We know that for an ideal gas with constant specific heat the entropy is conserved. Since in our consideration there are three subsystems, i.e. electrons, ions and dust grains, we suppose that the entropy of each subsystem is conserved. This in turn leads to Poisson's adiabatic relation among the density (n), temperature (T) and pressure (p) of different species undergoing adiabatic expansion or compression, which for the case of non-relativistic temperatures is expressed as

$$\frac{p_\alpha}{n_\alpha^{5/3}} = c_\alpha. \tag{14}$$

Using (13) and (14) we can express $p_{\rm e}$, $p_{\rm re}$ and $p_{\rm i}$, $p_{\rm ri}$ through the density of dust grains in the following manner:

$$p_{\rm e} = p_{0\rm e}\left(\frac{n_{\rm d}}{n_{0\rm d}}\right)^{5/3}, \quad p_{\rm i} = p_{0\rm i}\left(\frac{n_{\rm d}}{n_{0\rm d}}\right)^{5/3}, \tag{15}$$

where $p_{0\alpha} = n_{0\alpha} T_{0\alpha}$, $n_{\rm d}$ is the number density of dust grains, $n_{0\alpha}$ and $T_{0\alpha}$ are the equilibrium number densities and temperatures, respectively, and

$$p_e^{\rm r} = \frac{\alpha_{\rm r}}{3} T_{0e}^4 \left(\frac{n_{\rm d}}{n_{0\rm i}}\right)^{8/3}, \quad p_{\rm i}^{\rm r} = \frac{\alpha_r}{3} T_{0\rm i}^4 \left(\frac{n_{\rm d}}{n_{0\rm d}}\right)^{8/3}. \tag{16}$$

Substituting (15) and (16) into (7), we obtain (6)–(11) as a closed set of MHD equations.

Now we investigate the propagation of waves by taking the magnetic field into account along with thermal radiation and gravitational effects. We suppose that the external magnetic field is directed along the z-axis while the gravitational force along x-axis. We linearize the set of equations (6)–(11) and look for a plane wave solution in the form that all fluctuating quantities are proportional to $e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}$ and finally we obtain the linear dispersion relation

$$\omega^4 - \omega^2\{k^2(V_{\rm A}^2 + u_{\rm j}^2)\} + k^4(V_{\rm A}^2 u_{\rm j}^2 \cos^2\theta) = 0, \tag{17}$$

where $V_{\rm A} = B_0/(4\pi\rho_{\rm d})^{1/2}$ is the Alfvén velocity of the grains and $u_{\rm j} = (V_{\rm d}^2 - (\Omega^2/k^2))^{1/2}$ is the Jeans velocity $V_{\rm d}^2 = V_{\rm sd}^2 + V_{\rm rd}^2$ where

$$V_{\rm sd} = \left(\frac{5}{3}\frac{n_{0\rm e}k_{\rm B}T_{0\rm e} + n_{0\rm i}k_{\rm Bi}T_{0\rm i}}{\rho_{\rm d}}\right)^{1/2}$$

is the dust acoustic velocity,

$$V_{\rm rd} = \left(\frac{8}{9}\frac{\alpha_{\rm r}}{\rho_{\rm d}}(T_{0\rm e}^4 + T_{0\rm i}^4)\right)^{1/2}$$

is the dust radiation velocity, $\Omega = (4\pi G\rho_{\rm d})^{1/2}$ is the Jeans frequency, and $V_{\rm d}$ is the total dust velocity.

We begin by considering the case when the magnetic field is absent, i.e. $B_0 = 0$, then from (17) we obtain Jeans equation with radiation taken into account, i.e.

$$\omega^2 = k^2 u_j^2 = k^2\left(V_{\rm d}^2 - \frac{\Omega^2}{k^2}\right). \tag{18}$$

As an example, we chose plasma parameters which are typical for photodissociation regions which separate HII regions from dense molecular clouds (Mamun and Shukla 2001). The parameters here are $n_{\rm i0} = 2\times10^{-3}$ cm^{-3}, $n_{\rm d0} = 5\times10^{-7}$ cm^{-3}, $n_{\rm e} = 10^{-3}$ cm^{-3}, $Z_{\rm d} = 2.0\times10^3$, $m_{\rm d} = 10^{-11}$ g, $B_0 = 10$ μG, $T_{\rm e} = 30$ K, $T_{\rm i} = 10$ K and $T_{\rm d} = 1$ K. For these parameter values $\eta_{\rm e} \approx 10^{-9}$, $\eta_{\rm i} \approx 5\times10^{-7}$ and $\eta_{\rm d} \approx 10^{-8}$ which shows that the conditions for the radiation being important are fulfilled for each subsystem of the plasma that we consider. For an instability we thus have the condition $V_{\rm d}^2 - \Omega^2\lambda^2/(2\pi)^2 < 0$ or $\lambda > 2\pi V_{\rm d}/\Omega$ where $\lambda = 2\pi/k$. Using the above parameters we obtain $\lambda > 6.3\times10^3$ au. The MHD condition $kV_{\rm A} \ll \omega_{\rm cd}$ in turn leads to the result that the wavelength $\lambda > \lambda_{\rm m}$, where $\lambda_{\rm m} = 2\pi V_{\rm d}/\omega_{\rm cd} = 13.4$ au, i.e. for the above the parameters.We note here that MHD is valid if $\lambda \gg \lambda_{\rm m}$. The above inequalities of length show that for the region of length $\lambda_{\rm m} \ll \lambda < \lambda_{\rm j}$ the plasma is stable. In the expression for $V_{\rm d}^2$ the second term due to radiation is in this case much larger than the sound velocity, which leads to an increase in the wavelength of the Jeans instability,

$$\lambda \geqslant \lambda_{\rm r} = \frac{2\pi V_{\rm d}}{\Omega} \gg \lambda_{\rm s} = \frac{2\pi V_{\rm s}}{\Omega}.$$

Here $\lambda_{\rm r}$ and $\lambda_{\rm s}$ are the wavelengths associated with the radiation and dust acoustic velocities, respectively. This leads to the result that thermal radiation can very effectively stabilize the Jeans instability.

Now we consider the influence of the external magnetic field on the Jeans instability. For this purpose we solve (17) and obtain two solutions

$$\omega_{1,2}^2 = \frac{k^2(V_{\rm A}^2 + u_{\rm j}^2)}{2} \pm \frac{k^2}{2}((V_{\rm A}^2 + u_{\rm j}^2)^2 - 4V_{\rm A}^2u_{\rm j}^2\cos^2\theta)^{1/2}. \tag{19}$$

Furthermore, from (19) it follows that if the wave propagates along the magnetic field ($\theta = 0$), we obtain two independently propagating waves, namely the Alfvén and Jeans waves. If $\theta = \pi/2$, one of the roots vanishes and the other is given by

$$\omega^2 = k^2(V_{\rm A}^2 + u_{\rm j}^2). \tag{20}$$

In this case the magnetic field can stabilize the Jeans instability if

$$V_{\mathrm{A}}^2 > |u_{\mathrm{j}}^2| = \left| V_{\mathrm{d}}^2 - \frac{\Omega^2}{k^2} \right|. \tag{21}$$

For oblique propagation of waves, we will show that the magnetic field cannot influence the Jeans instability. Indeed, the product of the roots (19) is given by

$$\omega_1^2 \cdot \omega_2^2 = k^4 V_{\mathrm{A}}^2 u_{\mathrm{j}}^2 \cos^2\theta. \tag{22}$$

Now from (22) follows that, if $u_{\mathrm{j}}^2 < 0$, one of the root squares becomes negative, which means that this root has only one imaginary part and in this case the Jeans instability always develops.

In this paper we have investigated the effect of gravitation and thermal radiation on the Jeans instability for a magnetized dusty plasma. It has been shown that in the case when the magnetic field is absent radiation has a stabilizing effect on the instability. In the case when there is a finite ambient magnetic field it too stabilizes the Jeans instability when the MHD wave is perpendicularly propagating. However, for the case of oblique propagation no stabilizing effect is found to be associated with the magnetic field.

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