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Wake potential in a self-gravitating dusty plasma

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Abstract

Using a test particle approach, the dynamical wake potential has been examined in a homogeneous self-gravitating dusty plasma. The periodic oscillatory potential might lead to an alternative approach to the Jeans instability for the formation of dust agglomeration leading to gravitational collapse of the self-gravitating systems.

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Jeans gravitational instability is known to be the best model for the gravitational collapse of a self-gravitating dusty plasma system [1–5]. In most astrophysical systems, micron/submicron sized dust grains occur in addition to neutral gas and electrons and protons. Dust particles can acquire sufficient electric charge due to plasma currents, photoionization, secondary emissions, etc. [6]. Thus, a dusty plasma is formed, where the gravitational force of attraction among the heavier grains is comparable to the electromagnetic force in the plasma. It is now known that a multicomponent dusty plasma supports a great variety of electrostatic and electromagnetic waves. These waves can become unstable when some sort of free energy sources are available in the plasma. Several authors have recently studied the self-gravitating effects in dusty plasmas which lead to Jeans-type instability [1–5,7,8]. This instability is believed to play the vital role in the formation of astrophysical objects like stars, galaxies, etc.

In the cosmic environments, dust particles of different sizes and shapes are quite common [9,10]. The observed infrared and submillimeter radiation are attributed to the thermal emission from dust clouds heated by shock waves, the universal ultra-violet radiation, or stellar radiation. Dust particles of different size and shape are assumed to be formed by coagulation of smaller particulates in partially or fully ionized plasmas by some attractive forces [11], the details of which are not fully understood. However, it is thought that inelastic adhesive, and collective interaction between micron-sized dust particles give rise to kilometer-sized bodies, which are known as the planetesimals. Results from a microgravity aggregation experiment [12] flown onboard the spaceshuttle revealed the structure and growth of dust agglomeration. Specifically, Blum et al. [12] reported that a thermally aggregating swarm of dust particles evolves very rapidly and forms unexpected open-structure agglomerates.

On the other hand, generation of oscillatory wake potential is possible when the charged dust grains somehow acquire drift velocities on account of sudden local change of energy due to magnetic reconnection, shock waves, lightning, or some transient heavenly events. Wake potentials are believed to be the best mechanism for the formation of dust crystals and dust coagulation in the laboratory conditions [6,13]. In the astrophysical self-gravitating dusty plasmas, the dynamical wake potential may be formed due to the slow motion of the dust component and may lead to the coagulation/agglomeration of the grains of the self-gravitating plasmas leading to the formation of structures. In this Letter, we study the formation of the oscillating wake potential taking a dust

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grain as a test particle in a self-gravitating dusty plasma in the presence of a continuous dust drift. Such an attractive wake potential might be the origin of grain attraction giving rise to dust agglomeration.

We consider a self-gravitating dusty plasma consisting of electrons, ions, and charged dust grains in the background of neutral atoms. At equilibrium, the gravitational force among the grains is balanced by the electromagnetic force acting on them. The dusty plasma is assumed to satisfy the quasineutrality condition $n_{i0} = n_{e0} + Z_d n_{d0}$ where Z_d is the number of electronic charge residing on the grains and $n_{\alpha 0}$ with $\alpha = e, i, d$ is the number density of the α species. For mathematical simplicity, all grains are assumed to have the same radius a_0 with constant charge $(-Z_d e)$. The inter-grain distance r_0 is assumed to be smaller than the plasma Debye length $(a_0 \ll r_0 \ll \lambda_D)$ where λ_D is the effective Debye length of the self-gravitating dusty plasma.

In the presence of a low-frequency electrostatic wave, the electric field of the wave perturbation is $\mathbf{E} = -\nabla\phi$ where ϕ is the wave potential. Because of magnetic reconnection, shock waves, lightning, or any heavenly events, the electrons and ions are assumed to be hot and they satisfy the Boltzmann distributions

$$n_j = n_{j0} e^{\pm e\phi/T_j}, \quad j = e, i, \tag{1}$$

where T_j is the temperature in energy units of the jth species and e is the magnitude of the charge of an electron. Thus, the electron/ion susceptibilities are given by

$$\chi_{e,i}(\omega, \mathbf{k}) = \frac{1}{k^2 \lambda_{De,Di}^2}, \tag{2}$$

where $\lambda_{De,Di}^2 = T_{e,i}/4\pi n_{e0,i0} e^2$.

The dynamics of the cold dust grains in the presence of any electrostatic wave $(\omega, \mathbf{k})$, Lorentz force, and the gravitational force is governed by the dust momentum balance equation

$$\frac{\partial \mathbf{v}_d}{\partial t} = -\frac{q_d}{m_d}\nabla\phi - \nabla\psi - \nu_d \mathbf{v}_d, \tag{3}$$

the dust continuity equation

$$\frac{\partial n_d}{\partial t} + \nabla \cdot (n_d \mathbf{v}_d) = 0, \tag{4}$$

and the Poisson's equation in terms of the gravitational potential ψ and the dust mass density $m_d n_d$

$$\nabla^2 \psi = 4\pi G m_d n_d, \tag{5}$$

where n_d and $\mathbf{v}_d$ are the number density and the velocity of the dust grains, respectively, m_d is the mass of the dust grains, and G is the gravitational constant. In Eq. (3), ν_d is the average collision frequency of dust grains with neutral particles.

Solving Eqs. (3)–(5), we obtain the dust density perturbation for the cold and unmagnetized dust grains as

$$n_{d1} = -\frac{\chi_d k^2}{4\pi q_d}\phi, \tag{6}$$

where

$$\chi_d = -\frac{\omega_{pd}^2}{\omega(\omega + i\nu_d) + \omega_{Jd}^2}, \tag{7}$$

with $\omega_{Jd} = \sqrt{4\pi G m_d n_{d0}}$, the Jeans frequency.

The dielectric function of the unmagnetized self-gravitating dusty plasma in the presence of any electrostatic dust mode is given by

$$\epsilon(\omega, \mathbf{k}) = 1 + \frac{1}{k^2\lambda_D^2} - \frac{\omega_{pd}^2}{\omega(\omega + i\nu_d) + \omega_{Jd}^2}, \tag{8}$$

where $\lambda_D^{-2} = \lambda_{De}^{-2} + \lambda_{Di}^{-2}$. Eq. (8) represents the mixed Jeans-dust-acoustic wave in the unmagnetized self-gravitating dusty plasma.

Since we consider the charged dust grains to be cold, the dust-neutral collision frequency ν_d will be small. Thus, we can ignore the small damping of the possible dynamical wake potential behind the dust grains. Again, in the presence of the low-frequency electrostatic perturbation, the dust charge oscillates at the frequency of the perturbation. Consequently, the dust-charge-fluctuation causes a non-Landau-type damping known as the Tromsø damping of the electrostatic field [14]. The dust-charge-fluctuation damping also causes a modification of the dipole-like far-field potential [15]. However, outside the shielding cloud, the far-field potential falls off rapidly and the oscillatory wake potential might only dominate and act on the self-gravitating dust grains causing a possible attraction among themselves. We are interested to focus on the formation of the wake potential in the self-gravitating dusty plasma.

The electrostatic potential around an isolated test charged particulate in the presence of the electrostatic mode in a plasma is given by [16,17]

$$\phi(\mathbf{x},t)=\frac{q_t}{2\pi^2}\int\frac{\delta(\omega-\mathbf{k}\cdot\mathbf{v}_t)}{k^2\epsilon(\mathbf{k},\omega)}\exp[i\mathbf{k}\cdot\mathbf{r}]\,d\mathbf{k}\,d\omega, \tag{9}$$

where $\mathbf{r}=\mathbf{x}-\mathbf{v}_t t$, $\mathbf{v}_t$ is the velocity vector of a test charged particulate, and q_t is its charge.

Neglecting the dust-neutral collisions and the dust-charge-fluctuation effects, we may then rewrite Eq. (8) as

$$\epsilon(\omega,\mathbf{k})=\frac{1+k^2\lambda_D^2}{k^2\lambda_D^2}\left[1-\frac{\omega_k^2}{\omega^2+\omega_{Jd}^2}\right], \tag{10}$$

where

$$\omega_k^2=\frac{k^2C_d^2}{1+k^2\lambda_D^2},\quad C_d^2=\omega_{pd}^2\lambda_D^2. \tag{11}$$

The inverse of the dielectric function, $\epsilon(\omega,\mathbf{k})$ can be written as

$$\frac{1}{\epsilon(\omega,\mathbf{k})}=\frac{k^2\lambda_D^2}{1+k^2\lambda_D^2}\left[1+\frac{\omega_k^2}{\omega^2-(\omega_k^2-\omega_{Jd}^2)}\right]. \tag{12}$$

Substituting Eq. (12) into Eq. (9) and following the standard mathematical techniques [17,18], we obtain the total electrostatic potential as

$$\Phi=\Phi_{\mathrm{I}}+\Phi_{\mathrm{II}}, \tag{13}$$

where

$$\Phi_{\mathrm{I}}=\left(\frac{q_t}{r}\right)\exp\left(-\frac{r}{\lambda_D}\right), \tag{14}$$

is the Yukawa-type static Debye–Hückel screening potential with the effective screening length λ_D. We now use (ρ,θ,z) as the cylindrical coordinates of $\mathbf{r}$, where $r=(\rho^2+z^2)^{1/2}$. The additional part of the potential involving the collective effects between the electrostatic wave and a test dust ion, after evaluating θ- and ω-integrations, is

$$\begin{aligned}\Phi_{\mathrm{II}}(\rho,z,t)&=\frac{q_t}{\pi}\int\frac{k^2\delta(\omega-k_\parallel v_t)C_d^2\lambda_D^2J_0(k_\perp\rho)e^{ik_\parallel\xi}}{(1+k^2\lambda_D^2)^2[\omega^2-k^2C_d^2/(1+k^2\lambda_D^2)+\omega_{Jd}^2]}k_\perp\,dk_\perp\,d\omega\,dk_\parallel\\&=\frac{q_tC_d^2k_D^2}{\pi}\int\frac{k^2J_0(k_\perp\rho)e^{ik_\parallel\xi}k_\perp\,dk_\perp\,dk_\parallel}{(k^2+k_D^2)[(k_\parallel^2v_t^2+\omega_{Jd}^2)(k^2+k_D^2)-k^2C_d^2/\lambda_D^2]},\end{aligned} \tag{15}$$

where $\mathbf{v}_t\parallel\hat{z}$ is assumed and $k_D=1/\lambda_D$ and $\xi=z-v_t t$.

We now introduce a dimensionless notation $\mathbf{K}=\mathbf{k}\lambda_D$. Then, the above equation reduces to

$$\Phi_{\mathrm{II}}(\rho,\xi)=\frac{q_tM^{-2}}{\pi\lambda_D}\int\frac{K^2J_0(K_\perp\rho/\lambda_D)e^{iK_\parallel\xi/\lambda_D}K_\perp\,dK_\perp\,dK_\parallel}{(1+K^2)(K_\parallel^2+K_0^2)(K_\parallel^2-K_1^2)}, \tag{16}$$

where

$$K_{0,1}^2=\pm\frac{\{K_\perp^2+1+M^{-2}(f-1)\}}{2}+\left[\frac{\{K_\perp^2+1+M^{-2}(f-1)\}^2}{4}+\left[M^{-2}K_\perp^2-M^{-2}\left(K_\perp^2+1\right)f\right]\right]^{1/2}, \tag{17}$$

with $f=\omega_{Jd}^2/\omega_{pd}^2$ and $M=v_t/C_d$.

Taking $K_\perp<1$ and arbitrary M and carrying out the $K_\parallel$-integration, we obtain

$$\Phi_{\mathrm{II}}'(\rho',\xi')\equiv\frac{\Phi_{\mathrm{II}}(\rho',\xi')}{q_t/\lambda_D}=-2M^{-2}\int_0^\infty\frac{K_1J_0(K_\perp\rho')}{(K_1^2+K_0^2)(1+K_1^2)}\sin(K_1\xi')K_\perp\,dK_\perp, \tag{18}$$

where

$$K_{0,1}^2=\pm\frac{\{1+M^{-2}(f-1)\}}{2}+\left[\frac{\{1+M^{-2}(f-1)\}^2}{4}+\left[M^{-2}\left(K_\perp^2-f\right)\right]\right]^{1/2}. \tag{19}$$

We now consider two general parameter regimes of the dusty plasma.

First, we consider the absence of the self-gravitational effect, $f = \omega_{Jd}^2/\omega_{pd}^2 \approx 0$. For $M = v_t/C_d \gg 1$ and $f \ll 1$,

$$K_0^2 = 1 - M^{-2}, \qquad K_1^2 \simeq M^{-2} K_\perp^2, \tag{20}$$

and the wake potential turns out to be

$$\Phi_{\mathrm{II}}(\rho = 0, \xi) = \frac{2q_t}{1 - M^{-2}} \frac{\cos(|\xi|/L)}{|\xi|}, \tag{21}$$

where the effective length is $L \simeq M\lambda_D$. The condition for the excitation of this wake potential was first shown to be $v_t \gg C_d$ by Nambu et al. [19].

However, for the parameter regimes of the self-gravitating plasmas where $M^2 \ll 1$ and taking into account the finite self-gravitational effect with $\omega_{Jd}^2 < \omega_{pd}^2$, we can show the existence of the oscillatory wake potential as follows. We consider slow motion of the relatively massive dust particles, so that $v_t \ll C_d$ (i.e., $M \ll 1$). For the usual parameters of the self-gravitating plasmas [20], $\omega_{Jd} \ll \omega_{pd}$ (i.e., $f \ll 1$). Then, from Eq. (19), we obtain

$$K_0^2 \approx 1 - M^{-2}, \qquad K_1^2 \approx f.$$

The normalized wake potential turns out to be, from Eq. (18)

$$\frac{\Phi_{\mathrm{II}}(\xi)}{q_t/\lambda_D} = -\frac{M^{-2}\sqrt{f}}{1 - M^{-2}} \sin\left(\frac{\xi}{L_s}\right), \tag{22}$$

where the effective length $L_s = \lambda_D/\sqrt{f} = \lambda_D \omega_{pd}/\omega_{Jd}$ and $v_t \ll C_d$. Since the Jeans-dust-acoustic mode [cf. Eq. (8)] is a slow phase velocity wave, the slowly moving test particle ($v_t < C_d$) may only resonate with the wave producing the wake potential. However, in absence of the gravitational effect, the wake potential can be excited [cf. Eq. (21)] only with $v_t > C_d$.

Thus, periodic attractive centers are formed, where negatively charged dust grains may be accumulated forming dust coagulation/agglomeration with spacing of the order of $\lambda_D \omega_{pd}/\omega_{Jd}$. In laboratory conditions, the periodicity that is the lattice spacing of the dust crystal is of the order of λ_D. In the self-gravitating systems, the spacing is seen to increase by the factor $\omega_{pd}/\omega_{Jd} \gg 1$.

We have presented the possibility of attraction among the same polarity charged grains due to the collective interaction involving very low-frequency electrostatic wave in a self-gravitating dusty plasma. The slow motion of a test dust particle is in resonance with this extremely low-phase velocity wave producing the oscillatory wake potential which focuses the other dust grains to be accumulated.

The wake potential model presented here provides a qualitative possibility of how the same polarity charged grains might form agglomerates as seeds for gravitational collapse of a self-gravitating dusty plasma. This provides an alternative approach to the pure Jeans instability forming structures in self-gravitating dusty plasma systems. The other factors like plasma density inhomogeneity, collisions, ion drift and ambient magnetic field and its nonuniformity, etc. might also be relevant questions. These are the subjects of future investigations and the work in these lines is in progress.

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