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Drift ion acoustic shock waves in an inhomogeneous two-dimensional quantum magnetoplasma

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Linear and nonlinear propagation characteristics of drift ion acoustic waves are investigated in an inhomogeneous quantum plasma with neutrals in the background employing the quantum hydrodynamics (QHD) model. In this regard, a quantum Kadomtsev–Petviashvili–Burgers (KPB) equation is derived for the first time. It is shown that the ion acoustic wave couples with the drift wave if the parallel motion of ions is taken into account. Discrepancies in the earlier works on drift solitons and shocks in inhomogeneous plasmas are also pointed out and a correct theoretical framework is presented to study the one-dimensional as well as the two-dimensional propagation of shock waves in an inhomogeneous quantum plasma. Furthermore, the solution of KPB equation is presented using the tangent hyperbolic (tanh) method. The variation of the shock profile with the quantum Bohm potential, collision frequency, and ratio of drift to shock velocity in the comoving frame, v_*/u, are also investigated. It is found that increasing the number density and collision frequency enhances the strength of the shock. It is also shown that the fast drift shock (i.e., $v_*/u > 0$) increases, whereas the slow drift shock (i.e., $v_*/u < 0$) decreases the strength of the shock. The relevance of the present investigation with regard to dense astrophysical environments is also pointed out. © *2009 American Institute of Physics*. [DOI: 10.1063/1.3109663]

I. INTRODUCTION

The field of quantum plasmas has engendered a lot of interest in the plasma physics community owing to its wide domain of applicability. Numerous investigations have been carried out in dense astrophysical environments[1,2] in dusty plasmas[3,4] (such as white dwarfs and neutron stars), in microelectronic devices,[5] in intense laser beam produced plasmas,[6] in nonlinear optics,[7,8] etc., to understand the quantum effects on the behavior of linear and nonlinear wave propagations in these systems. The quantum plasmas are characterized by high densities and low temperatures in sharp contrast to the low densities and high temperatures that constitute the classical plasmas. When the plasma is cooled to extremely low temperatures, the de Broglie wavelength of the charge carriers becomes comparable to the dimension of the system under consideration. In such a situation, the plasma behaves like a Fermi gas and quantum mechanical effects are expected to play a significant role in the behavior of charged particles.[9–13] The thermal de Broglie wavelength for jth species is $\lambda_{Bj}=h/2\pi m_j v_{Tj}=a_j/\lambda_{Dj}$, where a_j $(=h/4q_j\sqrt{\pi m_j n_{j0}})$ characterizes the Bohr radius per unit number density n_{j0}, λ_{Dj} is the Debye length, and q_j is the charge. For classical regimes, $\lambda_{Bj} \ll \lambda_{Dj}$, while for quantum regimes $\lambda_{Bj} \gg \lambda_{Dj}$ and the quantum effects, therefore, could no longer be ignored.

The approaches that are commonly used for quantum plasmas are the Schrödinger–Poisson, the Wigner–Poisson, and the Dirac–Maxwell which describe the statistical and hydrodynamic behaviors of the plasma particles at quantum scales. These models are the quantum equivalent of fluid and kinetic models of the classical plasma physics. Manfredi[14] wrote a review article on the Schrödinger–Poisson and the Wigner–Poisson models in a collisionless quantum plasma. The quantum hydrodynamics (QHD) model is an extension of the classical fluid model in a plasma. The basic set of QHD equations describes the momentum and energy transport of the charged species. The departure from the classical model lies in the fact that an additional term, the so-called Bohm potential, is introduced in the equation of motion of the charged particles. In the limit that the quantum effects go to zero, the classical fluid equation of motion is retrieved in accordance with the correspondence principle.

The QHD model has also been employed to study the propagation of linear and nonlinear waves in inhomogeneous quantum plasmas. El-Taibany and Wadati[11] studied the dynamics of nonlinear quantum dust acoustic wave in a nonuniform quantum dusty plasma and found that the formation of solitons exhibited a dependence on a critical value of plasma parameters unlike a homogeneous plasma. Shukla and Stenflo[15] found new drift modes in nonuniform quantum magnetoplasmas and observed that the electron drift wave frequency was significantly modified by the electron Bohm potential term. Haque and Mahmood[16] studied the linear and nonlinear drift waves in inhomogeneous quantum plasmas with neutrals in the background. The authors found that the quantum corrections appreciably modifed the drift solitons and shocks in quantum magnetoplasmas. Recently, Haque and Saleem[17] proposed that monopolar and dipolar quantum vortices could appear in uniform dense plasmas.

It is known that shock waves can be excited in a dissipative nonlinear medium. There can be numerous dissipative processes in a plasma. The important ones are Landau damping, kinematic viscosity among the plasma constituents, as well as the collisions between charged particles and neutrals present in the system. However, when a medium has both dispersive and dissipative properties, the propagation of small amplitude perturbations can then be adequately described by Korteweg–de Vries–Burgers (KdVB) equation. The dissipative Burgers term in the nonlinear KdVB equation arises by taking into account the kinematic viscosity among the plasma constituents.[18–20] When the wave breaking due to nonlinearity is balanced by the combined effect of dispersion and dissipation, a monotonic or oscillatory dispersive shock wave is generated in a plasma.[20–23] It is well known that transverse perturbations would always exist in the higher-dimensional system. The presence of transverse perturbation introduces an anisotropy in the system which modifies the wave structure and the stability of the system.[24,25] In this paper, for the first time, the shock propagation in the presence of parallel perturbation is considered in an inhomogeneous quantum magnetoplasma. The parallel perturbation assumes the role of transverse perturbation in the present case and this point would be elaborated later in the paper.

In this paper, nonlinear coupling of drift and ion acoustic waves in an inhomogeneous quantum magnetoplasma is investigated. In this regard, a quantum Kadomtsev–Petviashvili–Burgers (KPB) equation is derived and its solution is presented using the tanh method. The dissipative effect appears due to the collisions of the ions with the neutrals in the background. The manuscript is organized as follows: In Sec. II, we present the basic set of nonlinear equations for the system under consideration. In Sec. III, linear dispersion relation of the quantum ion acoustic wave in an inhomogeneous quantum magnetoplasma is presented in different limits and discussed. In Sec. IV, nonlinear KPB equation is derived and its solution is presented using the tanh method. In Sec. V, stability analysis of the quantum KPB equation is presented. In Sec. VI, results are presented and discussed. Finally, in Sec. VII, the conclusion of the current investigation is presented.

II. SET OF NONLINEAR EQUATIONS

Consider an inhomogeneous quantum magnetoplasma composed of ions and electrons with neutrals in the background. The equilibrium magnetic field is in the z direction, whereas the density and temperature gradients are assumed to be in the x direction. The phase velocity of the wave is assumed to be $v_{Fi} \ll \omega/k \ll v_{Fe}$ (v_{Fi} and v_{Fe} are the ion and electron Fermi velocities, respectively). We, therefore, ignore the quantum statistical and Bohm potential contributions of ions. Using the QHD model, we can write down the governing equations as follows.

The equation of motion for electrons,

$$m_e n_e(\partial_t + \mathbf{v}_e \,.\, \nabla)\mathbf{v}_e = -e n_e\left(\mathbf{E} + \frac{1}{c}\mathbf{v}_e \times \mathbf{B}_0\right) - \nabla p_e + \frac{\hbar^2 n_e}{2m_e}\nabla\left(\frac{\nabla^2\sqrt{n_e}}{\sqrt{n_e}}\right), \quad (1)$$

where $\mathbf{E} = -\nabla\phi$ is the electrostatic field (ϕ is the electrostatic potential) and n_e, m_e, and e are the electron density, mass, and charge, respectively. In Eq. (1) two quantum effects, i.e., quantum diffraction and quantum statistics, are also included. The quantum diffraction effects appear due to the wave nature of particle in a quantum plasma which is taken into account by the term proportional to $\hbar^2$ also known as Bohm potential. However, quantum pressure is obtained by using the quantum statistics which takes into account the fermionic nature of electrons. For three-dimensional Fermi gas, electron pressure is defined as[26]

$$p_e = \frac{\hbar^2(3\pi^2)^{2/3}}{5m_e} n_e^{5/3}. \quad (2)$$

The parallel component of Eq. (1) for inertialess electrons gives

$$e\partial_z\phi - \frac{1}{n_e}\partial_z p_e + \frac{\hbar^2}{2m_e}\partial_z\left(\frac{\nabla^2\sqrt{n_e}}{\sqrt{n_e}}\right) = 0. \quad (3)$$

Using the pressure in Eq. (3) and expanding it by Taylor series and integrating Eq. (3), after using the boundary conditions $n_e = n_{e_0}$ and $\phi = 0$ at $z \to \pm\infty$, we have

$$\left(\frac{\tilde{n}_e}{n_{eo}}\right) = \frac{3e\phi}{2k_B T_{Fe}} + \frac{3e^2\phi^2}{8k_B^2 T_{Fe}^2} + \frac{3\hbar^2}{4m_e k_B T_{Fe}}\left(\frac{\nabla^2\sqrt{n_e}}{\sqrt{n_e}}\right), \quad (4)$$

where T_{Fe} is the electron Fermi temperature, k_B is the Boltzmann constant, $\tilde{n}$ is the perturbed electron density, while n_{e_0} is the equilibrium density and $\tilde{n}_e < n_{e_0}$. The electron Fermi temperature and density are related by $T_{Fe} = (\hbar^2/2m_e k_B) \times (3\pi^2)^{2/3} n_{e_0}^{2/3}$.

Using the Taylor expansion for the Bohm potential term in Eq. (4) and backsubstituting into Eq. (4), we obtain

$$\left(\frac{\tilde{n}_e}{n_{e_0}}\right) = \frac{3e\phi}{2k_B T_{Fe}} + \frac{3e^2\phi^2}{8k_B^2 T_{Fe}^2} + \frac{9e\hbar^2}{16m_e k_B^2 T_{Fe}^2}\nabla^2\phi. \quad (5)$$

The equation of motion for ions is

$$m_i n_i(\partial_t + \mathbf{v}_i \,.\, \nabla)\mathbf{v}_i = e n_i\left(\mathbf{E} + \frac{1}{c}\mathbf{v}_i \times \mathbf{B}_0\right) - m_i n_i \nu_{in}\mathbf{v}_i, \quad (6)$$

where ν_{in} is the collisional frequency between ions and neutrals. The quantum force acting on the ions is small due to the large mass of ions as compared to the electrons and thus neglected in Eq. (6). The perpendicular component of the velocity from Eq. (6) can be written as

$$\mathbf{v}_{i\perp} = \frac{c}{B_0}(\hat{\mathbf{z}} \times \nabla\phi) - \frac{c\nu_{in}}{\Omega_{ci}B_0}\nabla_\perp\phi - \frac{c}{B_0\Omega_{ci}}\partial_t\nabla_\perp\phi, \quad (7)$$

where the usual limit $\partial_t \ll \Omega_{ci}$ ($\Omega_{ci} = eB_0/cm_i$ is the ion cyclotron frequency) has been used for low frequency drift waves and ($\perp$) means perpendicular to the magnetic field $\mathbf{B}_0$.

The parallel component of velocity from Eq. (6) can be written as

$$\hat{A}v_{iz} = -\frac{e}{m_i}\partial_z\phi, \quad (8)$$

where $\hat{A}$ is an operator defined as

$$\hat{A} = (\partial_t + \nu_{in} + \mathbf{v}_E . \nabla_\perp + v_{iz}\partial_z).$$

Using the Poisson equation

$$\nabla^2\phi = -4\pi e(n_i - n_e), \tag{9}$$

the perturbed ion number density from Eq. (9) is given by

$$\left(\frac{\tilde{n}_i}{n_{i_0}}\right) = \frac{3e\phi}{2k_B T_{Fe}} + \frac{3e^2\phi^2}{8k_B^2 T_{Fe}^2} + \frac{9e\hbar^2}{16m_e k_B^2 T_{Fe}^2}\nabla^2\phi - \frac{1}{4\pi e n_0}\nabla^2\phi. \tag{10}$$

Here, $n_{i_0}=n_{e_0}=n_0$ is the equilibrium plasma density.

The ion continuity equation is

$$\partial_t n_i + \nabla . (n_i \mathbf{v}_i) = 0. \tag{11}$$

Using Eqs. (7), (8), and (10) in Eq. (11), applying operator and assuming $\partial_x < \partial_z < \partial_y$, and finally multiplying the whole equation by $k_B T_{Fe}/e$, we have

$$\hat{A}\left\{\tfrac{3}{2}\partial_t\phi + a_1\partial_t\phi^2 - \lambda_{Fe}^2\partial_t\partial_y^2\phi + H^2\partial_t\partial_y^2\phi - \varrho_s^2\partial_t\partial_y^2\phi - \nu_{in}\varrho_s^2\partial_y^2\phi + \tfrac{3}{2}v_*\partial_y\phi - D_1\partial_y\phi^2\right\} - c_s^2\partial_z^2\phi = 0, \tag{12}$$

where $a_1=3e/8k_B T_{F_e}$, $\lambda_{F_e}=\sqrt{k_B T_{F_e}/4\pi e^2 n_0}$ is the electron Fermi wavelength, $H=\sqrt{9\hbar^2/16m_e k_B T_{F_e}}$ is quantum parameter, $v_*=(-2ck_B T_{F_e}/3eB_0)\kappa_n$ is the drift velocity, $\kappa_n=|d_x \ln n_0|$ is the inverse of the density scale length, $D_1=D_2[\kappa_n-\kappa_{T_{F_e}}]$, $D_2=3c/4B_0$, $\kappa_{T_{F_e}}=|d_x \ln T_{F_e}|$ is the temperature gradient, $c_s=\sqrt{k_B T_{Fe}/m_i}$ is the quantum ion acoustic speed, and $\varrho_s=\sqrt{k_B T_{Fe}/m_i\Omega_{ci}^2}$ is the ion Larmor radius at electron temperature in quantum plasma.

It is emphasized here that the term $a_1\partial_t\phi^2$ does not appear in Eq. (12) of Ref. 16 because the authors linearize the electron density and throw away the higher order contribution very early in their calculations; that later introduces nonlinearity in the system. Note that the coefficient of nonlinearity in Ref. 16 contains only the effects due to inhomogeneity and ignoring the density inhomogeneity would make the nonlinearity vanish in their work. A similar error was committed in Ref. 27 where the author discussed drift solitons and shocks in classical plasmas. The nonlinearity should not disappear with the disappearance of inhomogeneity as the KdV and KP equations are derived in homogeneous plasmas where the source of the nonlinearity is the convective derivative term. It is, therefore, imperative that the procedure given in this paper be followed to arrive at the correct equation. It is also worth mentioning that the nonlinearity coefficients obtained in Refs. 16 and 27 are in fact very small contributions as these contain the inhomogeneity term (which is generally considered small) multiplied by $\partial_y\phi^2$, making it even smaller. We note here that the relation between number density and Fermi temperature inhomogeneities of Ref. 16 is incorrect and thus their following results are incorrect. Here $|\kappa_n|>|\kappa_{T_F}|$ because the Fermi temperature is related to the density as $T_{Fe}=(\hbar^2/2m_e k_B)(3\pi^2)^{2/3}n_{e_0}^{2/3}$ which implies that $|\kappa_n|=\frac{3}{2}|\kappa_{T_F}|$.

Case 1: Consider the case when collisions dominate, i.e., $\partial_t \ll \nu_{in}$, then Eq. (12) takes the form

$$\frac{3}{2}\partial_t\phi + a_1\partial_t\phi^2 - \lambda_{Fe}^2\partial_t\partial_y^2\phi + H^2\partial_t\partial_y^2\phi - \varrho_s^2\partial_t\partial_y^2\phi - \nu_{in}\varrho_s^2\partial_y^2\phi + \frac{3}{2}v_*\partial_y\phi - D_1\partial_y\phi^2 - \frac{c_s^2}{\nu_{in}}\partial_z^2\phi = 0. \tag{13}$$

The last term of Eq. (13) can be ignored because ν_{in} (which is large in this case) appears in the denominator and also owing to the fact that the perturbation in the parallel direction is weak. Equation (13) therefore reduces to the standard KdVB form in a comoving frame of reference by taking $\xi=k(y-ut)$ as shown in Ref. 16. However, it must be pointed out that the coefficient A of Eq. (21) is different from Eq. (15) of Ref. 16 owing to the reasons elaborated in detail earlier in this paper.

Case 2: When $\partial_t \gg \nu_{in}$, Eq. (12) takes the form

$$\tfrac{3}{2}\partial_t^2\phi + a_1\partial_t^2\phi^2 - \lambda_{Fe}^2\partial_t^2\partial_y^2\phi + H^2\partial_t^2\partial_y^2\phi - \varrho_s^2\partial_t^2\partial_y^2\phi - \nu_{in}\varrho_s^2\partial_t\partial_y^2\phi + \tfrac{3}{2}v_*\partial_t\partial_y\phi - D_1\partial_t\partial_y\phi^2 - c_s^2\partial_z^2\phi = 0. \tag{14}$$

Equation (14) can be transformed into a form analogous to the KPB equation derived for a homogeneous classical plasma.

III. LINEAR ANALYSIS

On linearizing Eq. (14) and assuming the perturbation $\propto[ik_y y + ik_z z - \iota\omega t]$, the dispersion relation for the coupled quantum ion acoustic and drift mode reads as

$$\omega = \frac{(\omega_* - (2/3)i\nu_{in}k_y^2\varrho_s^2) \pm \sqrt{(\omega_* - (2/3)i\nu_{in}k_y^2\varrho_s^2)^2 + (8/3)c_s^2k_z^2S}}{2S}, \tag{15}$$

where $S=[1+(2/3)(\lambda_{Fe}^2+\varrho_s^2-H^2)k_y^2]$, $\omega_*=v_*k_y$ is the drift frequency, ω is the wave frequency, k_y, k_z are the wave numbers, and $S>0$ must hold. Note that the contribution of the acoustic wave is absent in Eq. (14) of Ref. 16 because the authors ignored the parallel motion of ions along the ambient magnetic field in order to arrive at the KdVB equation. It is also worth mentioning that in Ref. 16 the linear dispersion reduces to an oscillation if the transverse perturbation is ig-

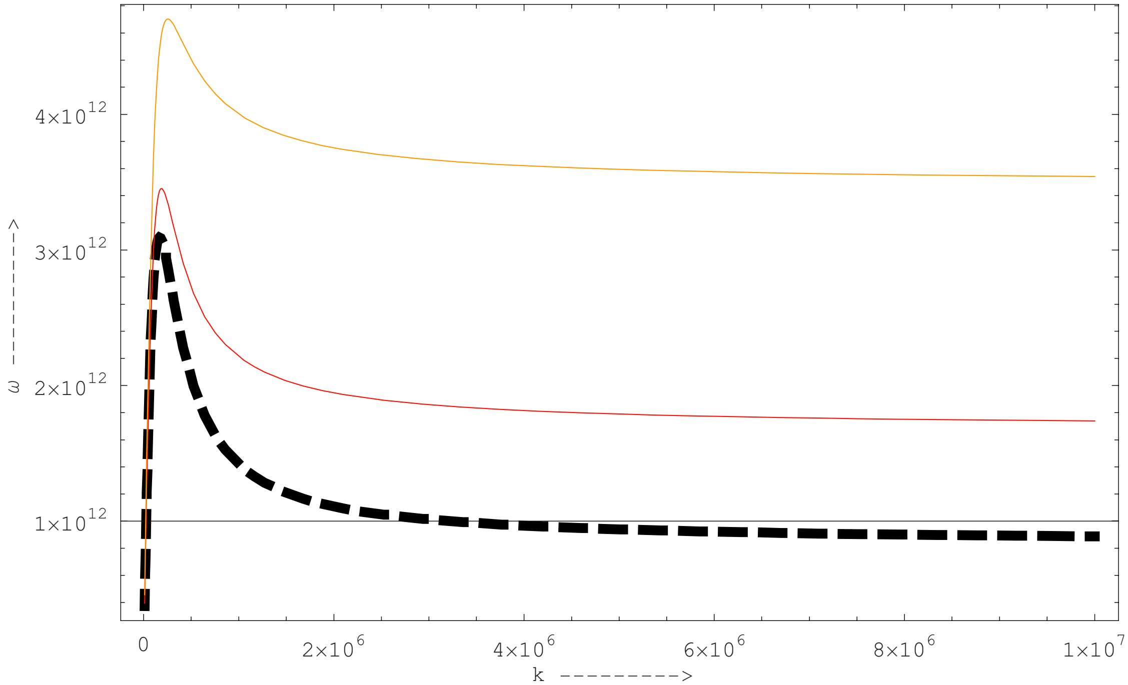

FIG. 1. (Color online) Variation of the real frequency of the coupled-drift ion acoustic wave with the obliqueness angle θ. Other parameters are $n_0=1.9\times10^{27}$ cm^{-3}, $B_0=10^9$ G, and $v_*/u=0.4$.

nored and goes to zero in the absence of background density gradient. However, here Eq. (15) does not vanish in any of these limits because the acoustic mode survives. Separating the real and imaginary parts (only the upper sign is taken into account as the lower sign yields nonphysical results) of Eq. (15), we obtain

$$\omega_r=\frac{\omega_*+\sqrt{\left(\omega_*^2+\frac{8}{3}c_s^2k_z^2S\right)}}{2S}, \tag{16}$$

$$\omega_i=\frac{\left(\frac{2}{3}\nu_{in}\varrho_s^2k_y^2+\frac{\frac{2}{3}\omega_*\nu_{in}\varrho_s^2k_y^2}{\sqrt{\left(\omega_*^2+\frac{8}{3}c_s^2k_z^2S\right)}}\right)}{2S}, \tag{17}$$

where $k_y=k\cos\theta$, $k_z=k\sin\theta$. ω_r and ω_i represent the real and imaginary parts of Eq. (15). We see from our Eqs. (16) and (17) and also from Figs. 1 and 2 that for small k the numerator dominates as it is proportional to k so correspondingly ω (real and imaginary) increases but for large k the denominator increases; as a result ω (real and imaginary) decreases. Figures 1 and 2 show the variation of real and imaginary frequencies (related to damping) of the coupled quantum ion acoustic and drift mode as a function of obliqueness. It is found that the real frequencies enhance, whereas the imaginary frequencies decrease with the increase in the obliqueness angle θ. By increasing the number density, quantum effects become greater; as a result real and imaginary frequencies increases with small k, while for large k they do not show any significant variation. The variation of the real frequency and the damping rate could similarly be found by varying the other plasma parameters.

In the absence of collisions between ions and neutral particles, i.e., $\nu_{in}=0$, Eq. (15) can be written as

$$\omega=\frac{\omega_*\pm\sqrt{\omega_*+(8/3)c_s^2k_z^2S}}{2S}. \tag{18}$$

In the absence of inhomogeneity in density and temperature, i.e., $\omega_*=0$, Eq. (18) can be written as

$$\omega=\frac{\pm(\sqrt{2/3})c_sk_z}{\sqrt{S}}. \tag{19}$$

It is evident from Eqs. (15)–(19) that quantum corrections affect the wave dispersion.

IV. DERIVATION OF KPB

In this section, KPB equation is derived for an ion acoustic drift shock wave in an inhomogeneous quantum magnetoplasma. It should be mentioned that no such work has been done to date. In order to find the localized solution, let us choose a coordinate ξ in the moving frame such that $\xi=\chi(y+\alpha z-ut)$, where χ is a nonlinear wave number, u is the velocity of the nonlinear structure moving with the frame, and α is the angle between wavefront normal and xy plane. Equation (14) in the transformed frame can be written as

$$\partial_\xi[a_2\partial_\xi\phi+a_3\partial_\xi\phi^2+a_4^2\chi^2\partial_\xi^3\phi+a_5\chi\partial_\xi^2\phi]+\alpha^2a_6\partial_\xi^2\phi=0, \tag{20}$$

where $a_2=(3/2)(1-v_*/u)$, $a_3=(a_1+D_1/u)$, $a_4=-(\lambda_{Fe}^2+\varrho_s^2-H^2)$, $a_5=\nu_{in}\varrho_s^2/u$, and $a_6=-c_s^2/u^2$. Equation (20) can be simplified further to obtain

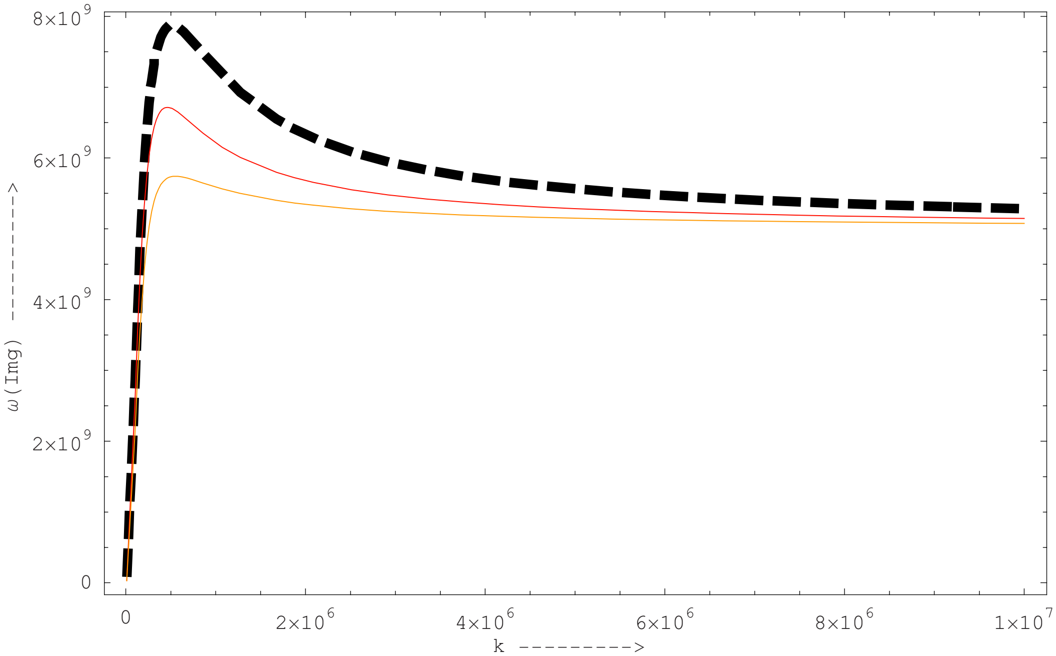

FIG. 2. (Color online) Variation of the imaginary frequency of the coupled-drift ion acoustic wave with the obliqueness angle θ. Other parameters are $n_0 = 1.9\times 10^{27}$ cm^{-3}, $B_0 = 10^9$ G, and $v_*/u = 0.4$.

$$\partial_\xi[\partial_\xi\phi + A\partial_\xi\phi^2 + B\chi^2\partial_\xi^3\phi + C\chi\partial_\xi^2\phi] + \alpha^2 D\partial_\xi^2\phi = 0, \quad (21)$$

where $A = a_3/a_2$, $B = a_4/a_2$, $C = a_5/a_2$, and $D = a_6/a_2$. Equation (21) is analogous to the KPB equation derived for the homogeneous plasmas (see, for instance, the papers by Xue[24,25]). The above equation admits shock solutions. It is interesting to note that contrary to its homogeneous counterpart, the KPB equation in an inhomogeneous plasma is predominantly in the transverse direction and weak in the parallel direction. This difference arises due to the drift approximation used in solving the inhomogeneous plasmas that assumes a stronger perturbation in the perpendicular direction by comparison with the parallel motion along the ambient magnetic field. Note that putting the dissipative coefficient C would reduce Eq. (21) to KP equation which admits soliton solutions. Similarly, putting the dispersive coefficients B and D equal to zero leads to Burgers equation. Finally, if the weak parallel perturbation is dropped, KdVB equation for a drift wave is retrieved. The ion acoustic contribution would vanish by dropping the parallel motion of ions as mentioned in the text earlier. However, it must be pointed out that the coefficient A of Eq. (21) is different from Eq. (15) of Ref. 16 owing to the reasons elaborated in detail earlier in this paper. It should be emphasized that the results of the KPB equation are shown and discussed here rather than the limiting cases unlike the earlier investigations in one dimension where only the limiting cases were discussed.[16,27]

There are a number of methods to solve the nonlinear partial differential equations (NLPDEs), for instance, inverse scattering method,[28] Hirota bilinear formalism,[29] Backlund transformation,[30] tanh,[31] etc. However, when the partial differential equation in a system is formed by the combined effect of dispersion and dissipation, the most convenient and efficient method to solve the NLPDE is the tanh method.[32]

Therefore, using the tanh method, we arrive at the following solution of the Eq. (21):

$$\phi(y,z,t) = \frac{-25B + 3C^2 - 25\alpha^2 BD}{50AB} + \frac{3C^2}{25AB}\tanh\left[\frac{C}{10B}(y + \alpha z - ut)\right] - \frac{3C^2}{50AB}\tanh^2\left[\frac{C}{10B}(y + \alpha z - ut)\right]. \quad (22)$$

V. STABILITY OF KPB

In order to check the stability of the KPB, we proceed as follows: Integrate Eq. (21) twice to obtain

$$Ay^2 + B\chi^2\partial_\xi^2 y + C\chi\partial_\xi y + (1 + \alpha^2 D)y = 0, \quad (23)$$

where $y = \varphi_1$. Appropriate boundary conditions are imposed, namely, $y \to 0$, $dy/d\xi \to 0$, $d^2y/d\xi^2 \to 0$ at $\xi \to -\infty$ to investigate the asymptotic behavior of Eq. (23) by linearizing it with respect to y.[33] Simplifying, we get

$$B\chi^2\partial_\xi^2 y + C\chi\partial_\xi y - (1 + \alpha^2 D)y = 0. \quad (24)$$

The solutions of Eq. (24) are proportional to $\exp(Wy)$, where

$$W = 5\left[1 \mp \sqrt{1 - \frac{4B(1 - \alpha^2 D)}{C^2}}\right].$$

It should be mentioned that in W, the signs of the coefficients B and D have been used. It should be noted that the quantum corrections appear in the dispersion coefficient B. Also note that there will be a stable shock if $4B(1 - \alpha^2 D)/C^2 \le 1$; else there will be an oscillatory shock.

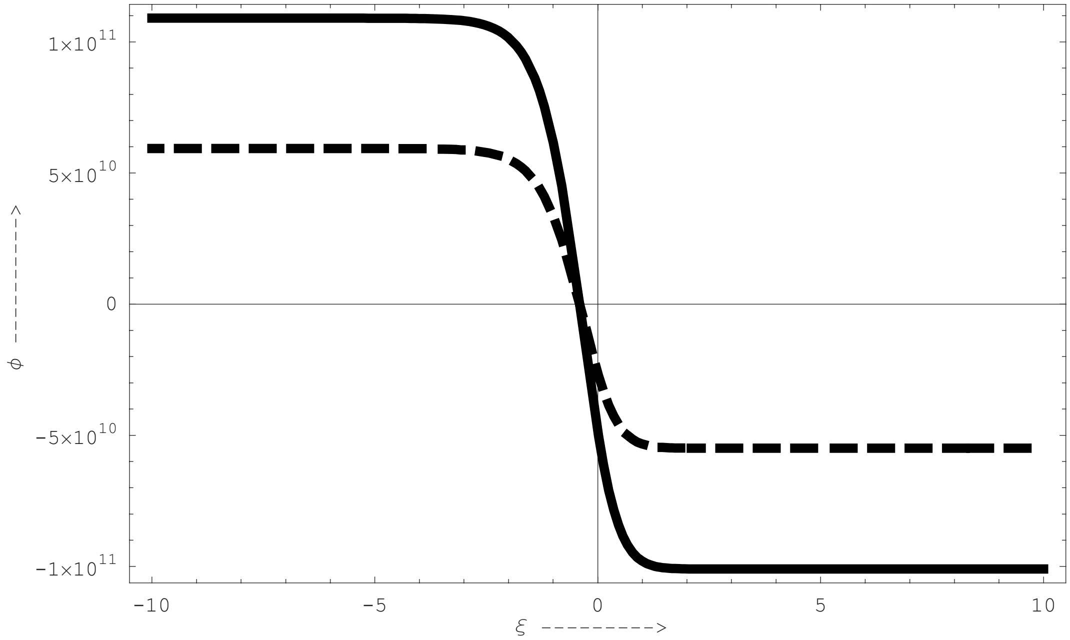

FIG. 3. Variation of the electrostatic potential ϕ for increasing number density, i.e., $n_0 = 1.9 \times 10^{27}$ cm^{-3} (dashed line) and $n_0 = 3 \times 10^{27}$ cm^{-3} (solid line). Other parameters are $B_0 = 10^9$ G, $v_*/u = 0.8$, $\nu_{in} = 10^{10}$ s^{-1}, and $\alpha = 10^{-4}$.

VI. RESULTS AND DISCUSSION

In this section, numerical investigation of the dependence of wave potential of the quantum drift ion acoustic shock wave on the quantum Bohm potential, collision frequency, and ratio of drift to shock velocity in the comoving frame, i.e., v_*/u is explored. In high density plasmas found in dense astrophysical objects like neutron stars and white dwarfs, the plasma densities are enormous and quantum effects may be important. For illustration, typical parameters are chosen which are representative of the plasma in dense astrophysical bodies, i.e., $n_0 \sim 10^{26} - 10^{29}$ cm^{-3} and $B_0 \sim 10^9 - 10^{14}$ G.[34,35] Graphical analysis of ion acoustic drift shock profile is presented by plotting the potential ϕ against different parameters affecting the wave. In Fig. 3, the variation of the wave potential with density is shown. It is found that increasing the number density enhances the strength of the shock. It should be noted that the expression of the quantum Bohm potential involves density and therefore the variation in density indirectly represents the change in the wave potential with the quantum Bohm potential term.

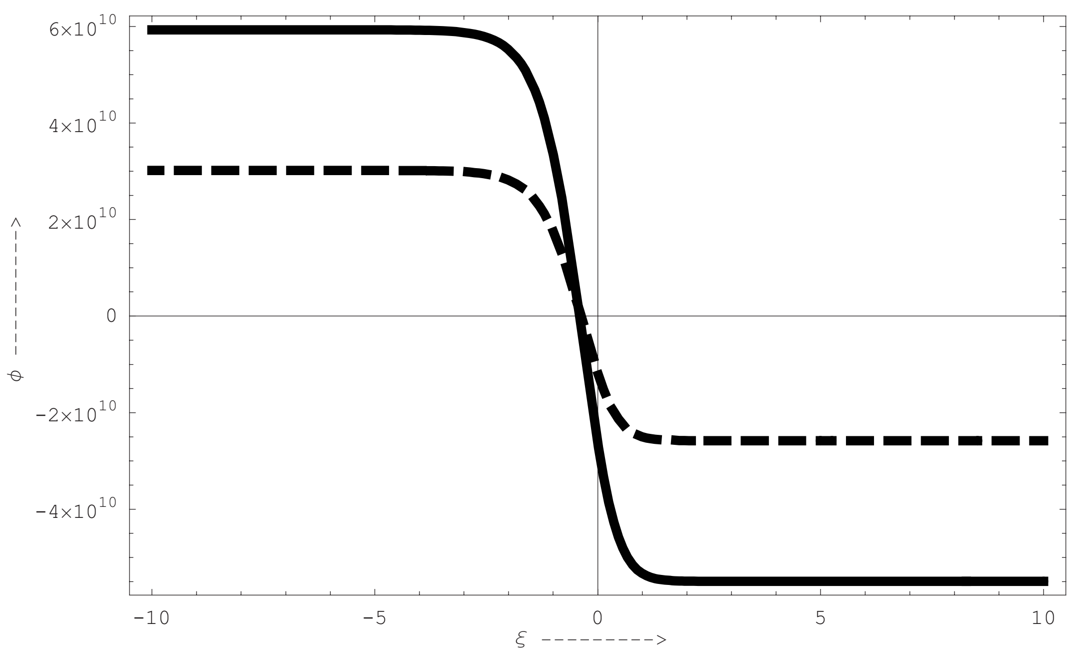

FIG. 4. Variation of the electrostatic potential ϕ for increasing collision frequency, i.e., $\nu_{in} = 7 \times 10^9$ s^{-1} (dashed line) and $\nu_{in} = 10^{10}$ s^{-1} (solid line). Other parameters are $n_0 = 1.9 \times 10^{27}$ cm^{-3}, $B_0 = 10^9$ G, $v_*/u = 0.8$, and $\alpha = 10^{-4}$.

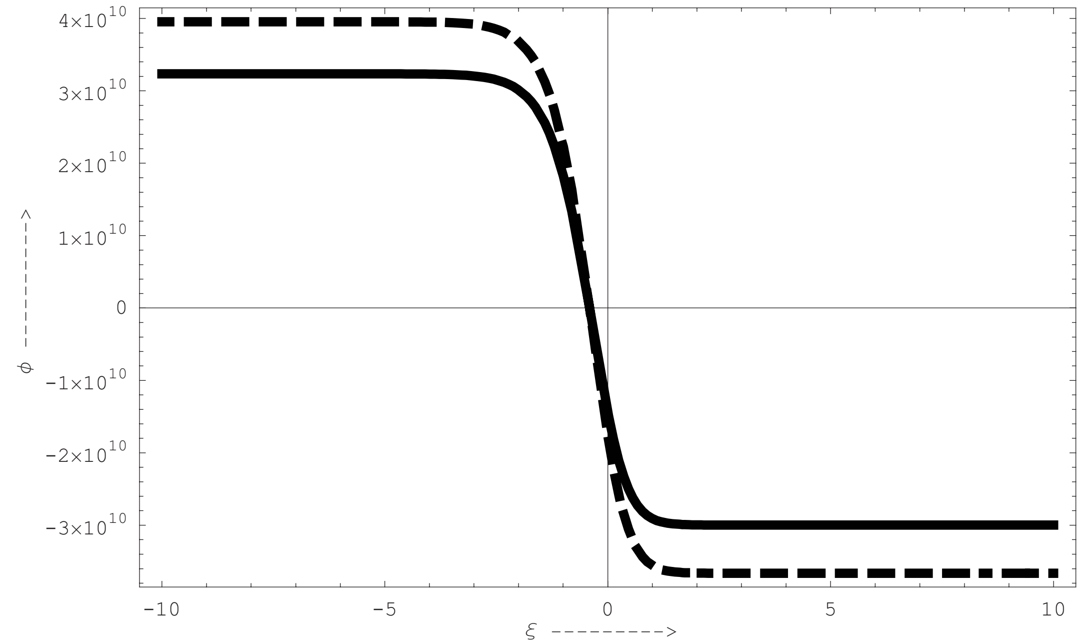

FIG. 5. Variation of the electrostatic potential ϕ for different values of v_*/u, i.e., $v_*/u>0$ (dashed line) and $v_*/u<0$ (solid line). Other parameters are $n_0=1.9\times 10^{27}$ cm^{-3}, $B_0=10^9$ G, $\nu_{in}=10^{10}$ s^{-1}, and $\alpha=10^{-4}$.

Figure 4 shows the effect of increasing collision frequency on the wave potential. It is found that an increase in the collision frequency enhances the shock strength. This is due to the fact that shock is formed due to dissipation and enhancing collision frequency is tantamount to increasing the dissipation in the system. Note that the second term that involves the tanh term in Eq. (22) is responsible for the shocklike structure as it destroys the balance between dispersion and nonlinearity unlike the ordinary KP which has a sech^2-type solution and admits solitary wave solution.

Figure 5 explores how the ratio of drift to shock velocity in the comoving frame, v_*/u, affects the shock structure. In this regard, two cases are considered, i.e., $v_*/u \gtrless 0$. It is found that for $v_*/u>0$ (fast drift shock), the shock strength increases whereas it decreases for the slow drift shock (which corresponds to the case when $v_*/u<0$). This is due to change of the sign of the coefficient appearing in Eq. (21).

VII. CONCLUSION

Linear and nonlinear propagation characteristics of drift ion acoustic shock waves in a two-dimensional (2D) inhomogeneous quantum magnetoplasma are investigated here using the QHD model. In this regard, a quantum KPB equation for an inhomogeneous plasma is derived, for the first time to the best of authors' knowledge, using the drift approximation. It is found that the ion acoustic mode couples with the drift wave if the parallel motion of ions is taken into account. Interestingly, it is noted that unlike the homogeneous plasmas, the KPB equation for inhomogeneous plasmas is derived by assuming weak parallel perturbations. Discrepancies in the earlier works have also been pointed out and a correct theoretical framework is presented to study the one-dimensional as well as the 2D propagation of shock waves in an inhomogeneous quantum plasma. Furthermore, the solution of KPB equation is presented using the tangent hyperbolic (tanh) method. The effects of quantum Bohm potential, collision frequency, and ratio of drift to shock velocity in the comoving frame on the shock profiles are numerically illustrated in Figs. 3–5. It is found that increasing the number density and collision frequency enhances the shock strength. Finally, it is found that the fast drift shock increases whereas the slow drift shock decreases the drift ion acoustic shock strength. The present study may be relevant to the study of dense astrophysical environments such as neutron stars and white dwarfs where the quantum effects are expected to play a significant role.

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