Physics of Plasmas

Nonlinear Landau damping of transverse electromagnetic waves in dusty plasmas

N. L. Tsintsadze, Rozina Chaudhary, H. A. Shah, and G. Murtaza

Citation: Phys. Plasmas 16, 043702 (2009); doi: 10.1063/1.3096715
View online: http://dx.doi.org/10.1063/1.3096715
View Table of Contents: http://pop.aip.org/resource/1/PHPAEN/v16/i4
Published by the American Institute of Physics.

Nonlinear Landau damping of transverse electromagnetic waves in dusty plasmas

N. L. Tsintsadze,[1,2,3] Rozina Chaudhary,[2,3] H. A. Shah,[2] and G. Murtaza[3]
[1]*E. Andronikashvili Institute of Physics, Tbilisi 0171, Georgia*
[2]*Department of Physics, Government College University, Lahore 54000, Pakistan*
[3]*Salam Chair in Physics, Government College University, Lahore 54000, Pakistan*

(Received 3 November 2008; accepted 18 February 2009; published online 3 April 2009)

High-frequency transverse electromagnetic waves in a collisionless isotropic dusty plasma damp via nonlinear Landau damping. Taking into account the latter we have obtained a generalized set of Zakharov equations with local and nonlocal terms. Then from this coupled set of Zakharov equations a kinetic nonlinear Schrödinger equation with local and nonlocal nonlinearities is derived for special cases. It is shown that the modulation of the amplitude of the electromagnetic waves leads to the modulation instability through the nonlinear Landau damping term. The maximum growth rate is obtained for the special case when the group velocity of electromagnetic waves is close to the dust acoustic velocity. © *2009 American Institute of Physics*.
[DOI: 10.1063/1.3096715]

I. INTRODUCTION

One of the important areas of plasma studies is the investigation of propagation of nonlinear waves and, more notably the study of nonlinear interaction of high frequency electromagnetic (EM) waves in an electron-ion plasma which has been investigated for a considerable period of time.[1–14] Davidson[15] considered nonlinear wave particle scattering in magnetized and unmagnetized plasmas and in the latter case he specifically discussed nonlinear Landau damping of electron plasma waves. Porkolab and Chang[16] considered instabilities and induced scattering due to nonlinear Landau damping of electrostatic plasma waves in a magnetized plasma. Electron beam acceleration and cross-field acceleration in relativistic magnetized plasmas was considered by Sugaya.[17,18] Using kinetic theory, Shatashvili and Tsintsadze[19] investigated the description of propagation of nonlinear Langmuir waves in a collisionless electron-ion plasma and showed that the effect of damping on ions is stronger than on electrons. They also derived a generalized set of Schrödinger equations, taking into account the nonlinear Landau damping and the relativistic effect; and demonstrated how the nonlinear Landau damping term leads to the modulation instability. Later, the modulational instability of magnetohydrodynamic waves due to nonlinear damping using full set of Zakharov equations was investigated.[20] Lerche and Schlickeiser[21] applied nonlinear Landau damping to the problem of heating of the interstellar medium. Medvedev *et al.*[22] studied nonlinear Landau damping and particle trapping in Alfven waves of finite amplitude. Prakash and Diamond[23] investigated nonlinear Landau damping and the bounced motion of protons in solar wind to describe the formation of dissipative structures and Hamza[24] examined the nonlinear Landau damping effects in Auroral (E-region) and Farley–Buneman turbulence.

In recent years a great deal of attention has been paid to the phenomena of collective processes in dusty plasmas.[25–30] More recently, the nonlinear interaction of the transverse EM field with the collisionless dusty plasma was considered.[31] It has been shown that in the region of localization of EM waves, the density of the grains increases and that the ions following the grains start clustering around them while electrons are pushed away from that region.

However to the best of our knowledge, influence of nonlinear Landau damping on the properties of the propagation of nonlinear transverse EM waves in dusty plasmas has not been investigated. We wish to consider the propagation of transverse EM waves in a collisionless dusty plasma. However it is clear that the linear damping of such waves is absent since the phase velocity $\omega_0/\mathbf{k}_0 > c$ (where c is the speed of light, ω_0 and $\mathbf{k}_0$ is the frequency and wave number of pump transverse EM wave) and there are no resonant particles here. On the other hand, nonlinear interaction of EM waves with the electron-ion (wave-wave-particle interactions) leads to the appearance of beat waves with frequencies $\omega_0-\omega_0'$ and wave vectors $\mathbf{k}_0-\mathbf{k}_0'$ which may resonate with the particles i.e.,

$$\frac{\omega_0-\omega_0'}{|\mathbf{k}_0-\mathbf{k}_0'|}=V_\alpha$$

and thus give rise to nonlinear Landau damping (here V_α is the thermal velocity of the species α).

In the present paper, we consider the nonlinear propagation of high frequency transverse EM waves in a collisionless dusty plasma, taking into account the nonlinear Landau damping. In order to consider one dimensional propagation of nonlinear EM waves, we obtain a set of generalized Zakharov equations with both local and nonlocal nonlinear terms. The nonlocal nonlinear terms arises as a consequence of nonlinear Landau damping. From this set we obtain the kinetic nonlinear Schrödinger (KNLS) equation which contains both local and nonlocal nonlinearities, which is investigated for two different cases. First we consider the subsonic regime and show that the local nonlinear term vanishes and compute the damping rate due to nonlinear damping. The second case we consider when the group velocity of the EM waves is taken to be approximately the same as the dust acoustic velocity and it is seen that the wave damps more rapidly compared to the first case. Finally we investigate the role of nonlinear Landau damping term for modulation instability.

The article is organized in the following fashion. In Sec. II, we present the basic mathematical formulation for a dusty plasma and derive expressions for density variations of the electrons, ions and dust. In Sec. III, we derive the KNLS equation with both local and nonlocal nonlinearities, and obtain nonlinear decrements for two special cases. In Sec. IV, modulation of the amplitude of the EM waves is discussed and the growth rate of the modulation instability through the nonlinear Landau damping is derived. Finally, conclusions are given in Sec. V.

II. BASIC EQUATIONS

To study the nonlinear Landau damping of the transverse EM waves in the three-component (electrons, ions and heavy charged dust grains) fully ionized collisionless isotropic plasma, we use the nonlinear Schrödinger equation and the average kinetic Vlasov equations (over the fast time scale) which contains the ponderomotive force term.[19] As the plasma is isotropic the contribution of electron current density is much larger in comparison with ion and dust current densities. Further we restrict our selves to the propagation of plane EM waves in one dimension (x coordinate) only. To investigate the behavior of transverse EM waves we write the Schrödinger equation and the Vlasov kinetic equations,[19]

$$2i\omega_0\left(\frac{\partial}{\partial t}+V_g\frac{\partial}{\partial x}\right)\mathbf{E}+c^2\frac{\partial^2\mathbf{E}}{\partial x^2}-\omega_{pe}^2\frac{\delta n_e}{n_{0e}}\mathbf{E}=0, \tag{1}$$

$$\frac{\partial}{\partial t}\langle f_\alpha\rangle+\left(V_x\frac{\partial}{\partial x}\right)\langle f_\alpha\rangle+\frac{1}{m_\alpha}\left[e_\alpha\langle E_L\rangle_x-\frac{\partial}{\partial x}\left(\frac{e^2|E|^2}{2m_\alpha\omega_0^2}\right)\right]\frac{\partial\langle f_\alpha\rangle}{\partial V_x}=0, \tag{2}$$

where α indicates plasma species $\alpha=e,i,d$; $V_g=k_0c^2/\omega_0$ is the group velocity of the EM waves, $\omega_{pe}=\sqrt{4\pi e^2 n_{0e}/m_{0e}}$ is the Langmuir frequency of electrons, $\langle f_\alpha\rangle$ is the averaged distribution function, $\mathbf{E}_{\text{tr}}$ is the transverse electric field, and $\langle E_L\rangle_x=-\partial\Phi/\partial x$ is the longitudinal electric field intensity due to the ponderomotive force $F_{\text{pond}}=-(\partial/\partial x)(e^2|E|^2/2m_\alpha\omega_0^2)$. The ponderomotive force is important only for electrons due to their small mass in Eq. (2) as

$$F_{\text{pond}}^e \gg F_{\text{pond}}^i \gg F_{\text{pond}}^d. \tag{3}$$

In order to define the density variation $\delta n_\alpha=\langle n_\alpha\rangle-n_{0\alpha}$ we will use the Zakharov approximation [i.e., we will linearize Eq. (2)] and obtain the linearized Vlasov equation

$$\left(\frac{\partial}{\partial t}+V_x\frac{\partial}{\partial x}\right)\delta f_\alpha-\frac{\partial}{\partial x}(e_\alpha\Phi+T_i|\Psi|_\alpha^2)\frac{\partial f_{0\alpha}}{\partial(m_\alpha V_x)}=0, \tag{4}$$

where $f_{0\alpha}$ is the equilibrium distribution function and assume it to be Maxwellian. We note here that we have written the ponderomotive potential in the following dimensionless form: $|\Psi|_e^2=(e_\alpha^2|E|^2/2m_e\omega_0^2T_i)$. Solving Eq. (4) in the Fourier representation

$$\{\delta f_\alpha,\Phi,|\Psi|_\alpha^2\}\sim\int\frac{dk}{2\pi}\int\frac{d\omega}{2\pi}e^{i(kx-\omega t)}\{\delta f_\alpha,\Phi,|\Psi|_\alpha^2\}, \tag{5}$$

we obtain

$$\delta f_\alpha(k,\omega,\mathbf{v})=-\left(\frac{e_\alpha\Phi(k,\omega)+T_i|\Psi(k,\omega)|_\alpha^2}{\omega-kV_x}\right)\frac{k\,\partial f_{0\alpha}}{\partial(m_\alpha V_x)}. \tag{6}$$

Consequently the density perturbation becomes

$$(\delta n_\alpha)_{k,\omega}=-\frac{n_{0\alpha}}{T_\alpha}[e_\alpha\Phi(k,\omega)+T_i|\Psi|_\alpha^2(k,\omega)]\left[1-I_{+\alpha}\left(\frac{\omega}{kV_{t\alpha}}\right)\right], \tag{7}$$

where $I_{+\alpha}(x)=xe^{-x^2/2}\int_{i\infty}^{x}d\tau e^{\tau^2/2}$, asymptotic expressions of which are[32] $I_{+\alpha}(x)=1+(1/x^2)+(3/x4)+\cdots-i\sqrt{\frac{\pi}{2}}xe^{-x^2/2}$ for $|x|\gg1$, ($|\text{Re}\,x|\gg|\text{Im}\,x|$) $I_{+\alpha}(x)=-i\sqrt{\pi/2}x$ for $|x|\ll1$.

We consider the phase velocities which satisfies the inequality

$$V_{dt}\ll\frac{\omega}{k}\ll V_{te},\quad V_{ti}. \tag{8}$$

Equation (7) thus takes the form for electrons, ions, and dust grains as

$$(\delta n_e)_{k,\omega}=\frac{n_{0e}e\Phi(k,\omega)}{T_e}-n_{0e}\frac{T_i}{T_e}|\Psi(k,\omega)|^2, \tag{9}$$

$$(\delta n_i)_{k,\omega}=-\frac{Z_ie\Phi(k,\omega)}{T_i}n_{0i}\left(1+i\sqrt{\frac{\pi}{2}}\frac{\omega}{kV_{ti}}\right), \tag{10}$$

$$(\delta n_d)_{k,\omega}=-n_{0d}\frac{Z_de\Phi(k,\omega)}{m_d}\frac{k^2}{\omega^2}. \tag{11}$$

Substituting δn_e and δn_i in the quasineutrality condition (for Fourier components)

$$Z_i\delta n_i(k,\omega)=\delta n_e(k,\omega)+Z_d\delta n_d(k,\omega), \tag{12}$$

we obtain the Fourier components of the potential field expressed through the variation of the density of dust grains and the ponderomotive potential as

$$e\Phi(k,\omega)=\frac{1}{1+i\Lambda}\left[n_{0e}\frac{T_i}{T_e}\gamma|\Psi(k,\omega)|^2-Z_d\gamma\delta n_d(k,\omega)\right], \tag{13}$$

where $\gamma=T_iT_e/(n_{0e}T_i+Z_i^2n_{0i}T_e)$ and $\Lambda=\sqrt{\pi/2}(Z_i^2n_{0i}\gamma/T_i)\times(\omega/k\mathbf{v}_{ti})$.

III. KINETIC NONLINEAR SCHRÖDINGER EQUATION AND NONLINEAR DECREMENT

In the present section we proceed to calculate the decrements for nonlinear Landau damping. From Eqs. (11) and (13), we obtain for δn_d

$$[\omega^2 + i\Lambda\omega^2 - k^2V_d^2]\frac{\delta n_d}{n_{od}} = -\frac{V_d^2}{Z_d n_{od}} n_{0e}\frac{T_i}{T_e}k^2|\Psi(k,\omega)|^2,$$

where $V_d^2 = \gamma Z_d^2 n_{0d}/m_d$. Taking the inverse Fourier transform of the above equation, we obtain

$$\left(\frac{\partial^2}{\partial t^2} - V_d^2\frac{\partial^2}{\partial x^2}\right)\frac{\delta n_d}{n_{od}} - \sqrt{\frac{1}{2\pi}}\frac{Z_i^2 n_{0i}}{T_i}\frac{\gamma}{V_{ti}}\frac{\partial^2}{\partial t^2}\wp \times \int_{-\infty}^{\infty}\frac{dx'}{x'-x}\wp\int_{-\infty}^{x'}dx''\frac{\partial}{\partial t}\frac{\delta n_d(x'',t)}{n_{0d}} = -\frac{V_d^2}{Z_d n_{od}}n_{0e}\frac{T_i}{T_e}\frac{\partial^2}{\partial x^2}|\Psi|^2. \tag{14}$$

Here, we have used the following formula $|k|/k = (1/i\pi)\wp\int_{-\infty}^{\infty}(dX/X)e^{ikX}$. Equation (14) is the generalization of the second of Zakharov's set of equations with the additional nonlocal term. Here $\wp$ denotes the principal value of the integral. In the first of Zakharov's equation, Eq. (1), it is necessary to express the variation of electron density through the ponderomotive of and density variation of dust grains $\delta n_d(x,t)$. For this we use the following Fourier transform:

$$\delta n_e(x,t) = \int\frac{dk}{2\pi}\int\frac{d\omega}{2\pi}\delta n_e(k,\omega)e^{i(kx-\omega t)}, \tag{15}$$

and further by using Eqs. (9) and (13), we obtain the perturbed electron density

$$\frac{\delta n_e(x,t)}{n_{0e}} = -\frac{Z_i^2 n_{0i}T_i|\Psi(x,t)|^2}{n_{0e}T_i + Z_i^2 n_{0i}T_e} - \frac{\gamma}{T_e}Z_d\delta n_d + \sqrt{\frac{1}{2\pi}}\frac{Z_i^2 n_{0i}}{T_e^2}\frac{n_{0e}\gamma^2}{V_{ti}}\wp\int_{-\infty}^{\infty}\frac{dx'}{x'-x}\wp \times\int_{-\infty}^{x'}dx''\frac{\partial}{\partial t}|\Psi(x'',t)|^2 - \sqrt{\frac{1}{2\pi}}\frac{Z_i^2 n_{0i}Z_d n_{od}}{T_i T_e}\frac{\gamma^2}{V_{ti}}\wp\int_{-\infty}^{\infty}\frac{dx'}{x'-x}\wp \times\int_{-\infty}^{x'}dx''\frac{\partial}{\partial t}\frac{\delta n_d(x'',t)}{n_{0d}}. \tag{16}$$

Equations (1) and (14) with Eq. (16) form a closed set of equations. We can see that the damping terms in Eq. (16) are much smaller than those in Eq. (14) and therefore we shall omit them. Substituting $\delta n_e(x,t)$ from Eq. (16) into Eq. (1) we obtain a new type of Schrödinger equation for a dimensionless function $\Psi = eE/\omega_0\sqrt{m_{0e}T_i}$,

$$2i\omega_0\left(\frac{\partial}{\partial t} + V_g\frac{\partial}{\partial x}\right)\Psi + c^2\frac{\partial^2\Psi}{\partial x^2} + \omega_{pe}^2\left[\frac{Z_i^2 n_{0i}\gamma}{T_e}|\Psi|^2 + \frac{\gamma}{T_e}Z_d\delta n_d\right]\Psi = 0. \tag{17}$$

Equations (14) and (17) are two simultaneous equations with two unknown functions Ψ and δn_d. We may rewrite Eq. (14) by using transformations

$$\Psi = \Psi(\xi,t), \quad \delta n_d = \delta n_d(\xi,t),$$

$$\xi = x - V_g t, \quad \frac{\partial}{\partial t} \ll V_g\frac{\partial}{\partial \xi} \tag{18}$$

as

$$\frac{\delta n_d}{n_{0d}} - \beta\wp\int_{-\infty}^{\infty}\frac{d\xi'}{\xi'-\xi}\frac{\delta n_d(\xi',t)}{n_{0d}} = \frac{1}{1-\frac{V_g^2}{V_d^2}}\frac{n_{0e}T_i}{Z_d n_{0d}T_e}|\Psi(\xi,t)|^2, \tag{19}$$

where

$$\beta = \sqrt{\frac{1}{2\pi}}\frac{Z_i^2 n_{0i}}{T_i}\gamma\left(\frac{V_g}{V_d}\right)^2\frac{1}{1-\frac{V_g^2}{V_d^2}}\frac{V_g}{V_{ti}}.$$

Multiplying both sides of Eq. (19) by the operator $M = 1 + \beta\wp\int d\xi'/(\xi'-\xi)$ and using the Poincare–Bertand formula (Ref. 19)

$$\wp\int_{-\infty}^{\infty}\frac{dx'}{x'-x}\wp\int_{-\infty}^{\infty}dx''\frac{F(x'')}{x''-x'} = (i\pi)^2F(x), \tag{20}$$

we obtain for δn_d,

$$\frac{\delta n_d}{n_{0d}} = \frac{1}{1-\frac{V_g^2}{V_d^2}}\frac{n_{0e}T_i}{Z_d n_{0d}T_e}|\Psi(\xi,t)|^2 + \frac{1}{1-\frac{V_g^2}{V_d^2}}\frac{n_{0e}T_i}{Z_d n_{od}T_e}\beta\wp\int_{-\infty}^{\infty}\frac{d\xi'}{\xi'-\xi}|\Psi(\xi',t)|^2. \tag{21}$$

Substituting δn_d into the Schrödinger Eq. (17), we obtain the KNLS equation with the nonlinear Landau damping term

$$2i\omega_0\frac{\partial\Psi}{\partial t} + c^2\frac{\partial^2\Psi}{\partial\xi^2} + \omega_{pe}^2H|\Psi|^2\Psi + \omega_{pe}^2\frac{\gamma n_{0e}T_i}{1-\frac{V_g^2}{V_d^2}}\frac{\beta}{T_e^2}\wp\int_{-\infty}^{\infty}\frac{d\xi'}{\xi'-\xi}|\Psi(\xi',t)|^2\Psi = 0, \tag{22}$$

where

$$H = \frac{\gamma}{T_e}\left(Z_i^2 n_{0i} + \frac{n_{0e}}{1-\frac{V_g^2}{V_d^2}}\frac{T_i}{T_e}\right). \tag{23}$$

In the subsonic regime $V_g^2 \ll V_d^2$ with $T_e > T_i$, H is reduced to $H \sim (T_i/T_e)$, while in the supersonic regime, H reduces to zero provided $Z_i^2 n_{0i} + n_{0e}(T_i/T_e) = (V_g^2/V_d^2)Z_i^2 n_{0i}$. In the latter case, the local nonlinear term disappears from Eq. (22). The last term of Eq. (22) represents nonlinear Landau damping of transverse EM waves for ions.

To derive the nonlinear Landau damping rate, we assume in Eq. (22) $\Psi = \Psi_0 e^{i(k\xi-\omega t)}$+c.c. with Ψ_0 constant, then from Eq. (22) follows the expression of nonlinear decrement

$$\mathrm{Im}\ \omega = -\omega_p \frac{\pi\omega_{pe}}{2\omega_0 T_e^2} \frac{\gamma n_{0e} T_i \beta}{1 - \frac{V_g^2}{V_d^2}} |\Psi_0|^2. \tag{24}$$

Since β contains the factor $1/[1-(V_g^2/V_d^2)]$, the above expression does not depend on whether we consider the subsonic or supersonic processes.

Now we shall consider the case when the dust acoustic velocity V_d is almost equal to the group velocity V_g, i.e., $V_g \sim V_d$ so that the dust grains density variation from Eq. (14) becomes

$$\wp \int_{-\infty}^{\infty} \frac{d\xi'}{\xi' - \xi} Z_d \delta n_d = -\sqrt{2\pi} n_{0e} \frac{T_i}{T_e} \sqrt{\frac{m_d}{m_i} \frac{Z_i^2 n_{0i}}{Z_d^2 n_{0d}}} |\Psi|^2. \tag{25}$$

Multiplying both sides of Eq. (25) by the operator $M = \wp \int_{-\infty}^{\infty} d\xi'/(\xi'-\xi)$ and using Eq. (20), we obtain expression for δn_d as

$$Z_d \delta n_d = \frac{\sqrt{2}}{\pi^{3/2}} n_{0e} \frac{T_i}{T_e} \sqrt{\frac{m_d}{m_i} \frac{Z_i^2 n_{0i}}{Z_d^2 n_{0d}}} \wp \int_{-\infty}^{\infty} \frac{d\xi'}{\xi' - \xi} |\Psi(\xi',t)|^2. \tag{26}$$

Substituting Eq. (26) into the nonlinear Schrödinger Eq. (17), and assuming that $T_e > T_i$, we obtain

$$\begin{aligned} &2i\omega_0 \frac{\partial \Psi}{\partial t} + c^2 \frac{\partial^2 \Psi}{\partial \xi^2} + \omega_{pe}^2 \frac{T_i}{T_e} |\Psi|^2 \Psi \\ &\quad + \omega_{pe}^2 \frac{\sqrt{2}}{\pi^{3/2}} \frac{n_{0e}}{Z_i^2 n_{0i}} \left(\frac{T_i}{T_e}\right)^2 \sqrt{\frac{m_d}{m_i} \frac{Z_i^2 n_{0i}}{Z_d^2 n_{0d}}} \wp \\ &\quad \times \int \frac{d\xi'}{\xi' - \xi} |\Psi(\xi',t)|^2 \Psi = 0. \end{aligned} \tag{27}$$

The last term in Eq. (27) describes the nonlinear Landau damping of EM waves on ions.

Again assuming $\Psi = \Psi_0 e^{i(k\xi - \omega t)} + \mathrm{c.c.}$ with Ψ_0, we obtain nonlinear Landau damping rate

$$\mathrm{Im}\ \omega = -\omega_{pe} \frac{\pi\omega_{pe}}{\omega_0} \sqrt{\frac{2}{\pi} \frac{n_{0e}}{Z_i^2 n_{0i}}} \left(\frac{T_i}{T_e}\right)^2 \sqrt{\frac{m_d}{m_i} \frac{Z_i^2 n_{0i}}{Z_d^2 n_{0d}}} |\Psi_0|^2. \tag{28}$$

Comparing Eq. (28) to Eq. (24) we see that the EM waves having group velocity equal to the dust acoustic velocity, damps more rapidly i.e., transfer energy to ions takes more effectively.

IV. MODULATION INSTABILITY

The presence of the nonlinear Landau damping term in Eq. (27), drastically change the physical picture of modulational instability. We note here that if the nonlocal nonlinear term is absent, then for modulation instability to occur the local nonlinear term should always be positive. We would like to show that in the presence of nonlinear Landau damping term in Eq. (22) or in Eq. (27) there is no restriction on the sign of the local and nonlocal terms in the afore mentioned equations. The nonlinear Landau damping term in Eqs. (22) and (27) as we will show leads to the development of a strong modulation instability.

To consider the modulation instability of the Eq. (27), we shall use Madelung's representation of the complex amplitude Ψ of the EM wave

$$\Psi = \Psi_0(\xi,t) e^{i\Theta(\xi,t)}, \tag{29}$$

where the amplitude Ψ_0 and the phase Θ are real. Substituting this into Eq. (27), we obtain the following set of equations:

$$\begin{aligned} &c^2 \frac{\partial^2 \Psi_0}{\partial \xi^2} - \left[2\omega_0 \frac{\partial \Theta}{\partial t} + c^2 \left(\frac{\partial \Theta}{\partial \xi}\right)^2 - \omega_{pe}^2 \frac{T_i}{T_e} |\Psi_0|^2\right] \Psi_0 \\ &\quad + \frac{\omega_{pe}^2 \sqrt{2}}{\pi^{3/2}} \frac{n_{0e}}{Z_i^2 n_{0i}} \left(\frac{T_i}{T_e}\right)^2 \sqrt{\frac{m_d}{m_i} \frac{Z_i^2 n_{0i}}{Z_d^2 n_{0d}}} \wp \\ &\quad \times \int_{-\infty}^{\infty} \frac{d\xi'}{\xi' - \xi} |\Psi_0(\xi',t)|^2 \Psi_0(\xi,t) = 0 \end{aligned} \tag{30}$$

and

$$\frac{\partial \Psi_0^2}{\partial t} + \frac{c^2}{\omega_0} \frac{\partial}{\partial \xi} \left(\Psi_0^2 \frac{\partial \Theta}{\partial \xi}\right) = 0. \tag{31}$$

We note that Eqs. (30) and (31) are analogous to the equation of motion (of plasma particles) with a friction term and the continuity equation respectively, and may be considered as equation of motion and continuity for the wave. Now we linearize using $\Psi_0 = a_0 + \delta a$ and $\Theta = \Theta_0 + \delta\Theta$, where a_0 and Θ_0 denote some equilibrium values and δa, $\delta\Theta$ are small perturbations. After linearization of Eqs. (30) and (31) we seek plane wave solutions proportional to $\exp i(k\xi - \omega t)$. We can see that Θ_0 is time dependant and is expressed as

$$\Theta_0(t) = \frac{\omega_{pe}^2}{2\omega_0} \frac{T_i}{T_e} a_0^2 t. \tag{32}$$

This expression describes the nonlinear correction on frequency shift of the frequency ω_0 of the EM waves.

For the perturbation δa and $\delta\Theta$, we obtain following dispersion equation:

$$\omega^2 = -\frac{k^2 c^2}{4\omega_0^2} \left[2\omega_p^2 \frac{T_i}{T_e} a_0^2 - k^2 c^2 + iQ\right]. \tag{33}$$

We note here that in order to obtain iQ, we linearize the last term in Eq. (30) by using $\Psi_0 = a_0 + \delta a \exp i(k\xi - \omega t) + \mathrm{c.c.}$ and considering only the principal part ($\wp$) of the integration and by using the identities $\wp \int_{-\infty}^{\infty} d\xi'/(\xi' - \xi) = 0$, and $\wp \int_{-\infty}^{\infty} d\xi'/[(\xi' - \xi)] \exp ik(\xi' - \xi) = i\pi$, we obtain Q, which is given by $Q = \omega_{pe}^2 \sqrt{8/\pi} (T_i/T_e)^2 \sqrt{m_d/m_i} \sqrt{n_{0e}^2 / Z_i^2 Z_d^2 n_{0i} n_{od}} a_0^2$, which represents the nonlinear Landau damping. If we neglect the nonlinear Landau damping term i.e., $Q=0$, then modulational instability occurs provided $(e^2 E^2 / m_0 \omega_0^2 T_e) > (k^2 c^2 / 2\omega_{pe}^2)$. However in the presence of nonlinear Landau damping, i.e., $Q \neq 0$ we obtain for the Re ω

$$\mathrm{Re}\ \omega = \frac{kc}{2\omega_0}\frac{1}{\sqrt{2}}\left[\sqrt{\left(2\omega_{pe}^2\frac{T_i}{T_e}a_0^2 - k^2c^2\right)^2 + Q^2} - \left(2\omega_{pe}^2\frac{T_i}{T_e}a_0^2 - k^2c^2\right)\right]^{1/2}, \quad (34)$$

for Im ω we have

$$\mathrm{Im}\ \omega = \frac{kc}{2\omega_0}\frac{1}{\sqrt{2}}\left[\sqrt{\left(2\omega_{pe}^2\frac{T_i}{T_e}a_0^2 - k^2c^2\right)^2 + Q^2} + \left(2\omega_{pe}^2\frac{T_i}{T_e}a_0^2 - k^2c^2\right)\right]^{1/2}. \quad (35)$$

From Eq. (35) and (34), we obtain the following three possibilities:

(i) if $2\omega_{pe}^2(T_i/T_e)a_0^2 \gg k^2c^2$ then Re $\omega \ll$ Im ω and we have purely growing mode in this case,

(ii) if $2\omega_{pe}^2(T_i/T_e)a_0^2 \ll k^2c^2$ then Re $\omega \gg$ Im ω and we obtain an oscillatory instability,

(iii) if $2\omega_{pe}^2(T_i/T_e)a_0^2 \simeq k^2c^2$, Re ω=Im ω, i.e., from Eq. (35) we note that when $2\omega_{pe}^2(T_i/T_e)a_0^2 \simeq k^2c^2$, the growth rate is due to the nonlinear Landau damping term

$$\mathrm{Re}\ \omega = \mathrm{Im}\ \omega_{\max} \sim \frac{kc}{2\omega_0}\frac{1}{\sqrt{2}}kc\sqrt{\frac{T_i}{T_e}}\left[\frac{1}{\pi}\frac{m_d}{m_i}\frac{n_{0e}^2}{Z_i^2Z_d^2n_{0i}n_{od}}\right]^{1/2}. \quad (36)$$

As we can see from above equation that for heavy dust grains case contribution of local nonlinearity is less than that made by the nonlocal nonlinear Landau damping.

V. CONCLUSIONS

In the present paper, nonlinear Landau damping of transverse EM waves in a collisionless isotropic dusty plasmas is investigated that to the best of our knowledge has not been considered before. In Sec. II, we have obtained expressions for the variations of densities and have shown that damping is important only for ions. In Sec. III, we have developed Schrödinger equation with local and nonlocal nonlinearities. It is shown that the high frequency transverse EM waves in a collisionless isotropic dusty plasma can damp through nonlinear Landau damping. Taking into account nonlinear Landau damping we have generalized set of Zakharov equations with nonlocal term, and then the KNLS equation with local and nonlocal nonlinearities is modified for special cases. It is shown in Sec. IV that maximum growth rate is obtained for the case when the group velocity of EM waves is close to the dust acoustic velocity.

ACKNOWLEDGMENTS

N.L.T. thanks the Salam Chair and Higher Education Commission of Pakistan. This work was partially supported by a GNSF grant, Project No. 69/07 GNSF/ST06/4-057.

[1] I. A. Akhiezer, I. A. Danelia, and N. L. Tsintsadze, Sov. Phys. JETP **19**, 208 (1964).
[2] V. Zakharov, Sov. Phys. JETP **35**, 908 (1972).
[3] N. L. Tsintsadze, Phys. Lett. **50A**, 33 (1974).
[4] J. F. Drake, P. K. Kaw, Y. C. Lee, G. Schmidt, C. S. Liu, and M. N. Rosenbluth, Phys. Fluids **17**, 778 (1974).
[5] P. K. Kaw, G. Schmidt, and T. Wilcox, Phys. Fluids **16**, 1522 (1973).
[6] E. A. Kuznetsov, Sov. Phys. JETP **39**, 1003 (1974).
[7] V. I. Berezhiani, N. L. Tsintsadze, and D. D. Tskhakaya, J. Plasma Phys. **24**, 15 (1980).
[8] H. P. Freund, C. S. Liu, and R. M. Kulsrud, J. Plasma Phys. **25**, 465 (1981).
[9] B. I. Cohen, Phys. Fluids **30**, 2676 (1987).
[10] L. N. Tsintsadze, K. Kusano, and K. Nishikawa, Phys. Plasmas **4**, 911 (1997).
[11] L. N. Tsintsadze and K. Nishikawa, Phys. Plasmas **3**, 511 (1996).
[12] L. N. Tsintsadze, K. Nishikawa, T. Tajima, and J. T. Mendonca, Phys. Rev. E **60**, 7435 (1999).
[13] L. N. Tsintsadze, D. K. Callebaut, and N. L. Tsintsadze, J. Plasma Phys. **55**, 407 (1996).
[14] P. K. Shukla, N. N. Rao, M. Y. Yu, and L. N. Tsintsadze, Phys. Rep. **138**, 1 (1986).
[15] R. C. Davidson, *Methods in Nonlinear Plasma Theory* (Academic, New York, 1972), Chaps. 8 and 14.
[16] M. Porkolab and R. P. H. Chang, Phys. Fluids **15**, 283 (1972).
[17] R. Sugaya, PFBPEIPhys. Fluids B **5**, 1 (1993).
[18] R. Sugaya, Phys. Plasmas **10**, 3939 (2003).
[19] N. L. Shatashvili and N. L. Tsintsadze, Phys. Scr., T **T2/2**, 511 (1982).
[20] T. Fla, E. Mjolhus, and J. Wyller, Phys. Scr. **40**, 219 (1989).
[21] I. Lerche and R. Schlickeiser, Astron. Astrophys. **383**, 319 (2002).
[22] M. V. Medvedev, P. H. Diamond, M. N. Rosenbluth, and V. i. Shevehenko, Phys. Rev. Lett. **81**, 5824 (1998).
[23] M. Prakash and P. H. Diamond, Nonlinear Processes Geophys. **6**, 161 (1999).
[24] A. M. Hamza, J. Nonlinear Math. Phys. **5**, 462 (1998).
[25] F. Verheest, *Waves in Dusty Space Plasmas* (Kluwer Academic, Dordrecht, 2000) and references therein.
[26] P. K. Shukla and A. A. Mamun, *Introduction to Dusty Plasma Physics* (Institute of Physics, Bristol, 2002) and references therein.
[27] D. A. Mendis, Plasma Sources Sci. Technol. **11**, A219 (2002).
[28] R. Bingham, U. de Angelis, V. N. Tsytovich, and O. Havens, Phys. Fluids B **28**, 23 (1991), and references therein.
[29] V. N. Tsytovich, G. E. Morfill, V. N. Tsytovich, and H. Thomos, Plasma Phys. Rep. **28**, 623 (2002).
[30] G. E. Morfill, V. N. Tsytovich, and H. Thomas, Plasma Phys. Rep. **29**, 2901 (2003).
[31] N. L. Tsintsadze, Z. Ehsan, H. A. Shah, and G. Murtaza, Phys. Plasmas **13**, 072103 (2006).
[32] A. F. Alexandrov, L. S. Bogdankevich, and A. A. Rukhadze, *Principles of Plasmas Electrodynamics* (Springer, Heidelberg, 1984).