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Nonlinear screening effect in an ultrarelativistic degenerate electron-positron gas

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Nonlinear screening process in an ultrarelativistic degenerate electron-positron gas has been investigated by deriving a generalized nonlinear Poisson equation for the electrostatic potential. In the simple one-dimensional case, the nonlinear Poisson equation leads to Debye-like (Coulomb-like) solutions at distances larger (less) than the characteristic length. When the electrostatic energy is larger than the thermal energy, this nonlinear Poisson equation converts into the relativistic Thomas–Fermi equation whose asymptotic solution in three dimensions shows that the potential field goes to zero at infinity much more slowly than the Debye potential. The possibility of the formation of a bound state in electron-positron plasma is also indicated. Further, it is investigated that the strong spatial fluctuations of the potential field may reduce the screening length and that the root mean square of this spatial fluctuating potential goes to zero for large **r** rather slowly as compared to the case of the Debye potential. *© 2009 American Institute of Physics*.
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I. INTRODUCTION

Relativistic thermodynamics and the determination of the screened potential of an electron-positron (e^-e^+) plasma is a research area of considerable interest for several stationary processes in astrophysical systems, where the plasma particle velocities are close to the speed of light and relativistic temperatures prevail. Astrophysical bodies such as active galactic nuclei,[1] accretion disks,[2] pulsar and neutron star atmospheres,[3] quasars, and black holes[4] have such characteristics. It is also known that the early prestellar period of the evolution of the Universe was presumably dominated by the relativistic electron positron having ultrarelativistic temperatures.[5] In the lepton epoch, which occurred 10^{-6} s $< t < 10$ s after the Big Bang, temperatures reached values of 10^9 K $< T < 10^{13}$ K, which caused the annihilation of nucleon-antinucleon pairs resulting in matter which constituted of electrons, positrons, and photons, in thermodynamic equilibrium. It was shown that in the ultrarelativistic limit the rest mass of electrons and positrons can be neglected, and thus they act as a photon gas.[6] Thus, the investigation of the structure of radiation electromagnetohydrodynamics for such a plasma becomes very important. Using the kinetic theory, a set of radiative electromagnetohydrodynamic equations for a relativistically hot electron positron and photon system was derived in Ref. 7 and in the particular case, when radiation was in thermal equilibrium, various discontinuities, including shock waves, were considered.[7–9] Nonlinear interaction of strong electromagnetic waves with an electron-positron plasma was considered in Ref. 9 and sound waves in an electron-positron plasma were investigated. More recently[10–15] in a relativistically hot electron-positron isothermal plasma, one-dimensional electromagnetic solitons were obtained. It is thought that spatial electrostatic fluctuations are the main cause of the origin of galaxies[5] and of the clusters of galaxies. In Ref. 16 the pulsar polar cap model is investigated, and in the one-dimensional case, it is shown that the magnetic field aligned electric field is screened beyond the electron-positron pair production front. The results are obtained and are analyzed both theoretically and numerically. Akhiezer and Merenkov[17] considered the transition of electron-positron into the bound state due to a radiative mechanism using quantum-electrodynamics perturbation theory. Experimental investigation of low temperature positron laboratory plasma has been carried out and different positron concentrations and related Debye lengths have been measured.[18,19] Using an ultraintense laser pulse, in Ref. 20 the possibility of the production of electron-positron pairs with the density 10^{21} cm^{-3} has been shown. In the above-mentioned references, fluctuations were not taken into account, which essentially determine the scattering processes of particles on a target plasma. Therefore, particle scattering becomes an important diagnostic tool. It may be noted that fluctuations are connected with the correlations between the particles.

In this paper, we present essential concepts, such as static nonlinear screening and self-energy for a nonlinear nonideal ultrarelativistic electron-positron plasma. We consider a strongly nonlinear plasma, when the mean values of kinetic and potential energies are of the same order, and show that three-dimensional screening of the potential becomes very weak. For the description of a strongly coupled degenerate electron-positron plasmas, a Poisson equation is derived. Further, we show that if the electrostatic energy $e\Phi$ is larger than thermal energy, Poisson's equation converts to the relativistic Thomas–Fermi equation.

In Sec. III we have considered the nonlinear Poisson equation in one-and three-dimensional cases and have ex-

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plicitly found the potential field as a function of coordinates. This is then compared with the Debye potential and it is seen that the potential obtained from the Poisson equation vanishes at infinity at different rates, as these potentials go to zero in the linear approximation, more slowly than the Debye potential. Finally, we have discussed the role of strong spatial fluctuations of the potential field and have shown that the root of correlation function of the potential is screened at shorter distances than the usual Debye length.

II. MATHEMATICAL PRELIMINARIES

In the present section we give a brief mathematical introduction to our problem. We consider a plasma consisting of electrons and positrons only.

However, we begin by noting that for the ultrarelativistic case when the temperature $T \gg m_0 c^2$, we must consider a gas of Fermi particles and antiparticles, obtained via creation and annihilation processes. Thus, pair production and annihilation can be regarded thermodynamically as a chemical reaction, $e^+ + e^- \rightleftarrows \gamma$, from this follows that the sum of the chemical potentials of electrons and positrons have to be same as the chemical potential of photons, i.e.,

$$\mu_- + \mu_+ = \mu_\gamma, \tag{1}$$

where μ_-, μ_+, and μ_γ are the chemical potentials of the electrons, positrons, and the photon gas, respectively.

In Planck's theory, the chemical potential μ_γ of the photon gas is zero. However, as was shown in Refs. 21–24 the equilibrium state is attained via Compton scattering, and thus in the system consisting of electrons, positrons, and a photon gas, the total number of photons is conserve, which means that the chemical potential of the photon gas is not zero. It should be emphasized that the changes dn_- and dn_+ in the number densities of electrons and positrons are related to one another in the following manner: $n_- - n_+ = n_- + dn_- - n_+ - dn_+ = n_- - n_+$, which implies that $dn_- = dn_+$ are due to creation and annihilation processes.

For the calculation of the difference in densities $(n_- - n_+)$, we have neglected the chemical potential μ_γ of the photon gas because in Planck's $\mu_\gamma = 0$. Further, statistical mechanics states that for an electron-positron plasma in an electrostatic field which is in equilibrium, the chemical potential of the positrons and electrons $\mu_\pm$ must be the same in magnitude at all points.[25,26] Assuming the chemical potentials $\mu_\pm$ to be constant, the energy of an electron or positron in electrostatic potential at point $\vec{r}$ can be written as

$$E_\mp = c\sqrt{p^2 + m_0^2 c^2} \mp e\varphi(r). \tag{2}$$

Here, the upper and lower signs refer to the electrons and positrons, respectively. In the ultrarelativistic case, Eq. (2) reduces to

$$E_\mp = cp \mp e\varphi(r). \tag{3}$$

As previously mentioned, in thermodynamic equilibrium the mean particle numbers will change via the continuously occurring creation and annihilation processes, but the difference $n_- - n_+$ remains unchanged. We now calculate this difference in densities by using the ultrarelativistic Fermi distribution function

$$n_- - n_+ = 2\int \frac{dp^3}{(2\pi\hbar)^3}\left\{ \frac{1}{\exp\left[\frac{cp - (\mu + e\varphi)}{T}\right] + 1} - \frac{1}{\exp\left[\frac{cp + (\mu + e\varphi)}{T}\right] + 1} \right\}. \tag{4}$$

Here, we have used the fact that the magnitude of the chemical potential of the electrons and positrons are equal and opposite, i.e., $\mu_+ = -\mu_- = \mu$. Introducing new variables $x = (cp - e\varphi)/T$ and $y = (cp + e\varphi)/T$ and preforming integration above, we obtain

$$n_- - n_+ = \frac{(\mu_- + e\varphi)}{3(c\hbar)^3}\left[T^2 + \frac{(\mu + e\varphi)^2}{\pi^2} \right]. \tag{5}$$

Subsequently, Poisson equation can be written as

$$\nabla^2 \varphi = 4\pi e \frac{(\mu_- + e\varphi)T^2}{3(c\hbar)^3}\left[1 + \frac{(\mu + e\varphi)^2}{\pi^2 T^2} \right]. \tag{6}$$

This equation is strongly nonlinear and its solution determines the screening potential φ.

Writing the potential energy and the chemical potential in dimensionless form in Eq. (6), we have

$$\nabla^2 \Phi = \frac{1}{r_0^2}(\Phi + H)\{1 + (\Phi + H)^2\}, \tag{7}$$

where $\Phi = e\varphi/\pi T$, $H = \mu/\pi T$, and

$$r_0 = \left[\frac{3}{4\pi}\frac{(c\hbar)^3}{e^2 T^2} \right]^{1/2}, \tag{8}$$

where r_0 is the characteristic length of the electron positron $(e^- e^+)$ pair plasma. Equation (7) has been derived without any simplifying assumption, and the potential Φ can be taken to be arbitrary at this point. Equation (7) shows us that the Coulomb interaction between particles is of special importance if the mean value potential energy is related as $\Phi \geq 1$. This condition describes a strongly correlated plasma.

III. NONLINEAR STATIC SCREENING

In this section we shall consider several different cases for the magnitudes of Φ and H for the solution of the nonlinear Poisson's equation [Eq. (7)].

A. Solution when potential is small and $H=0$

First, we consider a case for which the potential energy $\Phi \ll 1$, i.e., we consider a weakly correlated $e^- e^+$ plasma and $H \approx 0$ (the chemical potential is neglected) which is valid if we suppose that the number of electrons and positrons does not remain constant. In this case, Poisson's equation [Eq. (7)] reduces to

$$r_0^2\nabla^2\Phi=\Phi, \tag{9}$$

the solution of which is given by

$$\Phi=\frac{e_\alpha^2}{\pi T}\frac{e^{-r/r_0}}{r}, \tag{10}$$

where r_0 is the screening length given by the expression (8) and e_α refers to either the electrons or positrons. The length r_0 is large in comparison with the mean distances between particles, i.e., $r_0 \gg 1/n^{1/3}$, and is less than the usual Debye length. This screening length r_0 differs from the Debye, Thomas–Fermi, and Yukawa's screening lengths.

Itoh *et al.*[27] considered in the linear approximation $(\Phi \ll 1)$ contributions of the correlation effects in an electron-positron plasma to the thermodynamic quantities. Using the expression for the correlation energy,[25] in the case when the chemical potentials of particles is zero, the expressions of the Debye wave number and the correlation energy for arbitrary temperatures have been obtained. For ultrarelativistic temperatures, expressions of Refs. 27 and 28 and coincide with ours [see Eqs. (10) and (8) above] in the case when the electrostatic field Φ is small.

B. Nonlinear one-dimensional case $H=0$

In the presection we consider a one-dimensional nonlinear problem when $H=0$. The nonlinear Poisson's equation [Eq. (7)] in this case reduces to

$$r_0^2\frac{d^2\Phi}{dx^2}=\Phi+\Phi^3, \tag{11}$$

which for the boundary conditions that as $x\to\pm\infty$, $\Phi\to 0$ has the solution

$$\Phi=\frac{\sqrt{2}}{\pi T}\frac{e^2}{r_0}\sqrt{\coth^2\left[\frac{x}{r_0}\right]-1}. \tag{12}$$

This solution is quite different from the usual Debye and the Coulomb potential. However, in limiting cases this solution converges to Debye and Coulomb potential cases and these are given below, respectively.

For $x \gg r_0$, we obtain

$$\Phi=\frac{2^{3/2}}{\pi T}\frac{e^2}{r_0}e^{-x/r_0}, \tag{13}$$

which is the same as the one-dimensional Debye potential and for $x \ll r_0$, we get

$$\Phi=\frac{\sqrt{2}}{\pi T}\left(\frac{e^2}{x}\right), \tag{14}$$

which has the form of the Coulomb potential.

C. Three-dimensional solution

Presently, we consider the three-dimensional solution of Eq. (7) in two different situations: firstly, when the potential is large and, secondly, when the potential is small.

1. Case of large potential

Defining $\Psi=\Phi+H$ and assuming $\Psi \gg 1$ (a strongly correlated plasma), we obtain an equation similar to the relativistic Thomas–Fermi equation,

$$\frac{1}{r}\frac{d^2}{dr^2}(r\Psi)=\frac{\Psi^3}{r_0^2}. \tag{15}$$

It is well known that Thomas–Fermi model has successfully been applied not only to heavy atoms but also to other relativistic systems containing many particles. Such an equation can easily be derived at zero temperature for a relativistic dense electron gas. On the other hand, Eq. (15) is valid for an ultrarelativistic temperature in which case the creation and annihilation processes take place and also the screening length is different. In order to find the asymptotic solutions of Eq. (15), we introduce a new function $Z=r\Psi$, and a variable of coordinate $r=r_0e^{-t}$. Using the new variables, Eq. (15) reduces to

$$\frac{d^2Z}{dt^2}+\frac{dZ}{dt}=\frac{Z^3}{r_0^2}. \tag{16}$$

First, we consider asymptotic solution of this equation for $r \gg r_0$ (i.e., for $t \ll 0$). For this purpose, we follow Ref. 29 and employ the homologous transformation on Eq. (16), which leads to the solution

$$Z=\frac{Z_0}{\sqrt{1+2\left(\frac{Z_0}{r_0}\right)^2\ln\left[\frac{r}{r_0}\right]}}, \tag{17}$$

where $Z_0=r_0\Psi_0$, while Ψ_0 is the potential energy at the surface of the sphere of radius r_0. Thus, Ψ damps slowly as compared to the Debye potential.

Next we consider the case when $r \ll r_0$ (i.e., $t \gg 0$) the first term in Eq. (16) is larger than the second term on left hand side, and thus the solution reduces to

$$Z=\frac{Z_0}{1+\frac{Z_0}{r_0\sqrt{2}}\ln\left[\frac{r}{r_0}\right]}, \tag{18}$$

which shows that as $r\to 0$, Ψ goes faster to infinity as compared to the case of the Coulomb potential.

We emphasize here that the solution given by Eq. (18) does not lead to an infinite energy when $r\to\infty$ and this can be shown in the following manner. In general, the total electrostatic energy is given

$$U=\frac{1}{8\pi}\int E^2dV,$$

where E is the field produced by electron-positron plasma.

Subtituting $E=-\nabla\varphi$ using Gauss theorem and $\nabla\cdot E=-(\pi T/er_0^2)\Psi^3$ from Eq. (15), we obtain for the total energy, in our case, the following expression:

$$U = \frac{T}{8e}\int \Psi \nabla \cdot \mathbf{E} dV$$
$$= -\frac{\pi^2 T^2}{2e^2 r_0^2}\int_{r_c}^{\infty} r^2 \Psi^4 dr$$
$$= -\frac{\pi^2 T^2}{2e^2 r_0^2}\left[\int_{r_c}^{r_0} r^2\Psi^4 dr + \int_{r_0}^{\infty} r^2 \Psi^4 dr\right],$$

where $r_c = e^2/m_0c^2$ is the classical radius of the electron. Thus, it follows from the dependence of Ψ on r that the integrals above will not diverge at ∞.

2. Case of small potential

Now we shall consider the case when the chemical potential of particles is much larger than the interaction energy of electrons and positrons, i.e., $|\Phi| \ll H$. This condition allows us to neglect the potential energy term on the right hand side of Eq. (7) and we obtain

$$\nabla^2\Phi = \frac{H}{r_0^2}(1+H^2). \tag{19}$$

This equation shows that in a plasma consisting of electrons and positrons, the formation of bound states is possible in certain density and temperature ranges due to the attractive character of Coulomb forces. As a result, there may occur reactions of electrons and positrons giving rise to neutral compound particle such as positronium. To show this we will seek a solution of Eq. (19), which satisfies the boundary conditions $\Phi = \Phi_0$ and $d\Phi/dr = 0$ at $r=0$. We further suppose that at the center of the sphere of radius r_0 is a test positron, the distribution of electrons and other positrons inside the sphere is homogeneous, then we may define the potential energy at a point which is at distance $\mathbf{r}$ from the center in the following manner:

$$\Phi = \frac{H}{3}\frac{r^2}{2r_0^2} + U_0. \tag{20}$$

To calculate U_0, we note that the test positron is assumed to be at the origin, the potential of which is e/r. The interaction energy of all particles, homogeneously distributed in the volume of the sphere with the test positron at the center can be defined as

$$U_0 = -\frac{e^2}{\pi T V_0}\int \frac{d\vec{r}}{r} = -\frac{e^2}{\pi T V_0}\int_0^{r_0}\frac{1}{r}(4\pi r^2)dr = -\frac{3e^2}{2\pi T r_0}, \tag{21}$$

where $V_0 = \frac{4}{3}\pi r_0^3$ is the volume of the sphere and T is the temperature.

From Eqs. (20) and (21) follows that Φ is the potential energy of the three-dimensional harmonic oscillator, the frequency of which is given by

$$\omega_0^2 = \frac{\pi T H}{3r_0^2 m_0} \tag{22}$$

and thus

$$\Phi = \frac{1}{\pi T}\left(\frac{m_0\omega_0^2 r^2}{2} - \frac{3}{2}\frac{e^2}{r_0}\right). \tag{23}$$

Here, we have introduced an effective mass $m_0 = \pi TH/c^2$. For the estimation of the frequency ω_0, we supposed that m_0 is the rest mass of electrons (positrons) which is a function of the temperature and is given by[7] $m_0(T) = 4T/c^2$. Thus, if we assume that the chemical potential H, which is also a function of temperature, almost equals $m_0(T)c^2/\pi T$ for relativistic temperatures $T \sim 10^{10}$ K. Then, the electron orbits around the positron with a frequency $\omega_0 \sim 10^{20}\ \mathrm{s}^{-1}$. From Eq. (23) follows an interesting phenomenon when the two terms on the right hand side of Eq. (23) balance each other, giving

$$r_1 = \frac{3}{2}e\left(\frac{r_0}{T}\right)^{1/2}. \tag{24}$$

Thus, when $H \sim m_0(T)c^2/\pi T$ and the temperature $T \sim 10^{10}$ K, the radius r_1 where the potential energy given by Eq. (23) becomes zero is almost the same as the Compton length.

IV. RANDOMLY PHASED ELECTROSTATIC FIELD FLUCTUATIONS

As we mentioned in Sec. I, the spatial fluctuations of density, temperature, and electrostatic fields play a major role in the formation of galaxies and clusters of galaxies. We now consider spatial fluctuations of electrostatic fields that are randomly phased. Using the plasma fluctuation theory, we introduce the correlation function for a quantitative description of fluctuations of the electrostatic field energy Φ and assume it to be real, but its mean value, $\langle\Phi\rangle = 0$.

A spatial correlation function is defined as a mean value of the product of fluctuations of a quantity Φ at different points. The electron-positron plasma which we consider is homogeneous in space, then the quadratic space correlation function takes the form $\langle\Phi(r_1)\Phi(r_2)\rangle_r = \langle\Phi^2(r)\rangle_r$, where $r = r_2 - r_1$.

Since we consider a spatially homogeneous stationary plasma, Fourier transformation may be applied

$$\Phi(r) = \int \frac{d\vec{k}}{(2\pi)^3}a(\vec{k})e^{i\vec{k}\cdot\vec{r}} \tag{25}$$

with the inverse Fourier transform given by

$$a(\vec{k}) = \int d\vec{r}\Phi(\vec{r})e^{-i\vec{k}\cdot\vec{r}}. \tag{26}$$

The spectral distribution of the space correlation function can be defined for this case to be

$$\langle a^2\rangle_k = \int d\vec{r}\langle\Phi(\vec{r}_1)\Phi(\vec{r}_2)\rangle_r e^{-i\vec{k}\cdot\vec{r}}. \tag{27}$$

The mean value of the amplitude of the Fourier components $a(k)$ according to Ref. 25 is

$$\langle a(\vec{k})\rangle = 0. \tag{28}$$

The Fourier components $a(k)$ can be represented as complex amplitude, i.e., $a(\vec{k}) = |a(k)|e^{i\ \alpha}$, where α is the random phase.

Therefore, the expression (28) shows that phases of the Fourier components of fluctuating quantities are random and the averaging in Eq. (27) has been performed over random phases. Equation (27) is called the spectral density of the correlation function, which is an important relation, as it relates the mean value of the product of the Fourier components of $a(\vec{k})$ to the spectral distribution of the correlation function given by

$$\langle a(k)a(k_1)\rangle = (2\pi)^3\delta(k+k_1)\langle a^2(k)\rangle. \tag{29}$$

Now we shall apply the fluctuation theory to Eq. (7)

$$\nabla^2\Phi - \frac{\Phi}{r_0^2} = \frac{\Phi^3}{r_0^2} \tag{30}$$

and use the definitions stated above and rewrite this equation in the Fourier component $a(\vec{k})$ and obtain

$$-\left(k^2+\frac{1}{r_0^2}\right)a(k) = \frac{1}{r_0^2}\int \frac{d\vec{k}_1}{(2\pi)^3}a(\vec{k}_1) \times\int \frac{d\vec{k}_2}{(2\pi)^3}a(\vec{k}_2)a(\vec{k}-\vec{k}_1-\vec{k}_2). \tag{31}$$

Multiplying this equation by $a(k')$ and taking the average on both sides, we obtain for the spectral density of the correlation function

$$-\left(k^2+\frac{1}{r_0^2}\right)|a(\vec{k})|^2 = \frac{1}{r_0^2}\int \frac{d\vec{k}'}{(2\pi)^3}\int \frac{d\vec{k}_1}{(2\pi)^3} \times\int \frac{d\vec{k}_2}{(2\pi)^3}\langle a(\vec{k}')a(\vec{k}_1)a(\vec{k}_2)a(\vec{k}_3)\rangle, \tag{32}$$

where $\vec{k}_3=\vec{k}-\vec{k}_1-\vec{k}_2$

On the right hand side of this equation, we have a quadruple correlation, which may be split up into products of binary correlators (in our approximation), i.e., the quadruple correlators can be represented as

$$\begin{aligned}\langle a(\vec{k}')a(\vec{k}_1)a(\vec{k}_2)a(\vec{k}_3)\rangle &= (2\pi)^3\delta(\vec{k}'+\vec{k}_1)\langle a(\vec{k}')a(\vec{k}_1)\rangle(2\pi)^3\delta(\vec{k}_2+\vec{k}_3)\\ &\times\langle a(\vec{k}_2)a(\vec{k}_3)\rangle + (2\pi)^3\delta(\vec{k}'+\vec{k}_2)\langle a(\vec{k}')a(\vec{k}_2)\rangle\\ &\times(2\pi)^3\delta(\vec{k}_1+\vec{k}_3)\langle a(\vec{k}_1)a(\vec{k}_3)\rangle + (2\pi)^3\delta(\vec{k}_2+\vec{k}_3)\\ &\times\langle a(\vec{k}_2)a(\vec{k}_3)\rangle(2\pi)^3\delta(\vec{k}_1+\vec{k}_2)\langle a(\vec{k}_1)a(\vec{k}_2)\rangle.\end{aligned} \tag{33}$$

Substituting this expression into integral of Eq. (32), we obtain for the binary correlator, the following equation.

$$\left[k^2+\frac{1}{r_0^2}+\frac{3}{\pi^2 r_0^2}\int_{-\infty}^{\infty}\frac{d\vec{k}'}{(2\pi)^3}|a(\vec{k}')|^2\right]|a(\vec{k})|^2 = 0. \tag{34}$$

Multiplying this equation by $e^{i\vec{k}\cdot\vec{r}}$ and integrating with respect to the wave vector $\vec{k}$, we obtain a Debye type of equation for $y(r)=\int d\vec{k}/(2\pi)^3|a(\vec{k})|^2e^{i\vec{k}\cdot\vec{r}}$,

$$\nabla^2 y(\vec{r}) - \frac{1}{r_0^2}(1+A)y(\vec{r}) = 0, \tag{35}$$

where $A=3/\pi^2\int_{-\infty}^{\infty}d\vec{k}'|a(\vec{k}')|^2$.

The spherically symmetric solution of Eq. (35) is

$$y(r) = \frac{C_1}{r}\exp\left(-\frac{\sqrt{1+A}r}{r_0}\right), \tag{36}$$

where C_1 is the constant of integration.

It is interesting to emphasize that the strong fluctuation of the electrostatic field defines the screening itself if $e^2\int d^3k/(2\pi)^3|\varphi(k)|^2 \gg T^2$. The root-mean-square fluctuation of $\langle\Phi^2(r)\rangle$ is given by

$$\sqrt{\langle\Phi^2(r)\rangle} = \frac{e^2\exp\left(-\frac{r}{2r_{\mathrm{eff}}}\right)}{\pi T\sqrt{rr_0}}, \tag{37}$$

where

$$r_{\mathrm{eff}} = \frac{r_0}{\sqrt{1+\frac{3}{\pi^2}\int d\vec{k}'|a(\vec{k}')|^2}} = \sqrt{\frac{r_0}{1+\frac{3}{\pi^4}\frac{e^2}{T^2}\int d\vec{k}'|\varphi(k')|^2}}. \tag{38}$$

The comparison of Eq. (37) with Eq. (10) shows that the root mean square goes to zero for large $\mathbf{r}$ as $\exp(-r/2r_{\mathrm{eff}})/\sqrt{rr_0}$ but rather slowly as compared to Debye potential energy. However, for $r \ll r_{\mathrm{eff}}$, the self-energy of the test particle behaves like $1/\sqrt{r}$ rather than $1/r$ as $r\to 0$.

V. CONCLUSION

The idea of screening has become a fundamental concept to treat many-particle system with Coulomb interactions. For a weak and small nonlinear correlation of the Coulomb interaction between particles, the Debye screening was considered in Ref. 30. Lifshitz and Pitaevskii[31] showed that the Debye screening drastically changes in the degenerate electron gas due to the quantum effect.

We have shown that in the strongly coupled ultrarelativistic electron-positron gas, the Coulomb interaction is of special importance when the mean kinetic and potential energies are of the same order. In this case, the strong nonlinear three-dimensional screening of the potential becomes very weak. By considering an ultrarelativistic degenerate electron-positron gas and taking into account the strong Coulomb interactions, the nonlinear differential equation was derived for the arbitrary potential field and have shown that this equation converts to the Thomas–Fermi relativistic equation when the electrostatic energy is larger than the thermal energy. The probability of bound state formation in electron-positron is also pointed. Finally, we note that, we have considered electron-positron pairs in equilibrium with black body radiation photons (zero chemical potential μ_γ). However, the equilibrium between pair plasma and photons can be established via the Compton effect (i.e., when then nonthermal radiation temperatures, T_-, $T_+ > T_\gamma$, where T_-, T_+, and T_γ are temperatures of electrons, positrons, and photons, respectively) and we can, even in this case, ignore the chemical potential μ_γ of photons if the chemical potential of the particles obeys the following inequality: $|\mu_-|, |\mu_+| \gg |\mu_\gamma|$.

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