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Effects of trapping and finite temperature in a relativistic degenerate plasma

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In the present work, we have undertaken, for the first time, investigation on the effect of trapping on the formation of solitary structures in relativistic degenerate plasmas. Such plasmas have been observed in dense astrophysical objects, and in laboratory these may result due to the interaction of intense lasers with matter. We have used the relativistic Fermi-Dirac distribution to describe the dynamics of the degenerate trapped electrons by solving the kinetic equation. The Sagdeev potential approach has been employed to obtain the arbitrary amplitude solitary structures both when the plasma has been considered cold and when small temperature effects have been taken into account. The theoretical results obtained have been analyzed numerically for different parameter values, and the results have been presented graphically. © *2011 American Institute of Physics*. [doi:10.1063/1.3646403]

I. INTRODUCTION

Nonlinear effects in relativistic classical plasmas have been considered for more than three decades. Early investigations in this area are given in the paper by Demchenko and El-Naggar[1] where the effect of nonlinear forced longitudinal oscillations in a relativistic plasma was considered. It was demonstrated that the nonlinear terms appearing in the equations of motion due to the relativistic nature of oscillations not only limit the oscillation amplitude in the resonance region but also lead to the development of a parametric instability. In Ref. 2 the authors considered transport phenomena in ultrarelativistic plasmas, later in Ref. 3 the authors studied parametric instabilities in a relativistic plasma and showed that in relativistic electron plasma ($T \gg m_{0e}c^2$) the electron mass oscillations in the external electrical field led to the instability of Langmuir and low frequency aperiodic oscillations, and in Ref. 4 the effect of ion dynamics on relativistic oscillations was considered. The investigation of solitary waves in relativistic plasmas has been taken up by many authors beginning with the work of Tsintsadze,[5] who considered ion acoustic solitary waves in the weak relativistic limit and obtained the Korteweg deVries (KdV) equation. Later Nejoh[6] also obtained the KdV equation for solitary waves with fully relativistic streaming ions. This approach was later extended by employing the Sagdeev potential approach which made possible the consideration of arbitrary amplitude waves.[7–9] More recently, Lee and Choi[10] used the fully relativistic two-fluid model in a flowing plasma to obtain the Sagdeev potential to investigate solitary structures. Such studies have proved useful in considerations related to laser plasma interactions[11] and space plasmas.[12]

More recently, investigations of degenerate quantum plasmas have gained much currency and these studies have covered different topics related to linear and nonlinear propagation of waves employing the quantum hydrodynamic model.[13–18] Degenerate plasmas are of interest on account of their applications in astrophysical environments, in modern lasers in their interaction with plasmas, and also in microelectronic devices. The relativistic effects are important in superdense astrophysical objects, such as white dwarf stars, which have number densities of the order of 10^{32} m^{-3} and beyond. The most common constituents of the interior of white dwarf stars are thought to be fully ionized helium, carbon, and oxygen.[19]

However, relativistic degenerate plasmas have not received much attention although such plasmas are of importance in different astrophysical situations, where particle velocities become comparable to the speed of light and relativistic temperatures are prevalent. Astrophysical objects such as active galactic nuclei, pulsar and neutron star magnetospheres, quasars, and accretion discs[20–23] are examples of where relativistic degenerate plasmas are dominant, and studies of relativistic plasmas and the accompanying nonlinear effects should be able to provide interesting new insights into phenomena in such plasmas. Recently in Ref. 24 we have investigated nonlinear static screening in ultrarelativistic electron positron plasmas by deriving a generalized nonlinear Poisson equation which was then investigated in different limiting situations, and the results were compared with the Debye and Coulomb screening results. The possibility of formation of bound structures and the role of electrostatic fluctuations were also discussed.[24]

The effect of adiabatic trapping as a nonlinear phenomena at the microscopic level was first introduced by Gurevich,[25] and it was seen that adiabatic trapping produced $3/2$ power nonlinearity instead of the usual quadratic one without trapping. Computer simulations[26] and experimental investigations[27] confirmed the existence of trapping as a microscopic phenomenon. The effect of trapping on the propagation characteristics of ion acoustic solitons was investigated using Maxwellian and non-Maxwellian distribution functions.[28] It was seen in both cases that solitary dynamics was considerably modified, especially in the latter case where spiky solitons were obtained. The effect of trapping on the formation of vortices in a classical plasma was considered, and a modified Hasegawa-Mima equation was derived and analyzed by considering the bounce frequencies of the trapped particles in shallow and deep potential wells, respectively.[29] The effect on the Sagdeev potential was also investigated. One of the first investigations in the area of quantum plasmas was carried out by Luque *et al*.[30] who considered quantum corrected electron holes by solving the Wigner-Poisson system perturbatively. Demeio[31] considered the effect of trapping on Bernstein-Greene-Kruskal equilibria by using a perturbative approach to solve the Wigner-Poisson system and to investigate the trapping effect in quantum phase space; however, in Refs. 30 and 31, the statistical nature of trapping was not investigated since the Wigner-Poisson equation was used, which accounts only for quantum diffraction effects.

Trapping in quantum plasma using the Gurevich approach has been considered very recently by us,[32] and the formation of solitary structures was investigated in the instances of fully degenerate plasma and when small temperature effects were taken into account. Jovanovich and Fedele[33] studied the nonlinear effects on a slow timescale by comparison with the electron plasma frequency in the plasma regimes characterized by the overlapping of the wavefunctions of individual electrons, by the presence of a large amplitude Langmuir pump wave, and whose temperature is higher than the Fermi temperature. It was shown that the electron trapping on closed orbits in phase space is strongly affected both by the classical nonlinear ponderomotive effects and by the quantum super-diffusion. Motivated by these developments, in the present work, we consider the effect of trapping in a relativistic degenerate plasma and the subsequent formation of solitary structures. In Sec. II, we present the general mathematical formalism for trapped particles by using the relativistic Fermi-Dirac distribution function for the electrons and the equations of motion and continuity for ions which are treated classically. The linear dispersion relation is derived and the limiting nonrelativistic and ultrarelativistic cases are also briefly discussed. In Sec. III, we derive the expression for the Sagdeev potential, and soliton formation is discussed. Throughout the text, the subscripts R, N, and U are used to differentiate between the relativistic, nonrelativistic, and ultrarelativistic cases, respectively.

II. BASIC SET OF EQUATIONS

We begin by deriving an expression for the number density of the adiabatically trapped electrons which are relativistic and degenerate. We follow Landau and Lifshitz[34] which outlines the method for obtaining the expression for the number densities of the trapped and free electrons, when the trapping of the electrons is due to the potential of the ions. The relativistic electron energy (which includes the rest mass energy) in the presence of a potential field φ is given by $\varepsilon = c\sqrt{p^2 + m_0^2c^2} + u$, where p and m_0 are momentum and rest mass of the electrons, respectively, and $u = -e\varphi$ is the potential energy of the well in which the electrons are trapped. Electrons with $\varepsilon > 0$ and $\varepsilon < 0$ are the free and trapped electrons, respectively. Trapping occurs when the condition $\varepsilon = 0$ is fulfilled.

In general, the normalized occupation number for the Fermi-Dirac distribution (after integration over the azimuthal and polar angles) is given by[35]

$$n_e = \frac{8\pi}{(2\pi\hbar)^3}\int_0^\infty \frac{p^2 dp}{e^{\frac{c\sqrt{p^2+m_0^2c^2}+u-\mu}{T}} + 1}. \tag{1}$$

Here $n_e(r,t)$ is the total number density and μ is the chemical potential. We now express the integral in terms of the energy ε and arrive at the following expression by setting $U = e\varphi + \mu$:

$$n_e(\vec{r},t) = \frac{8\pi}{(2\pi\hbar)^3 c^3}\int_{m_0c^2}^\infty \frac{\sqrt{\varepsilon'^2 - m_0^2c^4}\,d\varepsilon}{e^{(\varepsilon'-\mu')/T} + 1}, \tag{2}$$

where $\varepsilon' = \varepsilon - U$ and $\mu' = \mu - U$. Using the trapping condition $\varepsilon - U = 0$ and making a change of variables by substituting $z = \varepsilon' - \mu'/T$, we obtain the following expression (following Landau and Lifshitz[35]) for n_e:

$$n_e(r,t) = \frac{8\pi}{(2\pi\hbar)^3 c^3}\left[\int_{m_0c^2}^{\mu'} y\sqrt{y^2 - m_0^2c^4}\,dy - \left\{\begin{array}{l}\displaystyle\int_0^\infty \frac{(\mu'-Tz)\sqrt{(\mu'-Tz)^2 - m_0^2c^4}}{e^z+1}Tdz - \\ \displaystyle\int_0^\infty \frac{(\mu'+Tz)\sqrt{(\mu'+Tz)^2 - m_0^2c^4}}{e^z+1}Tdz\end{array}\right\}\right], \tag{3}$$

where $y = \mu' - Tz$ and $z = \mu' - m_0c^2/T$. Simplification of the above equation yields

$$n_e = \frac{8\pi}{(2\pi\hbar)^3 c^3}\int_{m_0c^2}^{\mu'} y\sqrt{y^2 - m_0^2c^4}\,dy + \frac{T^2(2\mu'^2 - m_0^2c^4)}{\pi^2\hbar^3c^3\sqrt{\mu'^2 - m_0^2c^4}}\int_0^\infty \frac{zdz}{e^z+1}. \tag{4}$$

The first integral in Eq. (4) is evaluated in a straightforward manner and represents the effect of trapped particles; however, the second integral is evaluated only approximately by taking the small temperature limit and expanding the functions in the numerator. Thus, upon integration we obtain for the normalized total number density

$$n_{eR} = \frac{\mu^3}{3\pi(\hbar c)^3} \times \left[\left\{ (1+\Phi_R)^2 - \varepsilon_0^2 \right\}^{3/2} + \frac{\pi^2 T_R^2}{6} \frac{\left\{ 2(1+\Phi_R)^2 - \varepsilon_0^2 \right\}}{\left\{ (1+\Phi_R)^2 - \varepsilon_0^2 \right\}^{1/2}} \right]. \tag{5}$$

Here $\mu = \varepsilon_{F0} + m_0 c^2$ is the chemical potential in the relativistic case (although this is not strictly correct when $T \neq 0$ but in the small temperature limit this is a reasonable approximation), $\varepsilon_{F0} = (3\pi^2 n_0)^{2/3}(\hbar^2/2m)$ is the Fermi energy, $\Phi_R = e\varphi/\mu$ is the normalized (relativistic denoted by the subscript R) potential, $T = T/\mu$ is the normalized temperature, and $\varepsilon_0 = m_0c^2/\mu$ is the total normalized energy. In the absence of the trapping potential Φ_R the electron number density $n_{eR} = n_{0R}$, where

$$n_{0R} = \frac{\varepsilon_{F0}^3}{3\pi(\hbar c)^3} \left[(1-\varepsilon_0^2)^{3/2} + \frac{\pi^2 T_R^2 (2-\varepsilon_0^2)}{6(1-\varepsilon_0^2)^{1/2}} \right], \tag{6}$$

for a relativistic Fermi Dirac distribution of electrons.

We now turn our attention to the ions which are taken to be cold and non-degenerate due to their heavy mass by comparison with the electrons. The ion equation of motion and continuity equation are

$$m_i n_i \left(\frac{\partial}{\partial t} + \vec{v}_i \cdot \vec{\nabla} \right) \vec{v}_i = -e\vec{\nabla}\varphi, \tag{7}$$

$$\frac{\partial}{\partial t} n_i + \vec{\nabla} \cdot n_i \vec{v}_i = 0, \tag{8}$$

and the set of equations is closed by the Poisson's equation

$$\nabla^2 \varphi = 4\pi(n_e - n_i). \tag{9}$$

In order to investigate the nonlinear properties of ion acoustic waves in a relativistic degenerate plasma, we simplify the procedure and consider a one dimensional case with propagation of the quantum ion acoustic waves taking place in the x-direction only by shifting to a comoving frame of reference by taking $\xi = x - ut$, where u is the velocity of propagation of the perturbation. We integrate Eqs. (8) and (9) by making use of the boundary conditions such that $\xi \to \infty$, v_i, $\varphi \to 0$, and $n_i \to n_0$, where n_0 is the background number density of the electrons and ions for a degenerate plasma, and obtain the following expression for the normalized number density of the ions

$$n_i = n_{0R}(1 - \Phi_R/M_R^2)^{-1/2}, \tag{10}$$

where n_0 is the equilibrium number density given by expression (6) and M_R is the given by $M_R = \sqrt{m_i u^2/2\mu}$.

We now substitute Eqs. (5) and (10) in the Poisson's equation Eq. (9) and obtain

$$\frac{d^2\Phi_R}{d\xi^2} = \frac{4\pi e^2 n_{0R}}{\mu} \left[\frac{\left\{ (1+\Phi_R)^2 - \varepsilon_0^2 \right\}^{3/2} + \frac{\pi^2 T^2}{6} \frac{\left\{ 2(1+\Phi_R)^2 - \varepsilon_0^2 \right\}}{\left\{ (1+\Phi_R)^2 - \varepsilon_0^2 \right\}^{1/2}}}{n_{0R}} - \left(1 - \frac{\Phi_R}{M_R^2} \right)^{-1/2} \right]. \tag{11}$$

Before we proceed further, we linearize Eqs. (5), (9), and (11), and using the definition of the M_R given above and taking the velocity $u = \omega/k$ we obtain the expression for the linear dispersion relation of ion acoustic waves in a plasma having relativistically degenerate electrons

$$\omega = k c_{sR} \sqrt{\frac{1}{1 + k^2 \lambda_{TF,R}^2}}, \tag{12}$$

where the relativistic Thomas Fermi length $\lambda_{TF,R}$ is given by

$$\lambda_{TF,R} = \sqrt{\frac{\mu}{4\pi e^2 n_{0R} A_R}}, \tag{13}$$

and the relativistic sound velocity c_{sR} is given by

$$c_{sR} = \sqrt{\frac{\mu}{m_i A_R}}, \tag{14}$$

and

$$A_R = \frac{3\left[(1-\varepsilon_0^2)^{1/2} + \frac{\pi^2 T^2 (2 - 3\varepsilon_0^2)}{18(1-\varepsilon_0^2)^{1/2}} \right]}{(1-\varepsilon_0^2)^{3/2} + \frac{\pi^2 T^2 (2-\varepsilon_0^2)}{6(1-\varepsilon_0^2)^{1/2}}}, \tag{15}$$

where A_R contains the relativistic and temperature correction effects, which shows that the phase velocity has a complicated dependence on the temperature T and the relativistic energy ε_0. We can now define the Mach number using the linear dispersion relation (12) and the general definition of the Mach number we obtain

$$\mathsf{M}_R = M_R \sqrt{A_R}. \tag{16}$$

III. LIMITING CASES

We consider two limiting cases, i.e., the nonrelativistic case when $m_0 c^2 \succ\succ \varepsilon_{F0}$ and the ultrarelativistic case when

$m_0c^2 \prec\prec \varepsilon_{F0}$. Thus in the nonrelativistic limit Eq. (5) reduces to

$$n_{eN} = \frac{(2m_0\varepsilon_{F0})^{3/2}}{3\pi^2\hbar^3}\left[(1+\Phi_N)^{3/2} + \frac{\pi^2 T_N^2}{24}(1+\Phi_N)^{-1/2}\right]. \quad (17)$$

We note here that this is essentially the same expression as that obtained in our earlier paper[32] except for the difference in the normalization of T. Thus in the nonrelativistic limit the chemical potential $\mu = \varepsilon_{F0}$, and the normalized temperature and electrostatic potential are given by $T_N = \pi T/\varepsilon_{F0}$ and $\Phi_N = e\varphi/\varepsilon_{F0}$, respectively. The expression for the limiting nonrelativistic number density in the absence of the trapping potential Φ_N is then given by

$$n_{0N} = \frac{(2m_0\varepsilon_{F0})^{3/2}}{3\pi^2\hbar^3}\left[1 + \frac{\pi^2 T_N^2}{24}\right]. \quad (18)$$

Substituting expressions (9) and (15) in the Poisson equation (10) and using Eq. (16) we obtain

$$\frac{d^2\Phi_N}{d\xi^2} = \frac{4\pi e^2 n_{0N}}{\varepsilon_{F0}}\left[\frac{\{(1+\Phi_N)\}^{3/2} + \frac{\pi^2 T_N^2}{24}\{1+\Phi_N\}^{-1/2}}{n_{0N}} - \left(1 - \frac{\Phi_N}{M_N^2}\right)^{-1/2}\right]. \quad (19)$$

Linearizing Eq. (17) and using a plane wave solution and taking the definition of the in the nonrelativistic case to be $M_N = \sqrt{m_i u^2/2\varepsilon_{F0}}$, we obtain the expression for the linear dispersion relation for ion acoustic waves in a degenerate plasma with small temperature corrections, and this is given by

$$\omega = k c_{sN}\sqrt{\frac{1}{1 + k^2\lambda_{TF,N}^2}}. \quad (20)$$

Here, the nonrelativistic Thomas Fermi length $\lambda_{TF,N}$ is given by

$$\lambda_{TF,R} = \sqrt{\frac{\varepsilon_{F0}}{4\pi e^2 n_{0N} A_N}}. \quad (21)$$

The nonrelativistic sound velocity c_{sN} is given by

$$c_{sN} = \sqrt{\frac{2\varepsilon_{F0}}{m_i A_N}}. \quad (22)$$

The temperature corrections are included in the factor

$$A_N = \frac{3(1 + \pi^2 T^2/24)}{(1 - \pi^2 T^2/72)}. \quad (23)$$

The nonrelativistic Mach number is given by

$$\mathsf{M}_N = M_N\sqrt{A_N}. \quad (24)$$

In the ultrarelativistic case $m_0c^2 \prec\prec \varepsilon_{F0}$ the number density given by Eq. (5) becomes

$$n_{eU} = \frac{\varepsilon_{F0}^3}{3\pi^2\hbar^3 c^3}(1+\Phi_U)\left[(1+\Phi_U)^2 + \pi^2 T_U^2\right]. \quad (25)$$

The subscript U refers to the ultrarelativistic case and the Fermi energy in this case should be computed using ultrarelativistic number density. In general for the ultrarelativistic case the Fermi energy is replaced by the momentum p_F as $\varepsilon_{F0} = cp_F = (3\pi^2 n)^{1/3}\hbar c$. We note that the nature of the nonlinearity in the ultrarelativistic case is quite different from the relativistic and nonrelativistic cases as the fractional power indices are replaced by whole number indices. The number density in the absence of the trapping potential reduces to

$$n_{0U} = \frac{(2m_0\varepsilon_{F0})^{3/2}}{3\pi^2\hbar^3}\left[1 + \pi^2 T_U^2\right]. \quad (26)$$

Substituting Eqs. (10) and (22) in (9) with appropriate normalization yields the following expression for the ultrarelativistic Poisson equation

$$\frac{d^2\Phi_U}{d\xi^2} = \frac{4\pi e^2 n_{0U}}{\varepsilon_{F0}}\left[\frac{(1+\Phi_U)\left\{(1+\Phi_U)^2\right\} + T_U^2}{n_{0U}} - \left(1 - \frac{\Phi_U}{M_U^2}\right)^{-1/2}\right]. \quad (27)$$

Linearization of Eqs. (14) and (15) and the subsequent use of a plane wave solution yields the dispersion relation for the relativistic case

$$\omega = k c_{sU}\sqrt{\frac{1}{1 + k^2\lambda_{TF,U}^2}}. \quad (28)$$

Here $c_{sU} = \sqrt{\varepsilon_{F0}/m_i A_U}$ is the ultrarelativistic sound velocity and the ultrarelativistic Thomas Fermi length is given by $\lambda_{TF,U} = \sqrt{\frac{\varepsilon_{F0}}{4\pi e^2 n_{0U} A_U}}$, and the temperature dependent ultrarelativistic factor is given by

$$A_U = 3(1 + \pi^2 T_U^2/3)/(1 + \pi^2 T_U^2). \quad (29)$$

And the relativistic Mach number is

$$\mathsf{M}_U = M_U\sqrt{A_U}. \quad (30)$$

IV. SAGDEEV POTENTIAL

In this section, we derive an expression for the Sagdeev potential in the relativistic case and obtain conditions for the existence of solitary waves by taking into account small temperature corrections when $T \neq 0$.

Using the analogy of a particle in a potential well after integration and following Refs. 36 and 37, we obtain Eq. (11) in the following manner:

$$\frac{1}{2}\left(\frac{d\Phi_R}{d\xi}\right)^2 + V_R(\Phi_R) = 0, \quad (31)$$

where the relativistic Sagdeev potential is given by

$$V_R = -\frac{(-1+\epsilon_0^2)\left(\begin{array}{l}(-1+\epsilon_0^2)(-6-4\pi^2T_R^2+15\epsilon_0^2)+8\mathrm{M}_R^2\left(-\pi^2T_R^2(-2+\epsilon_0^2)+6(-1+\epsilon_0^2)^2\right)\\ +9\epsilon_0^4\sqrt{1-\epsilon_0^2}\,\mathrm{Log}\left[1+\sqrt{1-\epsilon_0^2}\right]\end{array}\right)}{4\left(18\epsilon_0^4+2\left(9+\pi^2T_R^2\sqrt{1-\epsilon_0^2}\right)-3\epsilon_0^2\left(12+\pi^2T_R^2\sqrt{1-\epsilon_0^2}\right)\right)}$$
$$-\frac{\left(-T_R^2(-2+\epsilon_0^2)+6(-1+\epsilon_0^2)^2\right)\left(\begin{array}{l}2M_R^2\sqrt{1-\frac{\Phi_R}{M_R^2}}+\frac{\sqrt{1-\epsilon_0^2}(1+\Phi_R)\sqrt{-\epsilon_0^2+(1+\Phi_R)^2}\left(4\pi^2T_R^2-15\epsilon_0^2+6(1+\Phi_R)^2\right)}{4\left(-T_R^2(-2+\epsilon_0^2)+6(-1+\epsilon_0^2)^2\right)}\\ +\frac{9\epsilon_0^4\sqrt{1-\epsilon_0^2}\,\mathrm{Log}\left[1+\Phi_R+\sqrt{-\epsilon_0^2+(1+\Phi_R)^2}\right]}{4\left(-\pi^2T^2(-2+\epsilon_0^2)+6(-1+\epsilon_0^2)^2\right)}\end{array}\right)}{18\sqrt{1-\epsilon_0^2}\left(\sqrt{1-\epsilon_0^2}+\frac{\pi^2T_R^2(-2+3\epsilon_0^2)}{18(-1+\epsilon_0^2)}\right)}. \tag{32}$$

Here the constants of integration have been evaluated by using the same boundary conditions as those used for the ions, i.e., when $\xi = \xi/\lambda_{TF,R} \to \infty$ $\Phi_R \to 0$. Furthermore, it is evident from Eq. (20) that $V_R(\Phi) = dV_R(\Phi)/d\Phi = 0$ when $\Phi = 0$, and following the argument given in Refs. 36 and 37, we see that in order to obtain solitary waves solutions from Eq. (9), (i) if $(d^2V_R/d\Phi_R^2)_{\Phi_R=0} < 0$, i.e., the fixed point is unstable at the origin and (ii) $V_R(\Phi_R) < 0$ when $0 < \Phi_R < \Phi_{R\max}$ for rarefactive solitary waves and for compressive solitary waves $V_R(\Phi_R) < 0$ when $0 > \Phi_R > \Phi_{R\min}$, where $\Phi_{R\max\,(\min)}$ is the maximum(minimum) value of the potential Φ_R for which $V_R(\Phi_R) = 0$. Below we discuss two special cases for the existence of solitary waves. The lower limit on the Mach number is obtained by Taylor expanding the Sagdeev potential given by Eq. (11) and by setting the coefficient of the quadratic term in $\Phi_R = 0$ which in our case yields

$$\mathbf{M}_{R,low} = \frac{\left((1-\varepsilon_0^2)^{3/2}+\frac{\pi^2T_R^2}{18}(2-3\varepsilon_0^2)\right)}{A_R}\sqrt{\frac{3\left((1-\varepsilon_0^2)^2+\frac{\pi^2T_R^2}{6}(2-\varepsilon_0^2)\right)}{2\left((1-\varepsilon_0^2)^2+\frac{\pi^2T_R^2}{18}(2-3\varepsilon_0^2)\right)}} = \sqrt{1/2}. \tag{33}$$

The upper limit of $\mathbf{M}_R$ is obtained from the first (ion) term in Eq. (10) such that it does not yield an imaginary result. In general the range $\mathbf{M}_R$ is obtained numerically for different values of ε_0 and T.

As in the preceding section we now turn our attention to the two limiting cases viz the nonrelativistic case and the ultrarelativistic case, respectively, for the Sagdeev potentials. In the nonrelativistic limit $m_0c^2 \succ\succ \varepsilon_{F0}$, the expression for the Sagdeev potential reduces to

$$V_N(\Phi_N) = -\frac{1}{(1-\pi^2T_N^2/72)}\left\{\begin{array}{l}\frac{\pi^2T_N^2}{18}(1+\Phi_N)^{1/2}+\frac{4}{15}(1+\Phi_N)^{5/2}+\frac{4M_N^2}{3}\left(1-\frac{\Phi_N}{M_N^2}\right)^{1/2}\left(1+\frac{\pi^2T_N^2}{24}\right)\\ -\frac{1}{90}(24+5\pi^2T_N^2)(1+5M_N^2)\end{array}\right\}. \tag{34}$$

For the relativistic case the lower limit on $\mathbf{M}_N$ is obtained as

$$\mathbf{M}_{N,lower} = \sqrt{\frac{3}{2}}. \tag{35}$$

And in the ultrarelativistic limit $m_0c^2 \prec\prec \varepsilon_{F0} \approx \varepsilon_{F0}$ the Sagdeev potential reduces to

$$V_U(\Phi_U) = -\frac{1}{3\left(1+\frac{\pi^2T_U^2}{3}\right)}\left\{(1+\pi^2T_U^2)\Phi_U+\frac{3}{2}(1+\pi^2T_U^2/3)\Phi_U^2 +\Phi_U^3+\frac{\Phi_U^4}{4}+2M_U^2(1+\pi^2T_U^2)\left(1-\frac{\Phi_U}{M_U^2}\right)^{1/2} -2M_U^2(1+\pi^2T_U^2)\right\}. \tag{36}$$

The lower limit on ultrarelativistic Mach number is

$$\mathbf{M}_{U,low} = \frac{1}{A_U}\sqrt{\frac{1}{6}\left(\frac{1+\pi^2 T_U^2}{1+\pi^2 T_U^2/6}\right)}. \tag{37}$$

We note that in both limiting cases the expressions for the Sagdeev potentials are less complicated than for the general relativistic case.

V. RESULTS AND DISCUSSION

In the present section we present some results of our numerical investigations. We have taken here the parameters of white dwarfs. There is a whole range of values for the number density and magnetic field in a white dwarf as a wide variety of them is found in nature. In the nonrelativistic case, we choose $n = 10^{26} - 10^{29}\ \mathrm{cm}^{-3}$ (Ref. 38) and for the relativistic and ultrarelativistic cases, we choose $n_i = 10^{30}$–$10^{32}\ \mathrm{cm}^{-3}$ (Ref. 38). We begin by considering the relativistic Sagdeev potential given by Eq. (26). In Fig. 1, in the upper panel, we have obtained plots for the Sagdeev potential V_R for different values of the electron temperature T, for fixed values of the Mach number ($\mathbf{M}_R = 0.8$) and normalized relativistic energy ($\varepsilon_0 = 0.3$). We see that as temperature increases the minimum of the Sagdeev potential decreases but the maximum values of Φ remain close to one another. The lower panel of Fig. 1 shows the corresponding soliton profiles for the same parameters. In Fig. 2 we have similar plots but here the electron temperature (normalized) and Mach number are kept fixed and the relativistic energy ε_0 is varied. It is observed

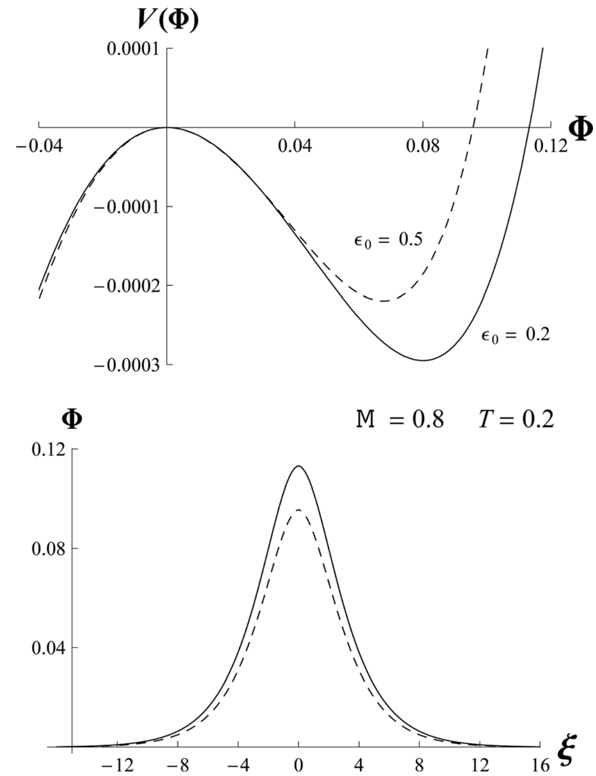

FIG. 2. Sagdeev potential (upper panel) and corresponding solitary structures (lower panel) for different values of ε_0 for fixed values of $M = 0.8$ and $T = 0.2$.

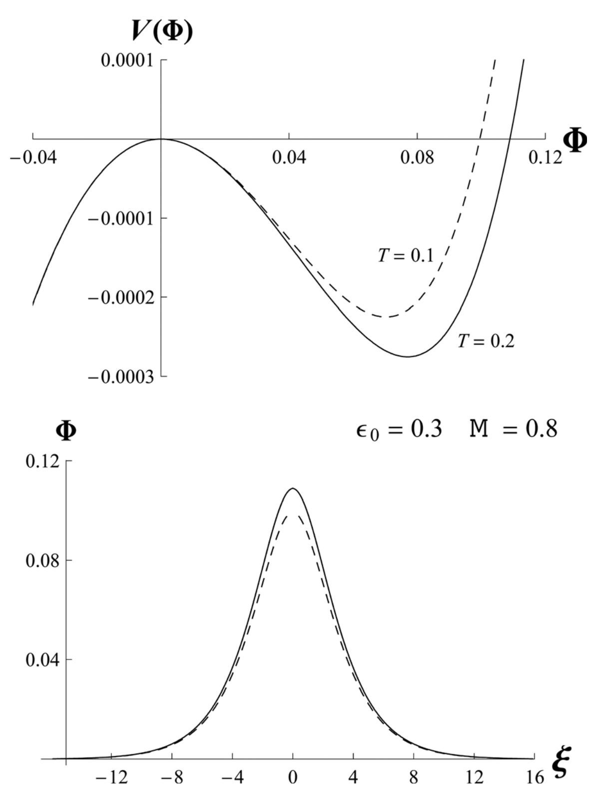

FIG. 1. Sagdeev potential (upper panel) and corresponding solitary structures (lower panel) for different values of T for fixed values of $\varepsilon_0 = 0.3$ and $M = 0.8$.

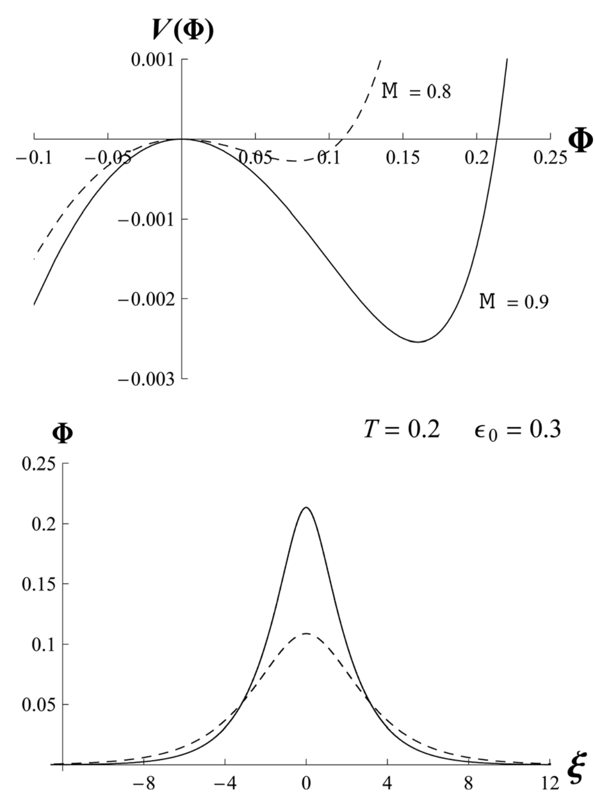

FIG. 3. Sagdeev potential (upper panel) and corresponding solitary structures (lower panel) for different values of M for fixed values of $T = 0.2$ and $\varepsilon_0 = 0.3$.

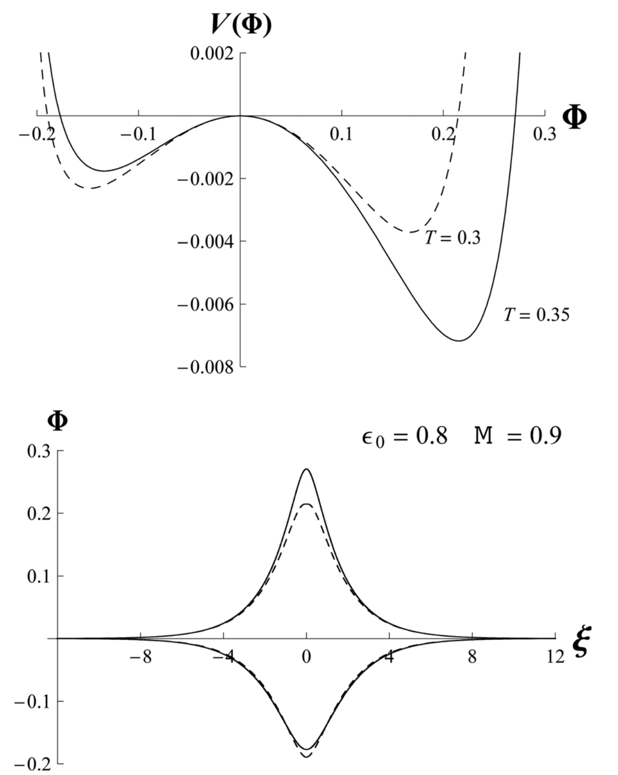

FIG. 4. Asymmetric Sagdeev potential (upper panel) and corresponding solitary structures (lower panel) for different values of T for fixed values of $\varepsilon_0 = 0.8$ and $M = 0.9$.

that an increase in the value of ε_0 increases the minimum of the Sagdeev potential, V_R. In Fig. 3 the values of the relativistic energy and electron temperature are fixed and the Mach number is varied. The Sagdeev potential shows a greater sensitivity for different values of the Mach number. It is seen that for a greater Mach number, the Sagdeev potential is deeper and the potential Φ_R is larger. In Fig. 4 we see that when the normalized electron temperature, i.e., $T_R \geq 0.5$ then both compressive and rarefactive solitons are obtained. A similar trend was observed in our previous paper.[32] It should be mentioned here that like the $T_R \leq 0.5$ cases, the depth of the Sagdeev potential and the corresponding potential Φ_R get modified with variations in the relativistic energy as well as the Mach number. Fig. 5 shows the variation of the Sagdeev

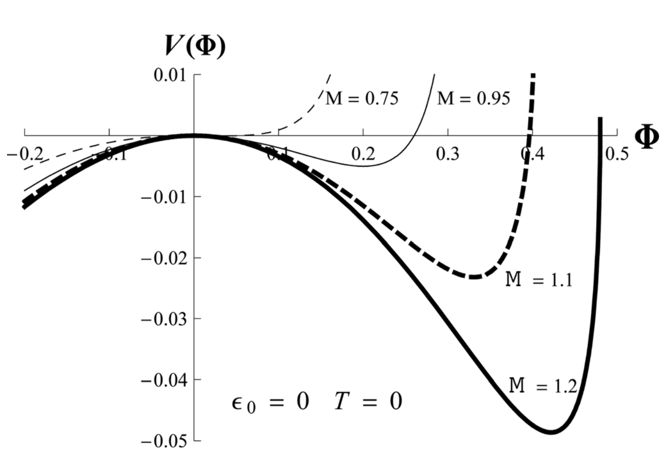

FIG. 5. Sagdeev potentials for different values of M without temperature effect, i.e., $T = 0$ in the ultra-relativistic limit, i.e., $\varepsilon_0 = 0$.

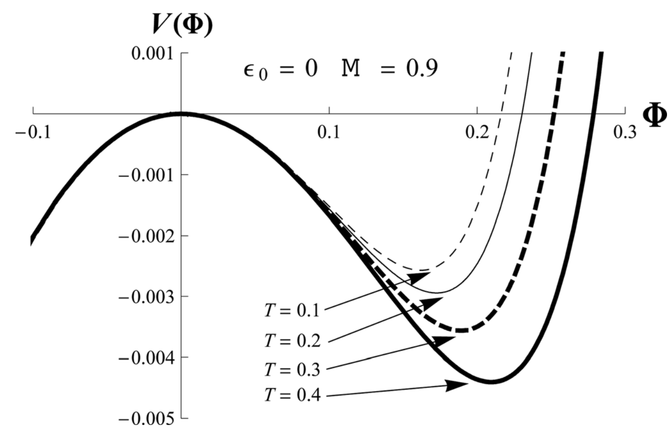

FIG. 6. Sagdeev potentials for different values of T for M = 0.9 in the ultra-relativistic limit, i.e., $\varepsilon_0 = 0$.

potential and the potential Φ_U for different values of ulralativistic Mach number M_U. It is found that the changing Mach number affects the depth of the Sagdeev potential and the value of the potential Φ_U appreciably. Fig. 6 shows that the inclusion of finite electron temperature enhances the depth of the Sagdeev potential and the values of the ultrarelativistic potential become larger. Finally, we would like to mention that the values of the potential have been found to be largest for non-relativistic, intermediate for ultra-relativistic, and least for the relativistic case.

In conclusion we have investigated the formation of solitary structures in degenerate relativistic plasma. Such investigations, to the best of our knowledge, have been carried out for the first time, and the results of our work should have importance for ultra strong laser plasma interactions and should help in understanding astrophysical observations of dense objects such as white dwarfs. We have numerically investigated our theoretical results for different parameter values such as the rest mass energy, Mach number, and the electron temperature. These results have been presented graphically illustrating the formation of solitary wave structures and their dependence on the different parameters mentioned above.

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