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Quantum modification of dust shear Alfvén wave in plasmas

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The low frequency, long wavelength electromagnetic waves, viz., shear Alfvén waves in quantum dusty magnetoplasmas, have been examined using quantum magnetohydrodynamic model. The magnetized electrons and ions, quantized electrons and magnetized/unmagnetized dust give rise to a modified dispersion relation of the shear Alfvén wave. This modification is significant which is also depicted through graphical representation. The importance and relevance of the present work to the space dusty plasma environments is also pointed out. © *2012 American Institute of Physics*. [doi:10.1063/1.3684641]

I. INTRODUCTION

Quantum plasmas have lately become main focus of plasma researchers because of their potential applications in ultrasmall electronic devices.[1–3] Several authors[4–15] have studied quantum effects in high density and extremely low temperature plasma environments. Later, Brodin *et al.*[16] showed that the quantum effect cannot be neglected even in a modest-density, high-temperature plasma which is normally regarded as a classical case.

Quantum plasmas are common in astrophysical environments[17] and can also be produced in the laboratory,[18] nanostructured materials, and quantum wells. It is important to note that Fermi-degenerate plasmas[19] may also arise when a pellet of hydrogen is compressed many times to its solid density. In quantum plasmas, the electron plasma frequency is extremely high due to its number density and far exceeds the electron collision frequency. Such properties lead to many new effects in quantum plasmas.[20–22]

Collective interactions[21,22] in dense quantum plasmas are treated employing different approaches. The quantum magnetohydrodynamic model (QMHD) considers a number of forces acting on the plasma species like the quantum force,[20,21] Bohm potential, and the Lorentz force. Dusty plasmas are rich in waves and instabilities. In the short wavelength and electrostatic limit, the physics of dusty plasmas has acquired great importance due to the discovery of dust-acoustic, dust-ion-acoustic, and dust-lower-hybrid waves.[23–36] However, in the long wavelength electromagnetic limit, not much work has been done yet.

In the recent past, several authors[37,38] clarified the role of dust on low-frequency electromagnetic waves. Using fluid model of plasmas, Shukla and Rahman[39] studied the shear Alfvén and other electromagnetic waves in nonuniform dusty magnetoplasmas. Reddy *et al.*[40] investigated low frequency Alfvén waves in multi-beam dusty plasmas with applications to comets and planetary rings. Salimullah and Rosenberg[41] first reported analytically the occurrence of dust kinetic Alfvén waves in a dusty plasma. Salimullah *et al.*[42] also studied the instabilities of the low-frequency electromagnetic, dust-lower-hybrid, and magnetosonic waves for nearly transverse propagation. Recently, Zubia *et al.*[43,44] have given a rigorous theory of kinetic and shear Alfvén waves in the presence of dust-charge-fluctuation damping and two potential theory. They, however, considered small but finite parallel wavenumber in their study in classical plasmas.

In this paper, we report the existence of a long wavelength electromagnetic wave with arbitrary parallel wavenumber, viz., a shear Alfvén wave using QMHD in a low-temperature quantum dusty magnetoplasma.

II. DISPERSION RELATION

We consider the propagation of an electromagnetic wave in a quantum dusty magnetoplasma containing electrons, ions, and relatively massive and charged dust grains in the presence of a static external magnetic field $\mathbf{B}_0 = \hat{z}B_0$.

The linearized equation of motion for plasma species can be written as

$$\frac{\partial \mathbf{v}_j}{\partial t} = \frac{q_j}{m_j}\underline{\mathrm{E}} + \mathbf{v}_j \times \omega_{cj}\hat{z} - \frac{\nabla P_{Fj}}{m_j n_{0j}} + \frac{\hbar^2}{4m_j^2 n_{0j}}\nabla(\nabla^2 n_j), \qquad (1)$$

where $\hbar = h/2\pi$ and $\omega_{cj} = q_j B_0/m_j c$, q_j, m_j, n_{0j}, and c are cyclotron frequency, the charge, mass, equilibrium number density of the jth species, and the velocity of light in a vacuum, respectively. Here, $q_e = -e$, $q_i = +e$, and $q_d = -Z_d e$, with e being the magnitude of electronic charge. In Eq. (1), we assume the plasma particles in a 3-dimensional Fermi gas satisfying the pressure law, $p_j = m_j V_{Fj}^2 n_j^3/3n_{0j}^2$,[45] where $V_{Fj}^2 = \frac{6}{5}\frac{T_{Fj}}{m_j}\left\{1 + \frac{5}{12}\pi^2\left(\frac{T_j}{T_{Fj}}\right)^2\right\}$ is the Fermi speed; k_B, T_{Fj}, T_j, and n_j are the Boltzmann constant, Fermi temperature, thermal temperature, and the number density with its equilibrium value n_{0j}, respectively. Thermal temperature of ions and dust is small as compared to the electrons and therefore ignored.

We assume that the propagation vector $\mathbf{k}$ of low frequency electromagnetic wave (ω, $\mathbf{k}$) lies in xz-plane, i.e., (k_x, 0, k_z) where $k_x = k\sin\theta$ and $k_z = k\cos\theta$, θ is the angle made by propagation direction $\mathbf{k}$ with the positive direction of the z-axis. The velocity components of jth species obtained from Eq. (1) are

$$v_{jx} = \frac{iq_j}{m_j\omega}\left[\frac{\omega^2}{\omega^2 G-\omega_{cj}^2}E_x + \frac{i\omega\omega_{cj}}{\omega^2 G-\omega_{cj}^2}E_y + \frac{V_{Fj}^{\prime 2}k_x k_z}{F(\omega^2 G-\omega_{cj}^2)}E_z\right], \tag{2}$$

$$v_{jy} = \frac{iq_j}{m_j\omega}\left[-\frac{i\omega\omega_{cj}}{\omega^2 G-\omega_{cj}^2}E_x + \frac{\omega^2 G}{\omega^2 G-\omega_{cj}^2}E_y - i\frac{\omega_{cj}V_{Fj}^{\prime 2}k_x k_z}{\omega F(\omega^2 G-\omega_{cj}^2)}E_z\right], \tag{3}$$

$$v_{jz} = \frac{iq_j}{m_j\omega F}\left[\frac{V_{Fj}^{\prime 2}k_x k_z}{(\omega^2 G-\omega_{cj}^2)}E_x + \frac{i\omega_{cj}V_{Fj}^{\prime 2}k_x k_z}{\omega(\omega^2 G-\omega_{cj}^2)}E_y + \left(1+\frac{V_{Fj}^{\prime 4}k_x^2 k_z^2}{F\omega^2(\omega^2 G-\omega_{cj}^2)}\right)E_z\right], \tag{4}$$

where

$$V'_{Fj} = V_{Fj}(1+\gamma_j)^{1/2}, \quad \gamma_j = \hbar^2 k^2/4m_j^2 V_{Fj}^2,$$
$$F = 1-\frac{V_{Fj}^{\prime 2}k_z^2}{\omega^2}, \quad G = \frac{\omega^2 - V_{Fj}^{\prime 2}k^2}{\omega^2 - V_{Fj}^{\prime 2}k_z^2}.$$

The current density of the plasma particles due to the electromagnetic dust shear Alfvén wave is

$$\mathbf{J} = \sum_j q_j n_{0j}\mathbf{v}_j. \tag{5}$$

After substitution of Eqs. (2)–(4) into Eq. (5), the current density becomes

$$\mathbf{J} = \underline{\underline{\sigma}}.\mathbf{E} \tag{6}$$

where $\underline{\underline{\sigma}}$ is the linear conductivity tensor due to the low frequency electromagnetic wave and is given by

$$\underline{\underline{\sigma}} = \sum_j \frac{iq_j^2 n_{0j}}{m_j\omega}\underline{\underline{\mathbf{K}}}_j, \tag{7}$$

where

$$\underline{\underline{\mathbf{K}}}_j = \begin{pmatrix} \frac{\omega^2}{\omega^2 G-\omega_{cj}^2} & \frac{i\omega\omega_{cj}}{\omega^2 G-\omega_{cj}^2} & \frac{V_{Fj}^{\prime 2}(k_x k_z)}{F(\omega^2 G-\omega_{cj}^2)} \\ -\frac{i\omega\omega_{cj}}{\omega^2 G-\omega_{cj}^2} & \frac{\omega^2 G}{\omega^2 G-\omega_{cj}^2} & -i\frac{\omega_{cj}V_{Fj}^{\prime 2}(k_x k_z)}{F\omega(\omega^2 G-\omega_{cj}^2)} \\ \frac{V_{Fj}^{\prime 2}(k_x k_z)}{F(\omega^2 G-\omega_{cj}^2)} & i\frac{\omega_{cj}V_{Fj}^{\prime 2}(k_x k_z)}{F\omega(\omega^2 G-\omega_{cj}^2)} & \frac{1}{F}\left(1+\frac{V_{Fj}^{\prime 4}(k_x^2 k_z^2)}{F\omega^2(\omega^2 G-\omega_{cj}^2)}\right) \end{pmatrix}. \tag{8}$$

The electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$ are related by the following curl equations:

$$\nabla\times\mathbf{B} = \frac{4\pi}{\mathrm{c}}\mathbf{J} + \frac{1}{\mathrm{c}}\frac{\partial\mathbf{E}}{\partial \mathrm{t}} \tag{9}$$

and

$$\nabla\times\mathbf{E} = -\frac{1}{\mathrm{c}}\frac{\partial\mathbf{B}}{\partial \mathrm{t}}. \tag{10}$$

Combining these curl equations, one may write

$$\underline{\underline{\mathbf{D}}}\,.\,\mathbf{E} = 0. \tag{11}$$

$\underline{\underline{\mathbf{D}}}$ gives the plasma dispersion relation due to electromagnetic dust shear Alfvén wave (ω, $\mathbf{k}$) and is defined by

$$Det[\underline{\underline{\mathbf{D}}}] = k^2\underline{\underline{I}} - \mathbf{kk} - \frac{\omega^2}{c^2}\underline{\underline{\epsilon}} = 0, \tag{12}$$

where $\underline{\underline{I}}$ is the unit dyadic and $\underline{\underline{\epsilon}} = \underline{\underline{I}} - \sum_j(\omega_{pj}^2/\omega^2)\underline{\underline{\mathbf{K}}}_j$. Here, $\omega_{pj} = (4\pi n_{0j}q_j^2/m_j)^{1/2}$ is the plasmas frequency of jth species. The matrix form of Eq. (12) is

$$Det[\underline{\underline{\mathbf{D}}}] = Det\begin{pmatrix} k_z^2 - \frac{\omega^2}{c^2}\epsilon_{xx} & -\frac{\omega^2}{c^2}\epsilon_{xy} & -k_z k_x - \frac{\omega^2}{c^2}\epsilon_{xz} \\ -\frac{\omega^2}{c^2}\epsilon_{yx} & k^2 - \frac{\omega^2}{c^2}\epsilon_{yy} & -\frac{\omega^2}{c^2}\epsilon_{yz} \\ -k_z k_x - \frac{\omega^2}{c^2}\epsilon_{zx} & -\frac{\omega^2}{c^2}\epsilon_{zy} & k_x^2 - \frac{\omega^2}{c^2}\epsilon_{zz} \end{pmatrix} = 0. \tag{13}$$

Here, we treat the electrons quantized and magnetized while the ions and the dust particles are nonquantum but magnetized. The components of the medium response function are

$$\epsilon_{xx} = 1 - \frac{\omega_{pe}^2}{\omega^2 G - \omega_{ce}^2} - \frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2} - \frac{\omega_{pd}^2}{\omega^2 - \omega_{cd}^2},$$

$$\epsilon_{yy} = 1 - \frac{\omega_{pe}^2 G}{\omega^2 G - \omega_{ce}^2} - \frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2} - \frac{\omega_{pd}^2}{\omega^2 - \omega_{cd}^2},$$

$$\epsilon_{zz} = 1 - \frac{\omega_{pe}^2}{\omega^2 F}\left(1 + \frac{V_{Fe}'^4 (k_x k_z)^2}{F\omega^2(\omega^2 G - \omega_{ce}^2)}\right) - \frac{\omega_{pi}^2}{\omega^2} - \frac{\omega_{pd}^2}{\omega^2},$$

$$\epsilon_{xy} = i\frac{\omega_{pe}^2}{\omega^2 G - \omega_{ce}^2}.\frac{\omega_{ce}}{\omega} - i\frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2}.\frac{\omega_{ci}}{\omega} - i\frac{\omega_{pd}^2}{\omega^2 - \omega_{cd}^2}.\frac{\omega_{cd}}{\omega},$$

$$\epsilon_{xz} = -\left(\frac{\omega_{pe}^2 V_{Fe}'^2 (k_x k_z)}{F\omega^2(\omega^2 G - \omega_{ce}^2)}\right), \quad \epsilon_{yz} = -i\left(\frac{\omega_{ce}\omega_{pe}^2 V_{Fe}'^2 (k_x k_z)}{F\omega^3(\omega^2 G - \omega_{ce}^2)}\right),$$

$$\epsilon_{yx} = -\epsilon_{xy}, \quad \epsilon_{zx} = \epsilon_{xz}, \quad \epsilon_{zy} = -\epsilon_{yz}.$$

The oblique shear Alfvén wave has both **k** and **E** in the same plane and therefore $E_y = 0$. Hence, for the low frequency, long wavelength mode in low β plasmas, the non-diagonal components of the dielectric tensor become negligibly small and therefore Eq. (13) can be written as[47–51]

$$Det[\underline{\mathbf{D}}] = Det\begin{pmatrix} k_z^2 - \frac{\omega^2}{c^2}\epsilon_{xx} & -k_z k_x - \frac{\omega^2}{c^2}\epsilon_{xz} \\ -k_z k_x - \frac{\omega^2}{c^2}\epsilon_{zx} & k_x^2 - \frac{\omega^2}{c^2}\epsilon_{zz} \end{pmatrix} = 0.$$

Or, the above equation can be written as follows:

$$\omega^2(\epsilon_{xx}\epsilon_{zz} - \epsilon_{xz}^2) - c^2 k_z^2 \epsilon_{zz} - c^2 k_x^2 \epsilon_{xx} - 2c^2 k_x k_z \epsilon_{xz} = 0. \quad (14)$$

A. Case 1: Magnetized dust

Firstly, we ignore the inertia of electrons as compared to the ions and dust particles. The dust particles are highly charged and therefore magnetized. Then, for frequency range $\omega^2 \ll \omega_{cd}^2 \ll \omega_{ci}^2$ and $\omega^2 \ll V_{Fe}'^2 k_z^2$, the components of medium response function gain following simplified form:

$$\epsilon_{xz} = -\frac{\omega_{pe}^2}{\omega_{ce}^2}\frac{k_x}{k_z}$$

$$\epsilon_{xx} = 1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} + \frac{\omega_{pi}^2}{\omega_{ci}^2} + \frac{\omega_{pd}^2}{\omega_{cd}^2},$$

$$\epsilon_{zz} = 1 + \frac{\omega_{pe}^2}{V_{Fe}'^2 k_z^2} + \frac{\omega_{pe}^2}{\omega_{ce}^2}\frac{k_x^2}{k_z^2} - \frac{\omega_{pi}^2}{\omega^2} - \frac{\omega_{pd}^2}{\omega^2},$$

Thus, we obtain from Eq. (14),

$$a\omega^4 + b\omega^2 + c = 0, \quad (15)$$

where

$$a = 1 + \lambda_{DFe}^2 k_z^2 + \rho_{Fe}^2 k_x^2 - \frac{v_{Ad}^2}{c^2}\rho_{Fe}^2 k_x^2 \frac{\omega_{pe}^2}{\omega_{ce}^2},$$

$$b = -\Big\{v_{Ad}^2 k_z^2(1 + \lambda_{DFe}^2 k_z^2 - \rho_{Fe}^2 k_x^2) + \lambda_{DFe}^2 k_z^2(\omega_{pi}^2 + \omega_{pd}^2 + c^2 k_x^2)\Big\}$$

$$c = v_{Ad}^2 \lambda_{DFe}^2 k_z^4(\omega_{pi}^2 + \omega_{pd}^2)$$

and

$$v_{Ad}^2 = \frac{B_0^2}{4\pi(n_{0d}m_d + n_{0i}m_i)}, \quad \lambda_{DFe}^2 = \frac{V_{Fe}'^2}{\omega_{pe}^2}, \quad \rho_{Fe^2} = \frac{V_{Fe}'^2}{\omega_{ce}^2}$$

are dust modified Alfvén speed, Debye length, and Larmour radius of Fermi electrons, respectively.

For $\lambda_{DFe}^2 k_z^2 \ll 1$, $\rho_{Fe}^2 k_x^2 \ll 1$, $v_{Ad}^2 \ll c^2$, $\omega_{pd}^2 \ll \omega_{pi}^2$, the simplified form of the coefficients a, b and c can be written as

$$a \approx 1, \quad b \approx -\Big\{v_{Ad}^2 k_z^2 + \lambda_{DFe}^2 k_z^2(\omega_{pi}^2 + c^2 k_x^2)\Big\},$$

$$c \approx v_{Ad}^2 \lambda_{DFe}^2 k_z^4 \omega_{pi}^2.$$

Equation (15) is a biquadratic equation which gives the quantum modified dispersion relation of dust shear Alfvén wave. After completing the square under the radical sign (discriminant) in the quadratic formula, we have

$$\omega^2 = \frac{k_z^2\Big\{v_{Ad}^2 + \lambda_{DFe}^2(c^2k_x^2 + \omega_{pi}^2)\Big\} + \sqrt{k_z^4\Big\{v_{Ad}^2 + \lambda_{DFe}^2(c^2k_x^2 - \omega_{pi}^2)\Big\}^2 + 4\omega_{pi}^2 c^2 k_x^2 \lambda_{DFe}^4 k_z^4}}{2}.$$

After ignoring the small term of discriminant according the given conditions, the above expression of dispersion relation of quantum modified dust shear Alfvén wave becomes

$$\omega^2 = v_{Ad}^2 k_z^2 + c^2 k_x^2 \lambda_{DFe}^2 k_z^2. \quad (16)$$

We know that the quantum effect for electromagnetic wave is geometry dependent and plays no role for parallel propagation. Consequently the standard dispersion relation of Alfvén wave in an incompressible plasmas is retrieved,

$$\omega = k v_{Ad}. \quad (17)$$

We also observe that, for general **k** if we choose $\hbar = 0$ (or $T_{Fe} = 0$), V_{Fe}', λ_{DFe}, and ρ_{Fe} are simply replaced by V_{te}, λ_{De}, ρ_e, respectively, and the dispersion relation of dust shear Alfvén wave in classical dusty magnetoplasmas is obtained,

$$\omega^2 = v_{Ad}^2 k_z^2 + c^2 k_x^2 \lambda_{De}^2 k_z^2. \quad (18)$$

Or Eq. (18) becomes

$$\omega^2 = v_{Ad}^2 k_z^2 (1 + \rho_{sd}^2 k_x^2), \tag{19}$$

where $\rho_{sd}^2 = (c^2/v_{Ad}^2)\lambda_{De}^2$ and Alfvén speed v_{Ad} is defined by the mass density of ions and dust particles.

B. Case 2: Unmagnetized dust

Secondly, we choose the frequency range $\omega_{cd}^2 \ll \omega^2 \ll \omega_{ci}^2$ and $\omega^2 \ll V_{Fe}'^2 k_z^2$, and attain from Eq. (14) the following dispersion relation:

$$A\omega^4 + B\omega^2 + C = 0, \tag{20}$$

where

$$A = \left(1 + \frac{v_A^2}{c^2}\right)\left(1 + \lambda_{DFe}^2 k_z^2 + \rho_{Fe}^2 k_x^2\right) - \frac{\omega_{pe}^2}{\omega_{ce}^2}\frac{v_A^2}{c^2}\rho_{Fe}^2 k_x^2,$$

$$B = -\left\{v_A^2 k_z^2 (1 + \lambda_{DFe}^2 k_z^2) + \omega_{dlh}^2 (1 + \lambda_{DFe}^2 k_z^2 + \rho_{Fe}^2 k_x^2) + c^2 k_x^2 \lambda_{DFe}^2 k_z^2 \left(1 + \frac{v_A^2}{c^2} - \frac{v_A^2}{c^2}\frac{\rho_{Fe}^2}{\lambda_{DFe}^2}\right) + \lambda_{DFe}^2 k_z^2 \omega_{pi}^2 \left(1 + \frac{v_A^2}{c^2} + \frac{\omega_{pd}^2}{\omega_{pi}^2}\right)\right\},$$

$$C = \lambda_{DFe}^2 k_z^2 \left(\omega_{ci}^2 \omega_{pd}^2 + v_A^2 k_z^2 \omega_{pi}^2 \left(1 + \frac{\omega_{pd}^2}{\omega_{pi}^2} + \frac{\omega_{pd}^2}{\omega_{pi}^2}\frac{k_x^2}{k_z^2} + \frac{\omega_{pd}^4}{c^2 k_z^2 \omega_{pi}^2}\right)\right).$$

Using same conditions, as described in case 1, the coefficients A, B, and C become

$$A \approx 1, \quad B = -\left\{v_A^2 k_z^2 + \omega_{dlh}^2 + c^2 k_x^2 \lambda_{DFe}^2 k_z^2 + \lambda_{DFe}^2 k_z^2 \omega_{pi}^2\right\}, \quad C = \lambda_{DFe}^2 k_z^2 \left(\omega_{ci}^2 \omega_{pd}^2 + v_A^2 k_z^2 \omega_{pi}^2\right).$$

Proceeding as in case 1, after completing the square inside the discriminant, the quantum modified dispersion relation of dust shear Alfvén wave is obtained as

$$\omega^2 = (1/2)\left\{v_A^2 k_z^2 + \omega_{dlh}^2 + c^2 k_x^2 \lambda_{DFe}^2 k_z^2 + \lambda_{DFe}^2 k_z^2 \omega_{pi}^2 + \sqrt{(v_A^2 k_z^2 + \omega_{dlh}^2 + c^2 k_x^2 \lambda_{DFe}^2 k_z^2 - \lambda_{DFe}^2 k_z^2 \omega_{pi}^2)^2 + 4\omega_{pi}^2 c^2 k_x^2 \lambda_{DFe}^4 k_z^4}\right\}$$

$$\omega^2 = v_A^2 k_z^2 + \omega_{dlh}^2 + c^2 k_x^2 \lambda_{DFe}^2 k_z^2. \tag{21}$$

In Eq. (21), when the quantum effect is eliminated by taking $k_x = 0$, the standard dispersion relation of dust Alfvén wave of Ref. 44 is retrieved

$$\omega^2 = k_z^2 v_A^2 + \omega_{dlh}^2, \tag{22}$$

where, $\omega_{dlh} = \omega_{pd}(\omega_{ci}/\omega_{pi})$ is dust lower hybrid frequency. For the dispersion relation of dust shear Alfvén wave in classical plasmas, Eq. (21) can be written as,

$$\omega^2 = v_A^2 k_z^2 + \omega_{dlh}^2 + c^2 k_x^2 \lambda_{De}^2 k_z^2. \tag{23}$$

Or Eq. (23) can be written as

$$\omega^2 = v_A^2 k_z^2 \left(1 + \rho_s^2 k_x^2 + \frac{\omega_{pd}^2}{c^2 k_z^2}\right), \tag{24}$$

where $\rho_s^2 = (c^2/v_A^2)\lambda_{De}^2$, Alfvén speed $v_A^2 = B_0^2/(4\pi n_{0i} m_i)$ being defined by the mass density of ions only.

For electron-ion plasmas, Eqs. (19) and (24) immediately reduce to the results of Hasegawa[46] and to the isotropic case of Bashir *et al.*[47]

III. GRAPHICAL REPRESENTATION AND DISCUSSION OF RESULTS

For graphical appreciation of the dispersion relation of the dust shear Alfvén wave in quantum plasmas, we have plotted ω as a function of k (Eqs. (16) and (21)), for the following typical parameters in interstellar and magnetospheric environments. $B_0 = 10^6$ G, $m_i = m_p$, $Z_d = (n_{0i} - n_{0e})/n_{0d}$,

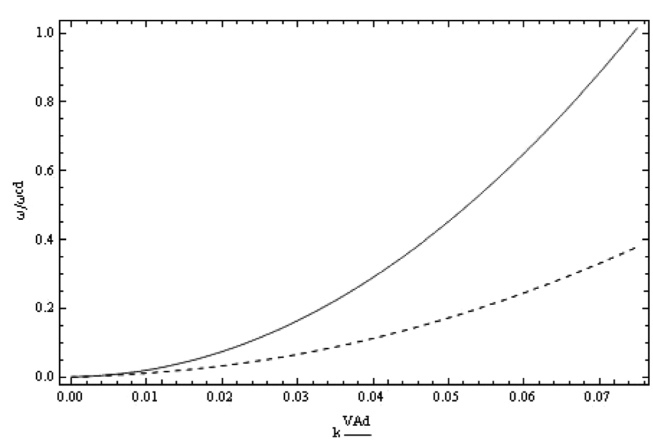

FIG. 1. ω versus k (case 1), for quantum (solid), nonquantum (dashed), and magnetized dust at $\theta = 5°$.

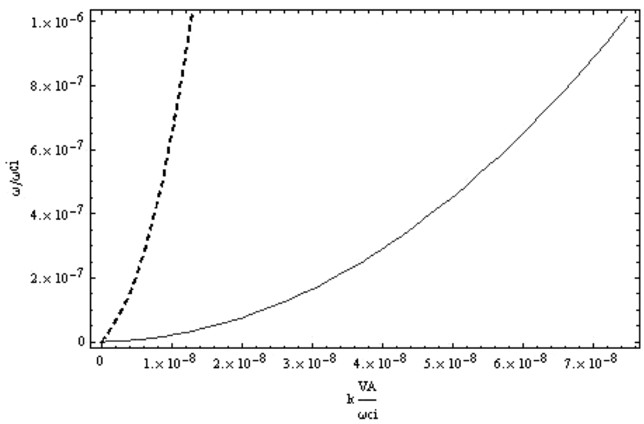

FIG. 2. ω versus k (case 1), the effect of dust (solid), and without dust (dashed) in a magnetized quantum plasmas at $\theta = 5°$.

$q_d = Z_d e$ esu, $n_{0e} = 10^{27}\,\text{cm}^{-3}$, $T_{Fe} = (3\pi^2 n_{0e})^{2/3}\hbar^2/(2m_e k_B)$, $\theta = 5°$, $m_d/m_i = 10^9$, $n_{0i} = 1.001 \times 10^{27}\,\text{cm}^{-3}$, $n_{0d} = 10^{-6} \times n_{0i}$ cm^{-3}.[45]

For magnetized dust: $k = 0$ to $3.25\,\text{cm}^{-1}$.

For unmagnetized dust:

$k = 0$ to $5.6 \times 10^4\,\text{cm}^{-1}$, and when there is no dust then $n_{0e} = n_{0i}$. Standard values are used for m_e, m_p, Planck constant, Boltzmann constant, electron charge, and ion charge in cgs system.

For the magnetized dust, ω versus k plot in Fig. 1 clearly describes the comparison of quantum and nonquantum dust shear Alfvén wave. Here the angle θ is fixed at 5°. The fact that quantization plays an important role in the propagation of wave at small spatial scale lengths is validated from this figure. Phase speed of the dust shear Alfvén wave in quantum plasmas is much larger and the difference increases as we go to the smaller wavelength.

The Alfvén speed due to the ion inertia is larger than the dust inertia. Therefore, the phase speed of shear Alfvén wave is small for dust and is large for nondust in a quantum dusty magnetoplasmas, shown in Fig. 2. Fig. 3 shows the angle variations between the propagation vector and the magnetic field.

As angle increases beyond some particular value, the wave goes to some what lower frequencies.

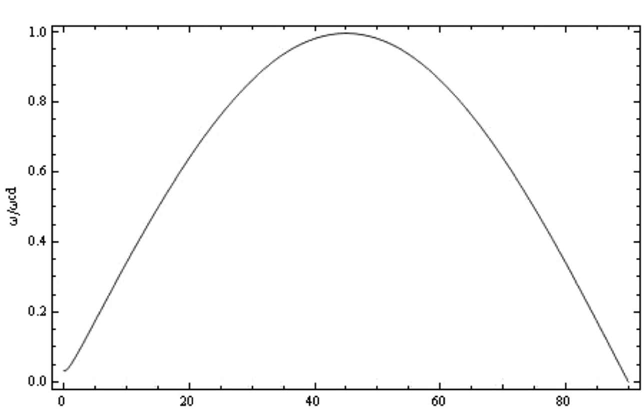

FIG. 3. ω versus k (case 1), variation of θ in a magnetized quantum dusty plasmas at $k = 1.36\,\text{cm}^{-1}$.

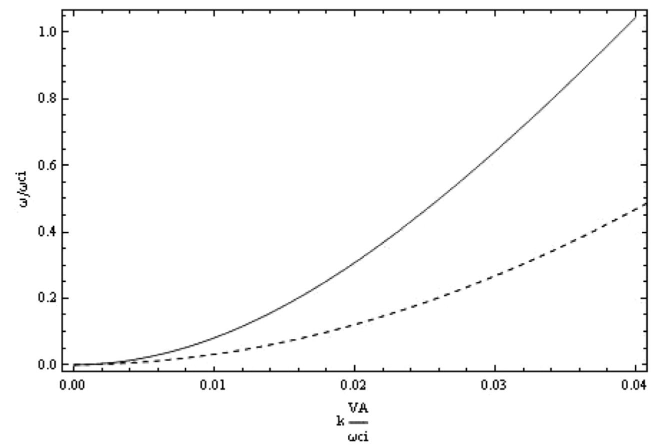

FIG. 4. ω versus k (case 2), for quantum (solid), nonquantum (dashed), and magnetized dust at $\theta = 5°$.

For unmagnetized dust case, Fig. 4 describes the behaviour of quantum modified shear Alfvén wave. The presence of dust and the quantum effect enhances the frequency. The quantized and nonquantized medium give almost similar response to the wave at long wavelengths (small k). At small wavelength (large k), the quantum behaviour of plasmas becomes significant. The bandwidth with respect to λ squeezes and ω increases in quantum plasmas. Fig. 5 examines the role of dust dynamics and shows how the presence of dust dynamics enlarges the spatial scale lengths. Physically, at long wavelength (small k) dust contributes to the wave motion. However, at small wave length (large k), dust effect is not visible as the wave does not observe the presence of dust and therefore behaviour of medium is the same with or without dust. Fig. 6 gives similar description as in Fig. 3.

This dust-modified shear Alfvén wave is a natural mode of any quantum dusty magnetoplasma containing electrons, ions, and dust in the presence of external static magnetic field. Electrons are quantized and magnetized while ions magnetized and dust particles are assumed magnetized/unmagnetized in quantum plasmas. The emission of red and infra-red light from spectra of stars, interstellar space, circumstellar cloud, zodiacal light from inner solar systems, etc., are evidence of dusty plasmas.

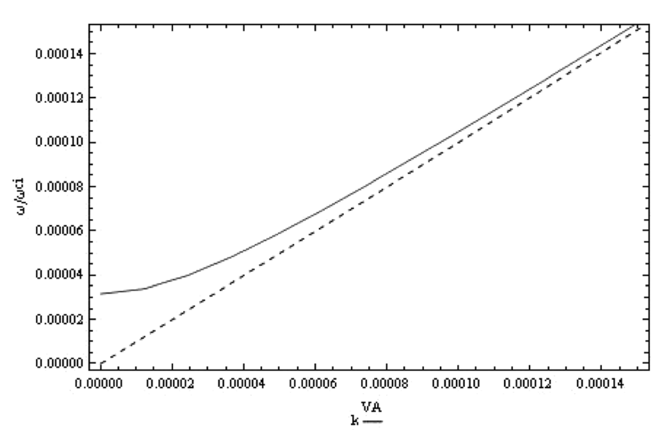

FIG. 5. ω versus k (case 2), with quantum effect, dust (solid) and without dust (dashed) at $\theta = 5°$.

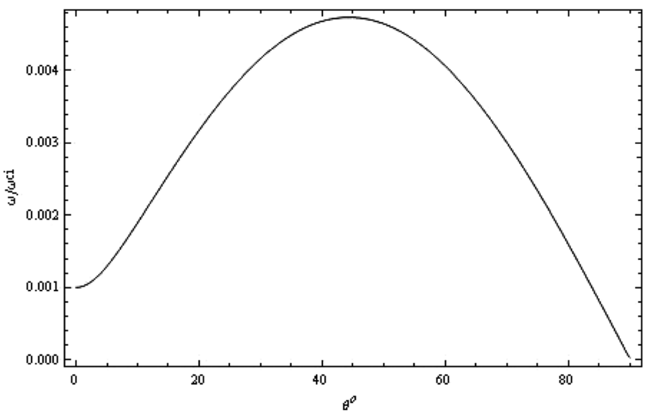

FIG. 6. ω versus k variation of θ in a magnetized quantum dusty plasmas at $k = 1.4 \times 10^3\,\mathrm{cm}^{-1}$.

Pure Alfvén waves propagate parallel to the magnetic field, and is not effected in quantized plasmas, while low frequency, shear Alfvén waves propagate at small angle and, therefore, modified by quantum effects. In summary, we have investigated the electromagnetic shear Alfvén wave in a quantum dusty magnetoplasma. This new frequency involving the dynamics of quantum and magnetized electrons, magnetized ions, and dust particles will give rise to a limit for the electromagnetic wave propagation in quantum dusty plasma environments in the presence of external (ambient) static magnetic field. The parametric cascading of long wavelength electromagnetic waves can lead to the generation of electromagnetic noise in the emission spectra from space plasma environments. The possible observation of ultra-low-frequency long wavelength waves from the Earth-bound noctilucent dusty plasma clouds observed at the Earth's polar summer mesopause or auroral kilo-metric radiation may also be explained in terms of nonlinear interaction of these large amplitude low-frequency electromagnetic waves.

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