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Stability analysis of self-gravitational electrostatic drift waves for a streaming nonuniform quantum dusty magnetoplasma

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Using the quantum hydrodynamic model of plasmas, the stability analysis of self-gravitational electrostatic drift waves for a streaming non-uniform quantum dusty magnetoplasma is presented. For two different frequency domains, i.e., $\Omega_{0d} \ll \omega < \Omega_{0i}$ (unmagnetized dust) and $\omega \ll \Omega_{0d} < \Omega_{0i}$ (magnetized dust), we simplify the general dispersion relation for self-gravitational electrostatic drift waves, which incorporates the effects of density inhomogeneity $\nabla n_{0\alpha}$, streaming velocity $v_{0\alpha}$ due to magnetic field inhomogeneity ∇B_0, Bohm potential, and the Fermi degenerate pressure. For both frequency domains, the effect of density inhomogeneity gives rise to real oscillations while the ions streaming velocity v_{0i} as well as the effective electron quantum velocity v'_{Fe} make these oscillations propagate perpendicular to the ambient magnetic field. This oscillatory behavior of self-gravitational drift waves increases with increase in inhomogeneities and quantum effects while it decreases with increase in the gravitational potential. However, only for the unmagnetized case, the drift waves may become unstable under appropriate conditions giving rise to Jeans instability. The modified threshold condition is also determined for instability by using the intersection method for solving the cubic equation. We note that the inhomogeneity in magnetic field (equilibrium density) through streaming velocity (diamagnetic drift velocity) suppress the Jeans instability depending upon the characteristic scale length of these inhomogeneities. On the other hand, the dust-lower-hybrid wave and the quantum mechanical effects of electrons tend to reduce the growth rate as expected. A number of special cases are also discussed. © *2012 American Institute of Physics*. [http://dx.doi.org/10.1063/1.3698165]

I. INTRODUCTION

The problem of gravitational collapse of astrophysical nebulae for the formation of stars and galaxies has been a great challenge in astrophysics and cosmology. Jeans[1] first showed how a neutral fluid in a nebula containing micron-sized dust grains may become unstable due to its own self-gravity. This is the main mechanism for the large scale nebulae in the Universe to collapse to stars, galaxies, etc. or other structures and their evolution. It is also known that the Jeans instability is a relatively faster process, whereas the formation of heavenly objects takes place in billions of years.[2–7] Obviously, there should be a number of hindering effects on Jeans instability, which may explain the real phenomena behind the gravitational collapse.

It is believed that due to the presence of all-pervading ultra-violet photons, plasma currents or for some heavenly occurrences, the micron-sized dust grains of the collapsing systems can acquire electric charges, and thus, a self-gravitating dusty plasma under extreme conditions may be formed.[8–10] These plasmas may contain static and inhomogeneous ambient magnetic field, nonuniform densities, or even quantum effects under extreme conditions.

In recent years, there has been a growing interest in quantum plasmas because of their importance in microelectronics and electronic devices with nano-electronic components,[11,12] dense astrophysical systems,[13–15] and in laser-produced plasmas.[16–19] When a plasma is cooled to an extremely low temperature, the de Broglie wavelengths of the plasma particles could be comparable to the scale lengths, such as Debye length or Larmor radius, etc. in the system. In such plasmas, the ultracold dense plasma would behave as a Fermi gas and the quantum mechanical effects might play a vital role in the behavior of the charge carriers of these plasmas under the extreme conditions.

Extensive studies have been done over the years by taking into account a wide variety of effects so as to study Jeans instability. Shukla and Stenflo[20] examined the influence and the range of validity of quantum effects on Jeans instabilities of homogeneous and unmagnetized self-gravitating astrophysical quantum dusty plasma systems where the electromagnetic and gravitational forces on plasma charge carriers become comparable. Ren *et al.*[30,31] investigated the Jeans instability in a dense quantum plasma in the presence of two dimensional magnetic fields and the resistive effects with or without Hall current effects. Recently, Prajapati and Chhajlani[32] discussed the contributions of Hall current and viscosity of the medium to Jeans instability and its importance to astrophysical plasmas. The inhomogeneous ambient magnetic field, the nonuniform plasma density, and the quantum effect might play an important role in reducing the growth rate of the Jeans instability of the real physical plasmas. In an earlier paper,[21] we also examined Jeans instability in a homogeneous dusty plasma in the presence of an ambient magnetic field with quantum effect arising through the Bohm potential only.

In this paper, we present a detailed investigation showing how the inhomogeneous ambient magnetic field, the nonuniform density, and quantum effects influence the Jeans instability in a self-gravitational quantum dusty magnetoplasma. We also use the intersection method developed by Omar Khayyam[22] to solve the cubic equation and obtain minimum threshold condition for Jeans instability.

The plan of the paper is as follows. In Sec. II, we solve the quantum hydrodynamic fluid equations and the Poisson's equations for electrostatic and gravitational fields to obtain a general dielectric response function for the nonuniform quantum dusty magnetoplasma. In this section, we also derive the dispersion relation for Jeans instability and its limiting cases and retrieve the previously derived results of Jeans instability in homogeneous quantum dusty magnetoplasma[21] and of drift wave instability in a non-uniform dusty magnetoplasma.[23] Finally, a brief discussion of the results is given in Sec. III.

II. DIELECTRIC RESPONSE FUNCTION

We consider an infinitely extended inhomogeneous high density dusty magnetoplasma containing electrons, ions, and charged dust grains in the presence of an inhomogeneous ambient magnetic field $\mathbf{B}_0$. We choose the Cartesian coordinate system in such a way that $\mathbf{B}_0(x)$ is in the z-direction with the inhomogeneity along x-direction and the wave propagation vector $\mathbf{k}$ is in yz-plane. In order to satisfy the equilibrium conditions, given in Appendix A, we assume that the streaming velocity $\mathbf{v}_{0\alpha}$ is along y-axis and the density inhomogeneity along x-axis. Further, the charge quasi-neutrality condition is $n_{0e}(x) = n_{0i}(x) + Z_d n_{0d}(x)$, where $n_{0\alpha}(x)$ is the equilibrium inhomogeneous number density of α-species ($\alpha = e, i, d$), Z_d is the dust charge state. The density inhomogeneity produces diamagnetic drifts, and the magnetic field inhomogeneity causes uniform streaming of ions and electrons (not for heavy dust particles, i.e., $\mathbf{v}_{0d} = 0$) with $\mathbf{v}_{0e,i} = -\frac{c\partial B_0(x)/\partial x}{4\pi q_d n_{od}}\hat{y}$. Such plasmas may exist in the interiors and environments of astrophysical compact objects, e.g., white dwarfs and neutron stars/magnetars, supernovae, etc.[33,34] The governing linearized equations for electrostatic wave propagation in the quantum hydrodynamic (QHD) model[24–29] for the electrons, ions, and charged dust grains in the presence of the inhomogeneous ambient magnetic field $\mathbf{B}_0(x)$ are

$$m_\alpha n_{0\alpha}\left(\frac{\partial \mathbf{v}_{1\alpha}}{\partial t} + (\mathbf{v}_{0\alpha}.\nabla)\mathbf{v}_{1\alpha}\right) = n_{0\alpha}q_\alpha\left(\mathbf{E}_1 + \frac{1}{c}\mathbf{v}_\alpha \times \mathbf{B}_0\right) - k_B T_{F\alpha}\nabla n_{1\alpha} - m_\alpha n_{0\alpha}\nabla\psi_1 + \frac{\hbar}{4m_\alpha}\nabla(\nabla^2 n_{1\alpha}) \quad (1)$$

and

$$\frac{\partial n_{1\alpha}}{\partial t} + \mathbf{v}_{0\alpha}.\nabla n_{1\alpha} + \mathbf{v}_{1\alpha}.\nabla n_{0\alpha} + n_{0\alpha}\nabla\cdot\mathbf{v}_{1\alpha} = 0. \quad (2)$$

Poisson's equations for the perturbed electrostatic potential ϕ_1 and gravitational potential ψ_1 are

$$\nabla^2\phi_1 = -4\pi\sum q_\alpha n_{1\alpha} \quad (3)$$

and

$$\nabla^2\psi_1 = 4\pi G m_\alpha n_{1\alpha}, \quad (4)$$

where subscript 0 indicates equilibrium quantities while 1 is used for the perturbed quantities., summation over α is for the three species, i.e., $(\alpha = e, i, d), \hbar$ is Planck's constant divided by 2π and q_α, m_α, c, and G are the charge, mass, the velocity of light in vacuum, and gravitational constant, respectively. Here, we may take into account the quantum effects of all the species when they are considered extremely cold. In Eq. (1), we assume that the plasma particles in a zero-temperature Fermi gas satisfying the pressure $p_{F\alpha} = m_\alpha v_{F\alpha}^2 n_{1\alpha}$, where $v_{F\alpha} = (2k_B T_{F\alpha}/m_\alpha)^{\frac{1}{2}}$ is the Fermi speed; k_B and $T_{F\alpha}$ are the Boltzmann constant and Fermi temperature, respectively.

By assuming that perturbations have sinusoidal character, i.e.,

$$(E_1, B_1, n_{1\alpha}, \mathbf{v}_{1\alpha}) \approx e^{-i\omega t + i\underline{k}\cdot\underline{r}}, \quad (5)$$

we may rewrite the Eqs. (1)–(4) as

$$\mathbf{v}_{1\alpha} = \frac{q_\alpha \mathbf{k}\phi_1}{\omega^* m_\alpha} + \frac{i}{\omega^*}\mathbf{v}_{1\alpha}\times\mathbf{\Omega}_{0\alpha} + \frac{\mathbf{k}v_{F\alpha}'^2}{\omega^*}\left(\frac{n_{1\alpha}}{n_{0\alpha}}\right) + \frac{\mathbf{k}}{\omega^*}\psi_1, \quad (6)$$

$$\frac{n_{1\alpha}}{n_{0\alpha}} = \left(\frac{\mathbf{k}\cdot\mathbf{v}_{1\alpha}}{\omega^*}\right) + \frac{i v_{1\alpha}^x}{L_\alpha \omega^*}, \quad (7)$$

$$k^2\phi_1 = 4\pi\sum q_{0\alpha} n_{1\alpha}, \quad (8)$$

$$k^2\psi_1 = -4\pi G m_\alpha n_{1\alpha}. \quad (9)$$

Here, ω and $\mathbf{k}$ are the angular frequency and wavenumber vector, respectively. We have also defined

$$v_{F\alpha}'^2 = v_{F\alpha}^2 + \frac{\hbar^2 k^2}{4m_\alpha^2}, \quad \Omega_{0\alpha} = \frac{q_\alpha B_0}{m_\alpha c}\hat{z} \quad \text{and} \quad \omega^* = \omega - k_y v_{0\alpha}.$$

Using Eqs. (6)–(9) and after some straightforward calculations, we obtain the general dielectric susceptibility for a dusty magnetoplasma with magnetic field and density inhomogeneities as

$$\chi_\alpha = -\frac{\omega_{p\alpha}^2\left\{\frac{k_z^2}{(\omega^*)^2} + \frac{k_y^2}{(\omega^*)^2 - \Omega_{0\alpha}^2}\left(1 - \frac{\Omega_{0\alpha}}{\omega^* k_y L_\alpha}\right)\right\}}{k^2 - \left\{\frac{k_z^2}{(\omega^*)^2} + \frac{k_y^2}{(\omega^*)^2 - \Omega_{0\alpha}^2}\left(1 - \frac{\Omega_{0\alpha}}{\omega^* k_y L_\alpha}\right)\right\}(k^2 v_{F\alpha}'^2 - \omega_{J\alpha}^2)}, \quad (10)$$

where

$$\omega_{p\alpha}^2 = \frac{4\pi n_{0\alpha} q_\alpha^2}{m_\alpha}, \quad \omega_{J\alpha}^2 = 4\pi G m_\alpha n_{0\alpha} \quad \text{and} \quad L_\alpha = -\frac{n_{0\alpha}}{\partial n_{0\alpha}/\partial x}.$$

Using Eq. (10), the electrostatic dielectric response function is given by

$$\epsilon(\omega,k) = 1 + \sum_\alpha \chi_\alpha = 1 - \sum_\alpha \frac{\omega_{p\alpha}^2\left\{\frac{k_z^2}{(\omega^*)^2} + \frac{k_y^2}{(\omega^*)^2 - \Omega_{0\alpha}^2}\left(1 - \frac{\Omega_{0\alpha}}{\omega^* k_y L_\alpha}\right)\right\}}{k^2 - \left\{\frac{k_z^2}{(\omega^*)^2} + \frac{k_y^2}{(\omega^*)^2 - \Omega_{0\alpha}^2}\left(1 - \frac{\Omega_{0\alpha}}{\omega^* k_y L_\alpha}\right)\right\}(k^2 v_{F\alpha}'^2 - \omega_{J\alpha}^2)}. \tag{11}$$

Eq. (11) is the electrostatic dielectric response function, which incorporates the effects of inhomogeneities, quantum effects through both the Fermi potential and the Bohm potential and the effect of gravitational potential. On ignoring the inhomogeneities, this function immediately reduces to the previously derived response function.[21]

In the following section, we shall derive the dispersion relation for the self-gravitational electrostatic drift waves including the quantum effects for the electron dynamics, density inhomogeneity only for electrons and ions and the gravitational effects for dust grains. The quantum effects on ions and dust grains are neglected due to their heavier masses. Also, being insignificantly small, we can neglect the gravitational effects on electrons and ions. However, for dust grain, the self-gravitational effect is taken into account. Thus, for such plasmas satisfying the above conditions and assuming that the wave frequency and Doppler shifted frequency is much less than the gyro frequency of electron, i.e., $\omega, \omega^* \ll |\Omega_{0e}|$ and the phase velocity of the wave is less than the effective electron (quantum) velocity but greater than the ion one, i.e., $v_{Fi}' \ll \omega/k_z \ll v_{Fe}'$, we can write the dispersion relation of electrostatic waves, i.e., $\epsilon(\omega,k) = 0$ by simplifying Eq. (11) as

$$1 + \frac{\omega_{pe}^2}{k^2 v_{Fe}'^2} - \frac{\omega_{pi}^2}{k^2}\left[\frac{k_z^2}{(\omega^*)^2} + \frac{k_y^2}{(\omega^*)^2 - \Omega_{0i}^2}\left(1 - \frac{\Omega_{0i}}{k_y\omega^* L_i}\right)\right] - \frac{\omega_{pd}^2\left[\frac{k_z^2}{\omega^2} + \frac{k_y^2}{\omega^2 - \Omega_{0d}^2}\right]}{k^2 + \left[\frac{k_z^2}{\omega^2} + \frac{k_y^2}{\omega^2 - \Omega_{0d}^2}\right]\omega_{jd}^2} = 0. \tag{12}$$

The condition for the phase velocity (i.e., $v_{Fi}' \ll \omega/k_z \ll v_{Fe}'$) defines that electrons which run along magnetic lines will reach thermal equilibrium condition quickly whereas ions cannot reach thermal equilibrium and should be described by the drift equation.

In the following section, we will discuss two frequency domains, i.e., $\Omega_{0d} \ll \omega < \Omega_{0i}$ for unmagnetized dust and $\omega \ll \Omega_{0d} < \Omega_{0i}$ for magnetized dust grains.

A. For unmagnetized dust grains

For this case, we shall assume the intermediate frequency domain, i.e., the wave frequency lies between the gyro frequencies of dust and ions ($\Omega_{0d} \ll \omega < \Omega_{0i}$). Thus, the dispersion relation reduces to

$$1 + \frac{\omega_{pe}^2}{k^2 v_{Fe}'^2} - \frac{\omega_{pi}^2}{k^2}\left[\frac{k_z^2}{(\omega^*)^2} - \frac{k_y^2}{\Omega_{0i}^2}\left(1 - \frac{\Omega_{0i}}{k_y\omega^* L_i}\right)\right] - \frac{\omega_{pd}^2}{\omega^2 + \omega_{jd}^2} = 0. \tag{13}$$

If we assume that the propagation is predominantly perpendicular, i.e., $k_z^2 \ll k_y^2$, the above equation takes a simpler form

$$1 + \frac{\omega_{pe}^2}{k_y^2 v_{Fe}'^2} + f_i - \frac{\omega_i'}{\omega^*} - \frac{\omega_{pd}^2}{\omega^2 + \omega_{Jd}^2} = 0, \tag{14}$$

where

$$\omega_i' = \frac{\omega_{pi}^2}{\Omega_{0i} L_i k_y} \quad \text{and} \quad f_i = \frac{\omega_{pi}^2}{\Omega_{0i}^2}.$$

From Eq. (14), we notice that the drift wave may become unstable due to the presence of dust particles satisfying the certain threshold conditions for both the inhomogeneous plasma (i.e., $\omega_i' \neq 0$) and the homogeneous (i.e., $\omega_i' = 0$). In the absence of dust particles, we only obtain the real oscillations in the former case while in the latter no wave would exist. We also note that for the plasma system satisfying above assumptions (see Appendix A and Eq. (14)), the effect of streaming will appear only if the inhomogeneities in both the ambient magnetic and the equilibrium density are present.

We may rewrite the above Eq. (14) as

$$0 = \omega^3 - (k_y v_0 + \omega_i'/f_i F)\omega^2 + \omega(\omega_{Jd}^2 - \omega_{dlh}^2/F) - \left\{(k_y v_0 + \omega_i'/f_i F)(\omega_{Jd}^2 - \omega_{dlh}^2/F) + (\omega_{dlh}^2/F)(\omega_i'/f_i F)\right\}, \tag{15}$$

where

$$F = 1 + \frac{1}{f_i} + \frac{\omega_{pe}^2}{f_i k_y^2 v_{Fe}'^2}, \quad \omega_{dlh} = \frac{\omega_{pd}\Omega_{0i}}{\omega_{pi}}.$$

The above equation becomes cubic in ω due to inhomogeneities and gives three roots. We observe that one root remains always real whereas the other two roots can become complex if the discriminant of the cubic equation becomes negative. One of them would give the instability called Jeans instability for inhomogeneous streaming dusty magnetoplasma satisfying the threshold condition. Then, the other root would give damping along with the negative real frequency, which is unphysical and is thus ignored.

In order to get the exact threshold condition for Jeans instability, we use intersection method.[22] We split this cubic equation into a cubic and a parabolic function and then find out the intersection points for roots. The mathematical formulation of this intersection method is given in Appendix B. We note that the instability would occur only if the y-component of the vertex point of parabola (i.e., minima) is greater than zero. Thus, the resultant threshold condition in Eq. (B6) is given by

$$(\omega_{Jd}^2 - \omega_{dlh}^2/F) > 2(k_y v_{0i} + \omega_i'/f_i F)^2 \times \left(\sqrt{1 + \frac{(\omega_{dlh}^2/F)(\omega_i'/f_i F)}{(k_y v_{0i} + \omega_i'/f_i F)^3}} - 1\right). \tag{16}$$

The above threshold condition contains modification due to the inhomogeneities in the equilibrium density and in the ambient magnetic field. It is evident that both the density inhomogeneity and the streaming velocity not only reduce the growth rate of Jeans instability but also give rise to real oscillations. Similarly, the quantum effects through both the Fermi and the Bohm potentials also tend to stabilize the instability. Further, we observe that the gravitational potential reduces the real oscillations and that the ion streaming velocity v_{0i} and the effective quantum velocity v_{Fe}' causes the drift waves to propagate perpendicular to the ambient magnetic field.

We solve Eq. (15) numerically and present it graphically in Fig. 1 for typical parameters[34] (cgs system of units) for the interiors of the neutron stars, magnet stars, and white dwarfs, $m_e = 9.0 \times 10^{-28}$ g, $m_i = 12 m_p (m_p = 1.672 \times 10^{-24} g)$, $m_d \sim 10^{15} m_i, n_{oe} \simeq n_{oi} \sim 10^{27} \mathrm{cm}^{-3}, n_{od} = 10^{-6} n_{oi}, Z = 10^3$, $L_e = L_i \sim 1000$ cm, and $B_o = 10^9 G$. We note that for these parameters, the last term in curly brackets of cubic Eq. (15) becomes negligibly small and thus the equation reduces to

$$0 = \omega^3 - (k_y v_{0i} + \omega_i'/f_i F)\omega^2 + \omega(\omega_{Jd}^2 - \omega_{dlh}^2/F) - \{(k_y v_{0i} + \omega_i'/f_i F)(\omega_{Jd}^2 - \omega_{dlh}^2/F)\}, \tag{17}$$

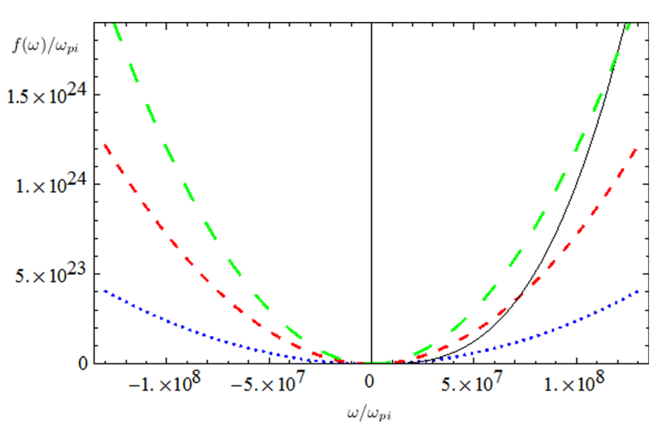

FIG. 1. The graph of $f(\omega)/\omega_{pi}$ vs ω/ω_{pi} for Eq. (15). Solid curve is for Eq. (2) and dotted curves are for Eq. (3) for set of parameters given after Eq. (16) with variation of streaming velocity as (i) dotted for $v_{0i} = 10^{-7}c$, (ii) small dashed for $v_{0i} = 3 \times 10^{-7}c$, (iii) large dashed for $v_{0i} = 5 \times 10^{-7}c$.

which may be factorized as

$$\{\omega^2 + (\omega_{Jd}^2 - \omega_{dlh}^2/F)\}\{\omega - (k_y v_{0i} + \omega_i'/f_i F)\} = 0. \tag{18}$$

The first root gives the growth rate of Jeans instability as

$$\gamma = (\omega_{Jd}^2 - \omega_{dlh}^2/F)^{\frac{1}{2}} \tag{19}$$

with the threshold condition

$$\omega_{Jd}^2 - \omega_{dlh}^2/F > \ 0 \ \text{ or} \frac{\left(\frac{\omega_{pe}}{\omega_{pd}}\right)\omega_{dlh}}{\left[\frac{\omega_{dlh}^2}{\omega_{Jd}^2} - \left(1 + \frac{\Omega_{0i}^2}{\omega_{pi}^2}\right)\right]^{\frac{1}{2}}} \geq k_y v_{Fe}'. \tag{20}$$

The second root gives real oscillations propagating with the streaming and the Fermi velocities as

$$\omega_r = k_y v_{0i} + \omega_i'/f_i F. \tag{21}$$

Eqs. (19) and (21) show that due to the presence of inhomogeneities, the real propagating oscillatory behavior is also observed with the Jeans instability. We observe that the dust-lower-hybrid wave and the quantum mechanical effects of electrons tend to reduce the growth rate. We also note that real frequency of self-gravitational drift waves increases with the increase in both the inhomogeneities and the quantum effects through the Fermi and the Bohm potentials.

The expression for the growth rate in Eq. (19) is different from the previously derived result[21] as it contains the contribution of quantum effects of electrons instead of dust grains. Thus, as a result, we obtain a new threshold condition on wavenumber for the instability to occur. For the chosen parameters, the general threshold condition given in Eq. (16) also immediately reduces to the above condition given in Eq. (20) because the second term in the square root becomes negligibly small.

Eq. (15) is plotted in Fig. 1 for the above set of parameters. We observe that the real frequency increases with the increase of streaming velocity. We also note that for these parameters, the values of the real frequency fulfill the condition of $\Omega_{0d} \ll \omega < \Omega_{0i}$.

For homogeneous plasma for which $\omega_i' \to 0$, Eq. (14) becomes

$$\omega^2 = -\omega_{Jd}^2 + \omega_{dlh}^2/F,$$

giving purely growing Jeans instability as

$$\gamma = (\omega_{Jd}^2 - \omega_{dlh}^2/F)^{\frac{1}{2}}.$$

If the gravitational potential is absent, Eq. (14)

$$1 + \frac{\omega_{pe}^2}{k_y^2 v_{Fe}'^2} + f_i - \frac{\omega_i'}{\omega^*} - \frac{\omega_{pd}^2}{\omega^2} = 0$$

or

$$1 - \frac{(\omega_i'/f_i F)}{\omega^*} - \frac{\omega_{dlh}^2/F}{\omega^2} = 0.$$

This is the same dispersion relation as derived by Salimullah *et al.*[23] and by letting $\omega = k_y v_{0i} + \delta$, where $\delta \ll k_y v_{0i}$, the above equation becomes

$$1 - \frac{(\omega_i'/f_i F)}{\delta} - \frac{\omega_{dlh}^2/F}{k_y^2 v_{0i}^2}\left(1 - \frac{2\delta}{k_y v_{0i}}\right) = 0.$$

The growth rate of drift wave is given by ($\omega = \omega_r + i\gamma$)

$$\gamma = \frac{\sqrt{|\omega_i^*| k_y^3 v_0^3}}{\sqrt{2}\omega_{pd}} \sqrt{1 - \frac{\omega_{pd}^2}{8|\omega_i^*| k_y v_0}\left(\frac{k_y^2 v_{0i}^2}{\omega_{pd}^2/F'} - 1\right)^2}, \tag{22}$$

where

$$F' = 1 + \frac{\omega_{pe}^2}{k^2 v_{Fe}'^2} + f_i.$$

This growth rate of drift wave is more general with the threshold condition

$$|\omega_i^*| > \frac{\omega_{pd}}{2\sqrt{2} k_y v_0}\left(\frac{k_y^2 v_{0i}^2}{\omega_{pd}^2/F'} - 1\right). \tag{23}$$

If we assume $k_y v_{0i} \sim \omega_{pd}/\sqrt{F'}$, it immediately reduces to the result of Salimullah *et al.*,[23] i.e.,

$$\gamma = \frac{\omega_{pi}(k_y v_{0i})^{\frac{3}{2}}}{\sqrt{2}\omega_{pd}\sqrt{k_y \Omega_{0i}|L_i|}}. \tag{24}$$

B. For magnetized dust grains

If the dust is also magnetized, the dispersion relation Eq. (12) takes the form

$$1 + \frac{\omega_{pe}^2}{k^2 v_{Fe}'^2} - \frac{\omega_{pi}^2}{k^2}\left[\frac{k_z^2}{(\omega^*)^2} - \frac{k_y^2}{\Omega_{0i}^2}\left(1 - \frac{\Omega_{0i}}{k_y \omega^* L_i}\right)\right] - \frac{\omega_{pd}^2\left[\frac{k_z^2}{\omega^2} - \frac{k_y^2}{\Omega_{0d}^2}\right]}{k^2 + \left[\frac{k_z^2}{\omega^2} - \frac{k_y^2}{\Omega_{0d}^2}\right]\omega_{jd}^2} = 0. \tag{25}$$

For perpendicular propagation (i.e., $k_z = 0$), the above equation reduces to

$$1 - \frac{(\omega_i'/f_i F)}{\omega^*} + \frac{\omega_{dlh}^2/F}{\Omega_{0d}^2 - \omega_{jd}^2} = 0$$

or

$$\omega = k_y v_{0i} + \frac{(\omega_i'/f_i F)}{\left(1 + \frac{\omega_{dlh}^2/F}{\Omega_{0d}^2 - \omega_{jd}^2}\right)}. \tag{26}$$

From the above equation, it is evident that for the magnetized dust grains, we only obtain stable oscillations and that these oscillations reduce with the increase in the gravitational potential.

III. SUMMARY OF RESULTS AND DISCUSSION

We present the stability analysis of self-gravitational electrostatic drift waves for a streaming non-uniform quantum dusty magnetoplasma by using the quantum hydrodynamic model of plasmas. Incorporating the effects of density inhomogeneity $\nabla n_{0\alpha}$, streaming velocity $v_{0\alpha}$ due to magnetic field inhomogeneity ∇B_0, Bohm potential, and the Fermi degenerate pressure, we first derive the general dispersion relation for self-gravitational electrostatic drift waves and then simplify it for two different frequency domains, i.e., $\Omega_d \ll \omega < \Omega_i$ (unmagnetized dust) and $\omega \ll \Omega_d < \Omega_i$ (magnetized dust).

For both frequency domains, the effects of density inhomogeneity give rise to real oscillations and the ion streaming velocity v_{0i} and the effective quantum velocity provide the source to propagate these oscillations perpendicular to the ambient magnetic field. This oscillatory behavior of self-gravitational electrostatic waves increases with increase in inhomogeneities and decreases with increase in quantum effects through both the Bohm potential and Fermi potential and in gravitational potential.

We also note that for the magnetized case, the available free energy through density inhomogeneity and the streaming velocity is not sufficient to make the drift waves unstable. However, only for the unmagnetized case, the electrostatic drift waves may become unstable under appropriate conditions. These unstable self-gravitational drift waves give rise to Jeans instability, and by using the intersection method for solving cubic equation, we also determine the minimum threshold condition for Jeans instability. We also note that the inhomogeneity in the ambient magnetic field (equilibrium density) through streaming velocity (diamagnetic drift velocity) suppresses the Jeans instability depending upon the characteristic scale length of these inhomogeneities. On the other hand, the dust-lower-hybrid wave and the quantum mechanical effects of electrons tend to reduce the growth rate.

For the unmagnetized case, we further simplify the dispersion relation for growth rate in some limiting cases, e.g., in the absence of gravitational potential, we obtain a new dispersion relation for the growth rate of drift wave instability in non-uniform quantum dusty magnetoplasma, which incorporates the quantum effects through electrons. We also retrieve the growth rates of Jeans instability in a homogeneous dusty magnetoplasma.

Our results in this paper may be useful for the study of Jeans instability and the possible drift waves for the nonuniform streaming dusty quantum plasmas, which may occur, e.g., in dense astrophysical systems, i.e., the interiors of white dwarfs and neutron stars,[13–15] laser-produced plasmas,[16–19] and in the laboratory plasmas.[11,12]

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APPENDIX A: ZEROTH ORDER EQUATIONS

Assuming that there is no external electric field $\mathbf{E}_0$, constant streaming velocity $\mathbf{v}_0$, and equilibrium density is space dependent only, we may write the momentum equation as

$$0 = \frac{q_\alpha}{c}(\mathbf{v}_{0\alpha} \times \mathbf{B}_0) - \left[\frac{\gamma k_B T_{F\alpha}}{n_{0\alpha}} - \frac{\hbar \nabla^2}{4 n_{0\alpha} m_\alpha}\right] \boldsymbol{\nabla} n_{0\alpha}. \quad \text{(A1)}$$

The equation of continuity as

$$\mathbf{v}_{0\alpha} \cdot \boldsymbol{\nabla} n_{0\alpha} = 0 \quad \text{(A2)}$$

and the Maxwell curl equation as

$$c\boldsymbol{\nabla} \times \mathbf{B}_0 = 4\pi \sum n_{0\alpha} q_{o\alpha} \mathbf{v}_{0\alpha}. \quad \text{(A3)}$$

Taking cross product of Eq. (A1) with $\mathbf{B}_0$, we obtain

$$\mathbf{v}_{0\alpha} = \frac{c[\gamma k_B T_{F\alpha}]}{q_\alpha n_{0\alpha}} \left(\frac{\mathbf{B}_0 \times \boldsymbol{\nabla} n_{0\alpha}}{B_0^2}\right). \quad \text{(A4)}$$

Using Eq. (4) in Eq. (A3), we get

$$\boldsymbol{\nabla} \times \mathbf{B}_0 = 4\pi \sum \gamma k_B T_{F\alpha} \left(\frac{\mathbf{B}_0 \times \boldsymbol{\nabla} n_{0\alpha}}{B_0^2}\right). \quad \text{(A5)}$$

Comparing Eqs. (A4), $\mathbf{v}_{0\alpha}$ takes the form

$$\mathbf{v}_{0\alpha} = \frac{c\gamma k_B T_{F\alpha}}{q_\alpha n_{0\alpha}} \left(\frac{\mathbf{B}_0 \times \boldsymbol{\nabla} n_{0\alpha}}{B_0^2}\right) = \frac{c}{4\pi q_\alpha n_{0\alpha}} \boldsymbol{\nabla} \times \mathbf{B}_0. \quad \text{(A6)}$$

APPENDIX B: INTERSECTION METHOD FOR CUBIC EQUATION

Eq. (15) may be rewritten as

$$\omega^3 - B\omega^2 + \omega C - G = 0, \quad \text{(B1)}$$

where

$$B = (k_y v_0 + \omega_i'/f_i F), \quad C = (\omega_{Jd}^2 - \omega_{dlh}^2/F),$$
$$D = (\omega_{dlh}^2/F)(\omega_i'/f_i F)$$

and

$$G = BC + D.$$

The coefficients B and D are always positive for our considered plasma system. For Jeans instability, the coefficient C is also positive. Therefore, all coefficients are kept positive in the above equation.

Now we use the intersection method for a cubic function

$$f(\omega) = \omega^3 \quad \text{(B2)}$$

and the parabolic function

$$f(\omega) = B\omega^2 - \omega C + G. \quad \text{(B3)}$$

The above equation may be written as a standard equation of parabola

$$\left(\omega - \frac{C}{2B}\right)^2 = 4\left(\frac{1}{4B}\right)\left[y - \left(G - \frac{C^2}{4B}\right)\right] \quad \text{(B4)}$$

with the vertex point

$$(h, k) = \left(\frac{C}{2B}, G - \frac{C^2}{4B}\right).$$

We note that the instability will occur when the y-intercept of vertex point will be greater than zero, which is

$$G - \frac{C^2}{4B} > 0 \quad \text{(B5)}$$

or

$$(\omega_{Jd}^2 - \omega_{dlh}^2/F)^2 - 4(k_y v_{0i} + \omega_i'/f_i F)^2(\omega_{Jd}^2 - \omega_{dlh}^2/F)$$
$$- 4(k_y v_{0i} + \omega_i'/f_i F)(\omega_{dlh}^2/F)(\omega_i'/f_i F) < 0.$$

By solving the above quadratic equation, one root gives the threshold condition

$$(\omega_{Jd}^2 - \omega_{dlh}^2/F) > 2(k_y v_{0i} + \omega_i'/f_i F)^2 \times \left(\sqrt{1 + \frac{(\omega_{dlh}^2/F)(\omega_i'/f_i F)}{(k_y v_{0i} + \omega_i'/f_i F)^3}} - 1\right). \quad \text{(B6)}$$

Here, we have selected one root which gives the Jeans instability and neglected the other because that gives the negative real frequency, which is unphysical.

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