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Anomalous skin effects in a weakly magnetized degenerate electron plasma

G. Abbas,[1,a)] M. Sarfraz,[1] and H. A. Shah[2]
[1]Department of Physics, GC University Lahore, Katchery Road, Lahore 54000, Pakistan
[2]Forman Christian College University, Farozpur Road, Lahore 54600, Pakistan

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Fully relativistic analysis of anomalous skin effects for parallel propagating waves in a weakly magnetized degenerate electron plasma is presented and a graphical comparison is made with the results obtained using relativistic Maxwellian distribution function [G. Abbas, M. F. Bashir, and G. Murtaza, Phys. Plasmas **18**, 102115 (2011)]. It is found that the penetration depth for R- and L-waves for degenerate case is qualitatively small in comparison with the Maxwellian plasma case. The quantitative reduction due to weak magnetic field in the skin depth in R-wave for degenerate plasma is large as compared to the non-degenerate one. By ignoring the ambient magnetic field, previous results for degenerate field free case are salvaged [A. F. Alexandrov, A. S. Bogdankevich, and A. A. Rukhadze, *Principles of Plasma Electrodynamics* (Springer-Verlag, Berlin/Heidelberg, 1984), p. 90]. © *2014 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4894698]

I. INTRODUCTION

Collisionless wave penetration is a fundamental phenomenon that is important for a number of applications, in particular, as a mechanism of collisionless heating of magnetically confined, laser and low temperature (industrial) plasmas. The effect was first identified by Landau,[1] who discovered the temporal collisionless damping of an infinite (in space) wave. This is particularly known as Landau damping. He also considered the boundary value problem, yielding a spatial decay of the longitudinal wave launched from the plasma boundary. The spatial decay of the transverse (electromagnetic) wave due to the wave-particle interaction was studied later[2,3] and is usually referred to as the anomalous skin effect. In the normal skin effect regime in metallic plasmas, the wave vector of the electromagnetic wave obeys the relation $k_n \sim (i\omega/\nu)^{1/2}$. On the other hand, at low temperatures, the interaction between the current and the electric field becomes non-local, and the skin effect becomes anomalous.[4,5] The wave vector is independent of the electron collisional frequency ν and become proportional to $k_{as} \sim (i\omega)^{1/3}$. Here ω is the frequency of an electromagnetic wave. For ordinary metals, the electron gas obeys Fermi statistics. At the frequencies usually employed in radio engineering (e.g., $\omega \sim 10^{10}\,\mathrm{s}^{-1}$ and $\omega_p \approx 10^{15} - 10^{10}\,\mathrm{s}^{-1}$), the condition for anomalous skin effect $\omega \ll \omega_p$ is satisfied within a large margin.

The theory of anomalous skin-effect was originally developed for metals[6] and for plasmas.[7] The developed theoretical methods were naturally extended to metals in the presence of external magnetic field.[8,9] It was shown that the presence of the magnetic field weakly affects the anomalous skin effect in a certain region of fields and frequencies. The application of static magnetic fields to thin metal films has resulted in a variety of finite size and resonance effects.[10]

The anomalous skin effects in solid state plasmas have been studied by number of authors, e.g., Dragila[11] studied the surface waves in the regime of the anomalous skin effect. Mattie[12] numerically analyzed the laser energy absorption and transmission by the anomalous skin effect in hot plasmas and compared results with Weibel.[13] In some applications, e.g., in the interaction of ultrashort laser pulses with solid targets, the relativistic effects also become important in describing the anomalous skin effects.[14–16]

In an earlier investigation, the anomalous skin effects in a classical relativistic parallel propagating weakly magnetized electron plasma waves have been discussed using relativistic Maxwellian distribution function.[18] Subsequently, the high frequency parallel and perpendicular propagating modes were investigated in a weakly magnetized relativistic degenerate plasma having number densities of the order of 10^{26}–$10^{34}\,\mathrm{cm}^{-3}$ and an ambient magnetic field of the order of 10^9–10^{10}G. This investigation was carried out in the limits, i.e., $\omega > k.v$, Ω.[19,20] However, these have not been discussed yet to describe the anomalous skin effects for relativistic weakly magnetized degenerate plasmas. A useful formulation of anomalous skin effect has been given for non relativistic and ultrarelativistic unmagnetized degenerate plasma.[17] In the present investigation, we derive an explicit expression for the anomalous skin effects for weakly magnetized parallel propagating relativistic degenerate plasma waves and compare it with our previous results.[18] The modes may be useful to describe the linear behavior of the plasma wave absorption exist in the superdense objects having weakly magnetized plasma, i.e., $\Omega_0/\omega_{0p} < 1$.

II. DISPERSION RELATIONS OF RELATIVISTIC R-& L-WAVES IN A WEAKLY MAGNETIZED DEGENERATE PLASMA

In this section, we analyze anomalous skin effects in degenerate plasma. Earlier, Abbas *et al.*[19] discussed parallel propagating R-& L-waves in relativistic degenerate plasma using Vlasov Maxwell's model. The investigation was carried out in the high frequency (i.e., $\omega > \mathbf{k}.\mathbf{v}$) and weakly magnetized (i.e., $|\omega - \mathbf{k}.\mathbf{v}| > \Omega$) limits. In the high

a)Electronic mail: gohar.abbas@gcu.edu.pk

frequency limit $\omega > \mathbf{k.v}$, it is not possible to investigate space or time scale of damping. This is possible only when the phase velocity of wave becomes less than particle thermal velocity (Maxwellian plasma) or the Fermi velocity (Degenerate plasma) (i.e., $\omega < \mathbf{k}\cdot(\mathbf{v},\mathbf{v}_F)$. The dispersion relations for left and right circularly polarized waves are given by

$$\omega^2 - c^2k_z^2 + \Pi_{xx} \pm i\Pi_{xy} = 0, \tag{1}$$

where

$$\begin{aligned}\Pi_{xx} = \Pi_{yy} &= \frac{8\pi^2 e^2\omega^2}{k_z^2}\int_0^\infty dp\frac{p^2}{v}\frac{\partial f_o}{\partial|\mathbf{p}|}\left[1-\frac{\omega^2-k_z^2v^2}{2\omega k_z v}\log\left|\frac{\omega+k_zv}{\omega-k_zv}\right| - \frac{i\pi k_z v}{2\omega}\left(1-\frac{\omega^2}{k_z^2v^2}\right)H\left(1-\frac{\omega}{k_zv}\right)\right],\\ \Pi_{xy} = -\Pi_{yx} &= -i\frac{16\pi^2e^2\omega}{k_z^2}\int_0^\infty dp\frac{p^2\Omega}{v}\frac{\partial f_o}{\partial|\mathbf{p}|}\left[1-\frac{\omega}{2k_zv}\log\left|\frac{\omega+k_zv}{\omega-k_zv}\right| - \frac{i\pi k_z v}{4}\frac{\partial}{\partial\omega}\left(\left(1-\frac{\omega^2}{k_z^2v^2}\right)H\left(1-\frac{\omega}{k_zv}\right)\right)\right],\end{aligned} \tag{2}$$

where $H\left(1-\frac{\omega}{k_zv}\right)$ is the Heaviside function. Here, we mention that we have rewritten Eq. (2) given in Ref. 19 by including pole contribution under the limit $|\omega - \mathbf{k}\cdot\mathbf{v}| > \Omega$. Rewriting the above components in terms of relativistic energy $E = \gamma m_o c^2 = c\sqrt{p^2 + m_o^2c^2}$, we obtain

$$\begin{aligned}\Pi_{xx} = \Pi_{yy} = &\frac{8\pi^2e^2\omega^2}{c^3k_z^2}\int_{m_oc^2}^\infty E\sqrt{E^2-m_o^2c^4}\times\left[1-\frac{\omega^2E^2-k_z^2c^2\left(E^2-m_o^2c^4\right)}{2\omega k_z cE\sqrt{E^2-m_o^2c^4}}\log\left|\frac{E\omega+k_zc\sqrt{E^2-m_o^2c^4}}{E\omega-k_zc\sqrt{E^2-m_o^2c^4}}\right|\right.\\ &\left.-\frac{i\pi k_zc}{2\omega}\frac{\sqrt{E^2-m_o^2c^4}}{E}\left(1-\frac{\omega^2E^2}{k_z^2c^2\left(E^2-m_o^2c^4\right)}\right)\times H\left(1-\frac{\omega E}{k_zc\sqrt{E^2-m_o^2c^4}}\right)\right]\frac{\partial f_o}{\partial E}dE\\ \Pi_{xy} = -\Pi_{yx} = &-\frac{16\pi^2ie^2\omega}{ck_z^2}m_o\Omega_o\int_{m_oc^2}^\infty\sqrt{E^2-m_o^2c^4}\times\left[1-\frac{\omega E}{2k_zc\sqrt{E^2-m_o^2c^4}}\log\left|\frac{E\omega+k_zc\sqrt{E^2-m_o^2c^4}}{E\omega-k_zc\sqrt{E^2-m_o^2c^4}}\right|\right.\\ &\left.-\frac{i\pi k_zc}{4}\frac{\sqrt{E^2-m_o^2c^4}}{E}\frac{\partial}{\partial\omega}\times\left(\left(1-\frac{\omega^2E^2}{k_z^2c^2\left(E^2-m_o^2c^4\right)}\right)H\left(1-\frac{\omega E}{k_zc\sqrt{E^2-m_o^2c^4}}\right)\right)\right]\frac{\partial f_o}{\partial E}dE,\end{aligned} \tag{3}$$

where we have written velocity and derivative of momentum distribution function in terms of relativistic energy as $v = \frac{c}{E}\sqrt{E^2-m_o^2c^4}$, $\frac{\partial f_o}{\partial|\mathbf{p}|} = \frac{\partial f_o}{\partial E}\frac{c\sqrt{E^2-m_o^2c^4}}{E}$ and $\frac{\partial f_o}{\partial|\mathbf{p}|}dp = \frac{\partial f_o}{\partial E}dE$. In a fully degenerate plasma, the derivative of Fermi distribution (Heaviside function) is

$$\frac{\partial f_0}{\partial E} = -\frac{2}{(2\pi\hbar)^3}\delta(E_F - E). \tag{4}$$

Applying relativistic Heaviside distribution function Eq. (4) in Eq. (3), the polarization tensor components take the form as

$$\begin{aligned}\Pi_{xx} = \Pi_{yy} = &-\frac{3}{2}\frac{\omega^2}{c^2k_z^2}\frac{\omega_{0pF}^2}{\gamma_F}\frac{\gamma_F^2}{\gamma_F^2-1}\left[1-\frac{\omega^2-k_z^2c^2\left(\sqrt{\gamma_F^2-1}/\gamma_F\right)}{2\omega k_zc\left(\sqrt{\gamma_F^2-1}/\gamma_F\right)}\times\left(\log\left|\frac{\omega+k_zc\left(\sqrt{\gamma_F^2-1}/\gamma_F\right)}{\omega-k_zc\left(\sqrt{\gamma_F^2-1}/\gamma_F\right)}\right|\right)\right.\\ &\left.-\frac{i\pi k_zc}{2\omega}\left(\sqrt{\gamma_F^2-1}/\gamma_F\right)\times\left(1-\frac{\omega^2}{k_z^2c^2\left(\gamma_F^2-1\right)/\gamma_F^2}\right)H\left(1-\frac{\omega}{k_zc\left(\sqrt{\gamma_F^2-1}/\gamma_F\right)}\right)\right],\\ \Pi_{xy} = -\Pi_{yx} = &\,3i\frac{\omega}{c^2k_z^2}\frac{\omega_{0pF}^2}{\gamma_F}\frac{\gamma_F^2}{\gamma_F^2-1}\frac{\Omega_0}{\gamma_F}\left[1-\frac{\omega}{2k_zc\left(\sqrt{\gamma_F^2-1}/\gamma_F\right)}\times\left(\log\left|\frac{\omega+k_zc\left(\sqrt{\gamma_F^2-1}/\gamma_F\right)}{\omega-k_zc\left(\sqrt{\gamma_F^2-1}/\gamma_F\right)}\right|\right)\right.\\ &\left.-\frac{i\pi k_zc}{4}\left(\sqrt{\gamma_F^2-1}/\gamma_F\right)\times\frac{\partial}{\partial\omega}\left(\left(1-\frac{\omega^2}{k_z^2c^2\left(\gamma_F^2-1\right)/\gamma_F^2}\right)H\left(1-\frac{\omega}{k_zc\left(\sqrt{\gamma_F^2-1}/\gamma_F\right)}\right)\right)\right],\end{aligned} \tag{5}$$

where $\hbar = h/2\pi$ is a Plank's constant, $E_F = \gamma_F m_0 c^2$ is the Fermi energy, $\gamma_F = 1/\sqrt{1-\frac{v_F^2}{c^2}} = \sqrt{1+\frac{p_F^2}{m_0^2c^2}}$ is the relativistic factor for degenerate plasma, ω_{0pF} and Ω_0 are the non-relativistic plasma and cyclotron frequencies, respectively.

Inserting Π_{xx} and Π_{xy} from Eq. (5) in Eq. (1), we obtain the relativistic transverse permittivity given by

$$\epsilon_T \equiv \frac{c^2k_z^2}{\omega^2} = 1 - \frac{3}{2}\frac{\omega_{0pF}^2/\gamma_F}{k_z^2c^2\left(\gamma_F^2-1\right)/\gamma_F^2}\left[1 - \frac{\omega^2 - k_z^2c^2\left(\gamma_F^2 - 1/\gamma_F^2\right)}{2\omega k_z c\left(\sqrt{\gamma_F^2-1}/\gamma_F\right)}\log\left|\frac{\omega + k_z c\sqrt{\gamma_F^2-1}/\gamma_F}{\omega - k_z c\sqrt{\gamma_F^2-1}/\gamma_F}\right|\right.$$
$$\left. - \frac{i\pi k_z c\sqrt{\gamma_F^2-1}/\gamma_F}{2\omega}\left(1 - \frac{\omega^2}{k_z^2c^2\left(\gamma_F^2-1\right)/\gamma_F^2}\right)\times H\left(1 - \frac{\omega}{k_z c\sqrt{\gamma_F^2-1}/\gamma_F}\right)\right] \pm 3\frac{\omega_{0pF}^2/\gamma_F}{k_z^2c^2\left(\gamma_F^2-1\right)/\gamma_F^2}\frac{\Omega_0/\gamma_F}{\omega}$$
$$\times\left[1 - \frac{\omega}{2k_z c\sqrt{\gamma_F^2-1}/\gamma_F}\log\left|\frac{\omega + k_z c\sqrt{\gamma_F^2-1}/\gamma_F}{\omega - k_z c\sqrt{\gamma_F^2-1}/\gamma_F}\right| - \frac{i\pi k_z c\frac{\sqrt{\gamma_F^2-1}}{\gamma_F}}{4}\right.$$
$$\left.\times\frac{\partial}{\partial\omega}\left(\left(1 - \frac{\omega^2}{k_z^2c^2\left(\gamma_F^2-1\right)/\gamma_F^2}\right)H\left(1 - \frac{\omega}{k_z c\sqrt{\gamma_F^2-1}/\gamma_F}\right)\right)\right]. \quad (6)$$

A. Skin depth in relativistic un-magnetized case

We now proceed to evaluate the expression of skin depth for a fully relativistic degenerate unmagnetized plasma given by

$$\epsilon_T \equiv \frac{c^2k_z^2}{\omega^2} = 1 - \frac{3}{2}\frac{\omega_{0pF}^2/\gamma_F}{k_z^2c^2\left(\gamma_F^2-1\right)/\gamma_F^2}\left[1 - \frac{\omega^2 - k_z^2c^2\left(\left(\gamma_F^2-1\right)/\gamma_F^2\right)}{2\omega k_z c\left(\sqrt{\gamma_F^2-1}/\gamma_F\right)}\times\log\left|\frac{\omega + k_z c\sqrt{\gamma_F^2-1}/\gamma_F}{\omega - k_z c\sqrt{\gamma_F^2-1}/\gamma_F}\right| - \left(\frac{i\pi k_z c\sqrt{\gamma_F^2-1}/\gamma_F}{2\omega}\right)\right.$$
$$\left.\times\left(1 - \frac{\omega^2}{k_z^2c^2\left(\gamma_F^2-1\right)/\gamma_F^2}\right)H\left(1 - \frac{\omega}{k_z c\sqrt{\gamma_F^2-1}/\gamma_F}\right)\right]. \quad (7)$$

It is worth discussing here that in cylindrical geometry, we get plasma dispersion functions (which can be expanded for large/small arguments in order to get principle/pole contribution, respectively). On the other hand, in spherical geometry, we obtain logarithmic functions which also behave like plasma dispersion function. Here we note that in the limit $\frac{ck_z}{\omega}\left(\sqrt{\gamma_F^2-1}/\gamma_F\right) > 1$ implies $\frac{c^2k_z^2}{\omega^2} \gg \frac{\gamma_F^2}{\gamma_F^2-1}$, the Heaviside function $H\left(1 - \frac{\omega}{k_z c\sqrt{\gamma_F^2-1}/\gamma_F}\right) = 1$. The principle part containing the logarithmic terms yields $\sim 2\omega/\left(k_z c\sqrt{\gamma_F^2-1}/\gamma_F\right)$ + H.O.T. (Higher order terms) and therefore the resulting series is neglected under the condition $\frac{c^2k_z^2}{\omega^2} > 1$. Thus, the right hand side of the fully relativistic dispersion relation Eq. (7) takes the form

$$k_z^3 = i\pi\frac{3}{4}\frac{\omega\omega_{0pF}^2}{c^3\sqrt{\gamma_F^2-1}}. \quad (8)$$

Equation (8) is cubic in k_z having three roots, two of them are complex conjugate having form $(\pm a + ib)$ and one is purely imaginary. The space scale of damping $\mathrm{Im}(k_z)$ in degenerate plasma is obtained by the same positive imaginary parts of the two complex roots of the equation. The relativistic skin depth $1/\mathrm{Im}(k_z)$ for degenerate unmagnetized plasma yields

$$\lambda_{skF}^{rel} = 1/Im(k_z) = 2\left(\frac{4}{3}\frac{c^3\sqrt{\gamma_F^2}}{\pi\omega\omega_{0pF}^2}\right)^{\frac{1}{3}}. \quad (9)$$

The non-relativistic version of the above Eq. (9) discussed ahead is reported in Ref. 17. It is worth mentioning here a graphical comparison of the skin depths obtained using relativistic Maxwellian distribution function (Eq. (27) (Ref. 18)) and the above Eq. (9) for degenerate plasma. Fig. 1 illustrates the difference of the anomalous skin effects between degenerate and non degenerate plasma in a weakly relativistic (a = 10 (Solid line)), relativistic (a = 1 (Dashed)), and strongly relativistic (a = 0.1(dotted)) regimes. Where the purple graphical lines are for degenerate plasma and blue lines for non-degenerate (Maxwellian) plasma. From the figure, it can be observed that the skin effects for degenerate plasma are relatively low as compare to non degenerate

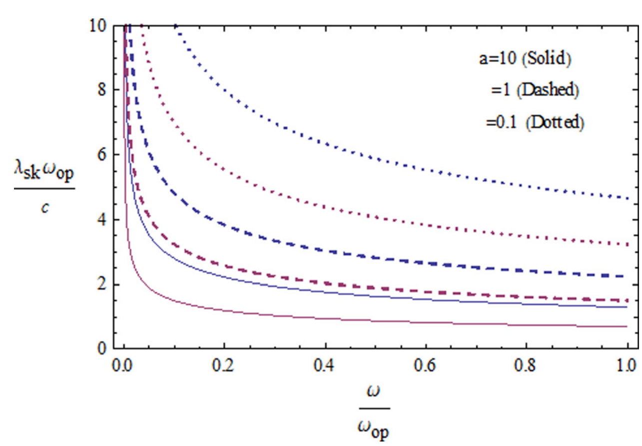

FIG. 1. A plot of normalized skin depth "$\frac{\lambda_{sk}\omega_{0p}}{c}$" vs normalized frequency "$\frac{\omega}{\omega_{0p}}$" for transverse wave at a_c, $a_F = 10$ (solid line), 1 (dashed line), 0.1 (dotted line) showing comparison of penetration depths of non-degenerate (blue lines) and degenerate (purple lines) plasma.

(Maxwellian) plasma. The difference between two penetration depths is less in the weakly relativistic regime and goes on increasing as we move from weakly relativistic to strongly relativistic regime. For convenience, we list a comparison of the different relativistic regimes. For non-degenerate case, the factor a_c decides the different temperature ranges for different relativistic regimes given in the following equation:

$$T = m_0 c^2 / a_c K_B, \tag{10}$$

whereas for degenerate case, a_F decides different density ranges by the equation

$$n = \frac{1}{a_F^3}\left(\frac{8\pi m_0^3 c^3}{3h^3}\right), \tag{11}$$

where a_c is the factor for classical plasmas and a_F is for degenerate plasmas. Note that the range $0 < (a_c, a_F) \leq 1$ corresponds to the strongly relativistic case whereas $(a_c, a_F) > 1$ related to weakly relativistic regimes.

Table I shows the temperature ranges T (MeV) for non-degenerate plasma and density ranges (n (cm^{-3})) for degenerate plasma regimes, where we have taken all the numerical values in cgs units.

Fig. 2 shows the density/temperature variation for weakly relativistic ($(a_c, a_F) > 1$) and for strongly relativistic ($0 < (a_c, a_F) < 1$) regimes. Here, we have normalized the variable density/temperature (n/T) with the relativistic density/temperature ($n_0 \sim 10^{29}\,\text{cm}^{-3}/T_0 \sim 0.51$ MeV) correspond to ($(a_c, a_F) = 1$). It can be observed that for greater values of a's, the degenerate plasma goes to non relativistic regime more abruptly as compared to non-degenerate plasma. Similar behavior can be observed for the strongly relativistic regime, i.e., in the range $0 < (a_c, a_F) < 1$.

B. Skin effect in relativistic weakly magnetized degenerate plasma

In some previous investigations,[19–21] the applied approximation $\frac{c^2k_z^2}{\omega^2} < 1$ is close to the cutoff frequency. While in the present case, the condition $\frac{c^2k_z^2}{\omega^2} > 1$ is suitable to discuss penetration depth. In the presence of weak magnetic field and under the conditions $c^2k_z^2 > \omega^2$, the transverse permittivity Eq. (6) takes the form

$$\frac{c^5k_z^5}{\omega_{0pF}^5} = \frac{3}{4} i\pi \frac{\omega/\omega_{0pF}}{\sqrt{\gamma_F^2 - 1}}\left(\frac{c^2k_z^2}{\omega_{0pF}^2} \pm 2(\omega/\omega_{0pF})(\Omega_0/\omega_{0pF})\frac{\gamma_F}{(\gamma_F^2 - 1)}\right). \tag{12}$$

Equation (12) is quintic in k_z. Solving equation numerically, we obtain five roots, one pure imaginary and four complex having the form $-ie, (\pm a' + ib'), (\pm c - id)$. Like unmagnetized case discussed before, the space scale of damping is obtained from the same positive imaginary parts of the two complex roots, i.e., $b' = \text{Im}k_z$. Moreover, the transverse skin depth obtained from the imaginary parts of the roots has additional weak magnetic field effects. We have plotted $\frac{\lambda_{sk}\omega_{0pF}}{c}$ vs $\frac{\omega}{\omega_{0pF}}$ in Figures 3 and 4 for weakly relativistic and strongly relativistic case, respectively. Both the figures show a comparison of weak magnetic field effects for degenerate and non degenerate plasma. Again like the unmagnetized case, the skin depth for R-& L-waves in degenerate plasma is less than the non-degenerate plasma. From Fig. 3, it is observed, that, the rise in the values of Ω_0/ω_{0pF} (from 0.01 to

TABLE I. Temperature/density ranges for different relativistic regimes defined by classical/degenerate factor a_c/a_F.

(a_c, a_F)	0.1	0.5	1	5	10
T (MeV)	5.116	1.023	0.511	0.10	0.0511
(n (cm^{-3}))	5.84×10^{32}	4.67×10^{30}	4.67×10^{29}	4.67×10^{27}	5.84×10^{26}

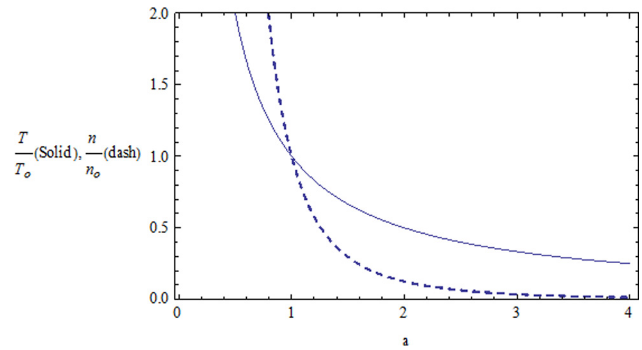

FIG. 2. Illustrates the density/temperature variation for the weakly relativistic ($(a_c, a_F) > 1$) and for the strongly relativistic ($0 < (a_c, a_F) < 1$) regimes.

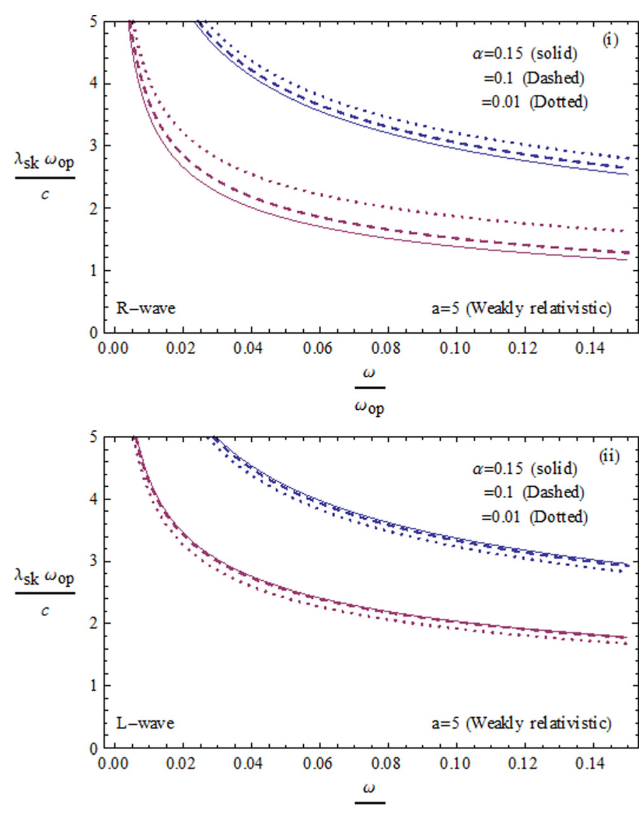

FIG. 3. A plot of weak magnetic field dependence $\alpha = \frac{\Omega_0}{\omega_{0p}} =$ (0.01 (dotted), 0.1, (dashed) and 0.15 (solid)) showing the penetration depth for (i) R-wave and (ii) L-wave in weakly relativistic case a_c, $a_F = 10$. Blue/purple lines are for non-degenerate/degenerate plasma, respectively.

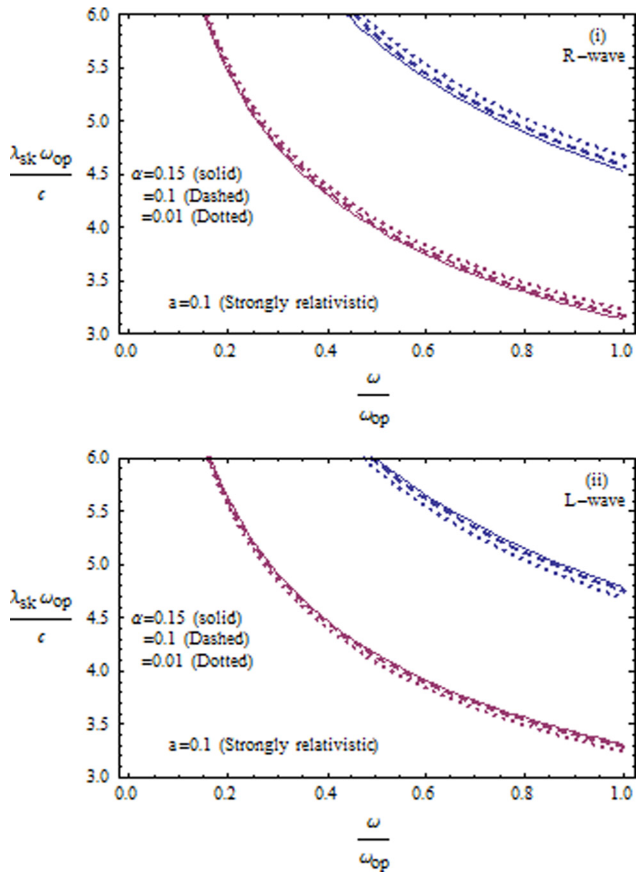

FIG. 4. A plot of weak magnetic field variation $\alpha = \frac{\Omega_0}{\omega_{0p}} = (0.01$ (dotted), 0.1, (dashed) and 0.15 (solid)) showing the penetration depth for (i) R-wave and (ii) L-wave in highly relativistic regime a_c, $a_F = 0.1$. Blue/purple lines are for non-degenerate/degenerate plasma, respectively.

0.15), the skin depth goes on decreasing in case of R-wave, while it tends to increase in case of L-wave. Furthermore, the quantitative reduction in the skin depth in R-wave for degenerate plasma is large as compared to the non-degenerate one. Whereas the relativistic effects suppress the magnetic field effects on plasma and resultantly we obtain minor shifts of the curves in the strongly relativistic regime in comparison with the weakly relativistic case (Fig. 3).

C. Limiting cases

In ultra-relativistic range ($p_F \gg m_0c$), the relativistic factor takes the form

$$\gamma_F = \sqrt{1 + p_F^2/m_0^2c^2} \simeq p_F/m_0c, \tag{13}$$

therefore the plasma and the cyclotron frequencies, respectively, become

$$\frac{\omega_{0pF}^2}{\gamma_F} = \frac{4\pi n_0 e^2 c}{p_F} = \omega_{pF}^2, \quad \frac{\Omega_0}{\gamma_F} = \frac{eB_0c}{p_F} = \omega_{cF}. \tag{14}$$

Using the above expressions (Eqs. (5) and (6)), the dispersion relations for R-& L- waves (Eq. (4)) therefore become

$$\frac{c^2k_z^2}{\omega^2} = 1 - \frac{3}{2}\frac{\omega_{pF}^2}{c^2k_z^2}\left[1 - \frac{\omega^2 - k_z^2c^2}{2\omega k_z c}\log\left|\frac{\omega + k_zc}{\omega - k_zc}\right| - \frac{i\pi k_zc}{2\omega}\left(1 - \frac{\omega^2}{k_z^2c^2}\right)H\left(1 - \frac{\omega}{k_zc}\right)\right] \pm 3\frac{\omega_{pF}^2}{c^2k_z^2}\frac{\omega_{cF}}{\omega} \times \left[1 - \frac{\omega}{2k_zc}\log\left|\frac{\omega + k_zc}{\omega - k_zc}\right| - \frac{i\pi k_zc}{4}\frac{\partial}{\partial\omega} \times \left(\left(1 - \frac{\omega^2}{k_z^2c^2}\right)H\left(1 - \frac{\omega}{k_zc}\right)\right)\right], \tag{15}$$

thus

$$\lambda_{skF}^{ultra-rel} = 1/Im(k_z) = 2\left(\frac{4}{3}\frac{c^3}{\pi\omega\omega_{pF}^2}\right)^{\frac{1}{3}}. \tag{16}$$

Similarly in the non-relativistic case ($p_F \ll m_0c \Rightarrow \gamma_F \simeq 1 + p_F^2/2m_0^2c^2 \approx 1$), Eq. (4) yields

$$\frac{c^2k_z^2}{\omega^2} = 1 - \frac{3}{2}\frac{\omega_{0pF}^2}{v_F^2k_z^2}\left[1 - \frac{\omega^2 - k_z^2v_F^2}{2\omega k_z v_F}\log\left|\frac{\omega + k_zv_F}{\omega - k_zv_F}\right| - \frac{i\pi k_zv_F}{2\omega}\left(1 - \frac{\omega^2}{k_z^2v_F^2}\right)H\left(1 - \frac{\omega}{k_zv_F}\right)\right] \pm 3\frac{\omega_{0pF}^2}{k_z^2v_F^2}\frac{\Omega_0}{\omega}\left[1 - \frac{\omega}{2k_zv_F}\log\left|\frac{\omega + k_zv_F}{\omega - k_zv_F}\right| - \frac{i\pi k_zv_F}{4}\frac{\partial}{\partial\omega}\left(\left(1 - \frac{\omega^2}{k_z^2v_F^2}\right)H\left(1 - \frac{\omega}{k_zv_F}\right)\right)\right]. \tag{17}$$

In the non-relativistic limit, the factor $\sqrt{\gamma_F^2 - 1} \approx v_F/c$, therefore

$$\lambda_{skF}^{non-rel} = 1/Im(k_z) = 2\left(\frac{4}{3}\frac{c^2v_F}{\pi\omega\omega_{0pF}^2}\right)^{\frac{1}{3}}. \tag{18}$$

The above results for non-relativistic and ultra-relativistic skin depth are in agreement with the results reported in Ref. 17.

III. RESULTS AND DISCUSSION

Using the Vlasov-Maxwell's model, we have derived the generalized polarization tensor for weakly magnetized degenerate electron plasma and have obtained relevant components for parallel propagation. Under the condition $c^2k_z^2 > \omega^2$, we have obtained relativistic anomalous skin effects in the transverse permittivity in both the field free and weakly magnetized case. We have presented a graphical comparison of our present results of the skin depths with the results reported previously for non degenerate plasmas. It is observed that the skin effects for degenerate plasma are relatively low as compared to the non degenerate (Maxwellian) plasma. The difference between two penetration depths becomes less in the weakly relativistic regime and goes on increasing in the strongly relativistic regime. In Figure 2, a general graphical comparison of the different relativistic ranges (weakly relativistic ($a = mc^2/T > 1$), strongly relativistic ($0 < a \leq 1$)) is shown for density/temperature variation for degenerate/non-degenerate plasma, respectively. It is observed that $a > 1$ the degenerate plasma goes to non relativistic regime more abruptly as compared to non-degenerate plasma. Similar behavior can be observed for the strongly relativistic regime, i.e., in the range $0 < a \leq 1$.

We have plotted $\frac{\lambda_{sk}\omega_{0p}}{c}$ vs $\frac{\omega}{\omega_{0p}}$ in Figures 3 and 4 for weakly relativistic and strongly relativistic case, respectively. Both the figures show comparison of weak magnetic field effects for degenerate and non degenerate plasma. Again like unmagnetized case, the skin depth for R-& L-waves in degenerate plasma is less than the non-degenerate plasma. Looking at magnetic field effects on skin depth in weakly magnetized plasma (Fig. 3), it is observed that the rise in the values of Ω_0/ω_{0p} (from 0.01 to 0.15), the skin depth goes on decreasing in case of R-wave, while it tends to increase in case of L-wave. Furthermore, the quantitative reduction in the skin depth in R-wave for degenerate plasma is large as compared to the non-degenerate one. Whereas the relativistic effects suppress the magnetic field effects on plasma and resultantly we obtain minor shifts of the curves in comparison with the weakly relativistic case (Fig. 4).

By switching off the magnetic field, our results reduce to those reported earlier[17] in the non-relativistic and ultra-relativistic limits.

Thus, we conclude that the skin depth for R-& L -waves decreases as we move from fully relativistic to weakly relativistic regime and that the ambient magnetic field enhances the skin effect for R- wave but reduces it for L- wave. Moreover the weak magnetic field effects are more pronounced in the weakly relativistic regime than in other relativistic regimes.

The available densities ($n \sim 10^{26}\,\mathrm{cm}^{-3}$(nonrelativistic) to $10^{31}\,\mathrm{cm}^{-3}$(highly relativistic)) corresponding to the magnetic field strengths ($B_0 = 10^{9-10}\,G$) satisfy the condition $\omega_{0p} > \Omega_0$ and cover a wide range of astrophysical environments like white dwarf and neutron stars. Depending upon the physical scenario, our results have wide range of applicability to all temperatures ranging from keV to MeV and to the magnetized plasmas having ratio $\frac{\Omega_0}{\omega_{0p}} < 1$. The white dwarf and pulsars consist of dense relativistic degenerate electron gas having number densities exceeding $10^{27}\,\mathrm{cm}^{-3}$ and above together with the magnetic fields of the order of 10^7 G and above.[22–24] The values can be considered in a weak-magnetic field approximations. These ranges can also be found in a number of environments like gamma ray burst afterglow plasmas.[25] In laboratory, these results can be applied to highly dense laser induced plasmas,[26] radio frequency field penetration effects into the plasma[27] and magnetically inductively coupled plasmas,[28] etc., where weak magnetic field effects may play a significant role.

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