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Dust heating by Alfvén waves using non-Maxwellian distribution function

K. Zubia,[1] H. A. Shah,[2] and P. H. Yoon[3,4]
[1]Department of Physics, Government College University, Lahore 54000, Pakistan
[2]Department of Physics, Forman Christian College, Lahore 54600, Pakistan
[3]Institute for Physical Science and Technology, University of Maryland, College Park, Maryland 20742, USA
[4]School of Space Research, Kyung Hee University, Yongin, Gyeonggi 446-701, Korea

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Quasilinear theory is employed in order to evaluate the resonant heating rate by Alfvén waves, of multiple species dust particles in a hot, collisionless, and magnetized plasma, with the underlying assumption that the dust velocity distribution function can be modeled by a generalized (r, q) distribution function. The kinetic linear dispersion relation for the electromagnetic dust cyclotron Alfvén waves is derived, and the dependence of the heating rate on the magnetic field, mass, and density of the dust species is subsequently investigated. The heating rate and its dependence on the spectral indices r and q of the distribution function are also investigated. It is found that the heating is sensitive to negative value of spectral index r. *© 2015 AIP Publishing LLC.*
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I. INTRODUCTION

Dusty plasma is a collection of micrometer sized solid objects immersed in a plasma consisting of electrons, ions, and neutrals. Most often, these small objects or dust particles are electrically charged. Charged dust particles are ubiquitous in space environments,[1–6] as well as in laboratory plasmas.[6–8] The charged dust particles introduce a mixture of new effects in a plasma that are associated with waves and instabilities[6] and cause different interesting phenomena, which occur in space and astrophysical environments, such as planetary atmospheres, cometary tails, solar corona, interplanetary space, interstellar medium, interstellar clouds, earth's environments such as the ionosphere and the magnetosphere,[1–5,9] as well as in laboratory experiments and in fusion devices, etc.[10]

The micrometer sized $(m_d/m_p \sim 10^6–10^8)$ dust grains having large charges $z_d \sim 10^2–10^5$ can be present in a plasma form a complex system or dusty plasmas.[1,3,11–14] Clearly, the field of dusty plasma promises to be a very rewarding topic of research for the next decade or so, and the emphasis is not only on developing the theory of complex collective and nonlinear process but also has many applications in astrophysics, space physics, environmental pollution protection, and energy research. Dusty grains can reform and dominate wave propagation, ionization balance, shock propagation, gradient and thermal instabilities in interstellar gases, plasma chemistry, supernova remnants, and planetary magnetospheres as well as in laboratory plasmas. Nonlinear effects at large amplitudes, such as solitons, shocks, double layers, and parametric instabilities, two-stream instability,[13,14] and scattering of Alfvén wave, in magnetized and unmagnetized, collisionless dusty plasmas have been investigated. A dust particle in steady state plasma collects on the average an equilibrium density of electrons and ions per unit time. The charging of a dust particle is a discrete process, and electrons and ions are captured by the dust particle at random times. The number of charges on a dust particle is fluctuating and can be considered a random variable whose mean value is given by the condition that total current to the dust particle is zero. Charge fluctuations may have some impact on the formation and growth of particulates in laboratory plasmas. Although the particulates are on the average negatively charged in low temperature laboratory plasmas, small particles may become neutral or even positively charged due to charge fluctuations. Choi and Kushner[15] proposed a growth model where the growth of large particles is fueled by the collection of small particles (clusters).

Dust charging effects have not been included in our consideration as these would only complicate the application of non-Maxwellian distribution functions to a quasilinear problem without adding anything significant to the results. Thus, we have taken the charge to be constant as determined from the quasi-neutrality condition. We have applied our results to laboratory plasmas, although our results are general enough. We have to be applied to space plasmas, where negatively charge dust is found in a situation with background turbulence.

Dusenbery and Hollweg[16] were amongst the first to obtain heating rates using quasilinear theory, and they parametrically studied the heating rate of minor ions with a parallel propagating Alfvén wave. Quasilinear theory has been used by many authors, e.g., Marsch *et al.*,[17] Isenberg and Hollweg,[18] Isenberg,[19] Hu and Habbal,[20] and Cranmer *et al.*,[21] to account for the heat dissipation rate in the energy equation of the MHD coronal hole heating model and to find the radial dependences of ion temperatures in solar coronal holes.

Sometime ago, Zhang and Li[22] used the quasilinear theory to determine the heating of different species of ions in a multi-ion laboratory plasmas, by employing Alfvén waves, which are also known to be the dominant waves in space environments. These waves are thought to be responsible for transport of electromagnetic energy in magnetized plasmas. However, we note that these authors did not properly take into account the electromagnetic nature of the Alfvén waves.

Collisionless heating[23] by Alfvén waves through ion cyclotron resonance has been used by many authors, e.g., Stoffels *et al.*[24] investigated the heating of a dusty plasma by using lasers, and their theoretical and experimental results showed a good agreement.

The physical mechanisms responsible for distribution functions in space plasmas may mainly be due to the background turbulence, which can be supposed to be a quasi-steady state and can play a significant role in shaping of distribution functions that are different from Maxwellian. Such non-Maxwellian distribution function can now be constructed from data of space plasma, and it has been seen that the shape of such distribution function shows the existence of high-energy tails or shoulders in their profiles. These deviations suggest that when we use Maxwellian distribution function, to describe the instabilities and waves, the quantitative ensuing results do not give good fits with observations.[25–27] In the natural space environment, examples of plasmas with non-thermal equilibrium are solar flares, solar wind, galactic cosmic rays distribution and radiation fields, planetary magnetospheres, and astrophysical plasmas.[27] Non-Maxwellian distributions have also been used to analyze and interpret spacecraft data in the Earth's magnetospheric plasma sheet, the solar wind.[25] Thus, an appropriate non-Maxwellian distribution function can help better resolve many problems relevant to astrophysical and space plasmas, as well as laboratory plasmas.

We have used a distribution function, which is a combination of Generalized Lorentzian (kappa)[25,28] and the Davydov-Druyvestyen[29] distribution function. The Davydov-Druyvestyen distribution function has been used for gas discharges in cold plasmas and is responsible for the nonlinear damping that may occur in such plasmas in quasi thermodynamic equilibrium. The distribution function that we propose to use depends on two spectral indices r and q, which are determined from empirical data.

This is a model distribution created in order to better fit experimental data.[27,30] Restrictions on the spectral indices r and q result from the normalization of the distribution function,[27] and are $q > 1$ and $q(r+1) > 5/2$ for real values of r and q.[27] This distribution reduces under limiting conditions to a kappa (κ-spectral index) distribution function if $r = 0$ and $q \to \kappa + 1$ and to a Maxwellian when $r = 0$ and $q \to \infty$. The spectral index q contributes to the high energy tails and r gives rise to flat or spiked top of the distribution function. Different modes of electron-ion plasma have been investigated using such distribution functions by Qureshi *et al.*[27] Such type of distribution functions has been shown to play an important role in laboratory and space plasmas[31] as stated above.

It may be noted here that flat top distribution was observed and reported by Feldman[32] for the first time, from data obtained near the Earth bow shock. Since then, there have been many observations of flat top distributions in Earth's immediate plasma environment, e.g., from the bow shock, magnetosheath, around magnetic reconnection regions.[33–36] Later, some theoretical work[37,38] was carried using the (r, q) distribution function to better model the observations.

The objective of this paper is to use a kinetic model for resonant heating of dust particles by Alfvén waves, which propagate along the magnetic field in the dust cyclotron frequency range by analyzing the wave particle interaction. Quasilinear theory is used in the Vlasov equation in order to find Alfvén wave heating of hot, magnetized, multi species dust laboratory plasma by using the non-Maxwellian (r, q) distribution function. In the numerical analysis, we investigate the heating of dust particle in the right handed circularly polarized electromagnetic dust cyclotron Alfvén waves (EMDAW). The heating rate dependence on different values of the spectral indices r and q is established.

The organization of the paper is as follows: In Sec. II, we have used the conventional quasilinear theory to evaluate the heating rate expression for the particles via the Alfvén waves in a collisionless and homogeneous dusty magnetoplasma in the presence of an external magnetic field, when a background plasma has a generalized (r, q) velocity distribution. The dispersion relation for the right handed circularly polarized EMDAW derived from the kinetic model is used in the expressions obtained for the heating rate. Finally, a brief discussion of the results is given in Sec. III.

II. MATHEMATICAL FORMALISM

We note that the plasma is considered neutral, and in our subsequent calculation, the charge quasi-neutrality condition is given by

$$n_{e0} = n_{i0} + \frac{q_{d0} n_{d0}}{e}, \tag{1}$$

where the equilibrium number densities of the electrons and ions are given by n_{e0} and n_{i0} and the dust equilibrium density is given by n_{d0}, and $q_{d0} = -z_d e$ is the equilibrium charge on an average dust grain, where z_d is the number of electronic charges on a grain and e is the electronic charge.[10]

We follow the standard formalism of kinetic theory to evaluate the heating rate of the particles through Alfvén waves propagating along the z-axis, which is parallel to the static, constant, and homogenous magnetic field $\mathbf{B}_0$.[22] In a collisionless and magnetized plasma, the Vlasov equation describes the evolution of the particle distribution $f_\alpha(\mathbf{r}, \mathbf{v}, t)$ for a multi component plasma, composed of electrons, ions, and dust grain species, i.e., $\alpha = e, i, d$. Using a standard procedure, we split the Vlasov equation into a set of the equations, the first of which describes the purely linear perturbations and is given by[39,40]

$$\frac{\partial \delta f_{\alpha,\mathbf{k}}(\mathbf{v},t)}{\partial t} + i\mathbf{k}\cdot\mathbf{v}\delta f_{\alpha,\mathbf{k}}(\mathbf{v},t) + \frac{q_\alpha}{m_\alpha c}(\mathbf{v}\times\mathbf{B}_0)\cdot\frac{\partial \delta f_{\alpha,\mathbf{k}}(\mathbf{v},t)}{\partial \mathbf{v}} + \frac{q_\alpha}{m_\alpha}\left[\delta\mathbf{E}_\mathbf{k}(t) + \frac{1}{c}(\mathbf{v}\times\delta\mathbf{B}_\mathbf{k})\right]\cdot\frac{\partial f_{\alpha,0}(\mathbf{v})}{\partial \mathbf{v}} = 0, \tag{2}$$

and the second is a non-linear (quasilinear) equation that describes wave-particle interactions and is given by[22]

$$\frac{\partial G_{\alpha,0}(\mathbf{v},t)}{\partial t} + \frac{q_\alpha}{m_\alpha}\sum_{\mathbf{k}\neq 0}\left[\delta\mathbf{E}_{-\mathbf{k}}(t) + \frac{1}{c}(\mathbf{v}\times\delta\mathbf{B}_{-\mathbf{k}})\right] \times \frac{\partial \delta f_{\alpha,\mathbf{k}}(\mathbf{v},t)}{\partial \mathbf{v}} = 0, \tag{3}$$

where q_α and m_α are the charge and mass of the particles, respectively, $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{B}(\mathbf{r}, t)$ are the perturbed electric and magnetic fields, respectively, $\mathbf{k}$ is the wave vector, and c is the speed of light, and $G_{\alpha,0}(\mathbf{v}, t)$ is defined by $G_{\alpha,0}(\mathbf{v}, t)$

$=f_{\alpha,0}(\mathbf{v})+\delta f_{\alpha,0}(\mathbf{v},t)$, which is the sum of the equilibrium part and a quasi-stationary equilibrium part $\delta f_{\alpha,0}(\mathbf{v},t)$ that is slowly varying in time. In general, $\delta f_{\alpha,\mathbf{k}}(\mathbf{v},t)=f_{\alpha,0}(\mathbf{v})+\delta f_{\alpha,0}(\mathbf{v},t)+\delta\tilde{f}_{\alpha,\mathbf{k}}(\mathbf{v},t)$, where the last term describes the rapidly varying fluctuations.

In the present work, we use the normalized isothermal (r,q) distribution function[27,30] given by

$$f_{\alpha,0}^{(r,q)}(\mathbf{v})=\frac{A_1}{\pi\Psi_\alpha^3}\left[1+\frac{1}{q-1}\left(\frac{v^2}{\Psi_\alpha^2}\right)^{r+1}\right]^{-q}, \tag{4}$$

where

$$A_1=\frac{3(q-1)^{-\frac{3}{2+2r}}\Gamma(q)}{4\Gamma\left(q-\frac{3}{2+2r}\right)\Gamma\left(1+\frac{3}{3+3r}\right)} \tag{5}$$

and

$$\Psi_\alpha=v_{T\alpha}A_2 \tag{6}$$

is the thermal velocity for the isotropic temperatures, respectively,[30] with

$$A_2=\sqrt{\frac{3(q-1)^{-\frac{1}{1+r}}\Gamma\left(q-\frac{3}{2+2r}\right)\Gamma\left(\frac{3}{2+2r}\right)}{\Gamma\left(\frac{5}{2+2r}\right)\Gamma\left(q-\frac{5}{2+2r}\right)}}. \tag{7}$$

We note here that $v_{T\alpha}=\sqrt{k_BT_\alpha/m_\alpha}$ is the Maxwellian thermal velocity of the particle of any species α of mass m_α and temperature T_α and Γ is the usual gamma function. The spectral indices r and q satisfy the constraints, $q>1$ and $q(1+r)>5/2$, which arise from the normalization and the definition of temperature for the (r,q) distribution function. The properties of this distribution function are discussed by Qureshi *et al.*[27] in detail.

For the time dependent distribution evolution from Eqs. (2) and (3), we can obtain the heating rate of the particle of any species by the following integration[40–44] using cylindrical coordinates as:

$$\frac{3}{4}k_BA_2^2\frac{dT_\alpha}{dt}=\frac{1}{2}k_BA_2^2\left(\frac{dT_{\alpha,\perp}}{dt}+\frac{1}{2}\frac{dT_{\alpha,\parallel}}{dt}\right)=\frac{m_\alpha}{2}\int d\mathbf{v}\,v^2\frac{\partial G_{\alpha,0}(\mathbf{v},t)}{\partial t}, \tag{8}$$

where k_B is the Boltzmann constant. We consider here the magnetic field aligned right-hand circularly polarized Alfvén waves. Although we started with an isothermal distribution function, but as we consider cyclotron damping of the Alfvén waves, the heating of the dust particles is in the perpendicular direction only. The parallel heating is possible only when obliquely propagating Alfvén waves are considered, because only then the parallel component of electric field appears.[30] Furthermore, we follow the general method used by Stix[39] and Ichimaru[44] for evaluating $\partial G_{\alpha,0}(\mathbf{v},t)/\partial t$. The expression obtained for $\partial G_{\alpha,0}(\mathbf{v},t)/\partial t$ [from Eqs. (2) and (3)] is substituted in Eq. (8), which yields

$$\frac{dT_{\alpha,\perp}}{dt}=\frac{-i4\pi q_\alpha^2}{3k_Bm_\alpha A_2^2}\sum_{\mathbf{k}\neq0}W_\mathbf{k}(t)\int d\mathbf{v}v^2\mathcal{L}\frac{1}{k_\parallel v_\parallel-\omega-\Omega_\alpha}\mathcal{L}f_{\alpha,0},$$
$$\mathcal{L}=\left(1-\frac{k_\parallel v_\parallel}{\omega}\right)\frac{\partial}{\partial v_\perp}+\frac{k_\parallel v_\perp}{\omega}\frac{\partial}{\partial v_\parallel}, \tag{9}$$

where $\Omega_\alpha=Z_\alpha eB_0/(m_\alpha c)$ is the cyclotron frequency of particles of species α and energy density of Alfvén waves defined as $W_\mathbf{k}(t)=|\delta\mathbf{E}_\mathbf{k}(t)|^2/8\pi$, where $W_\mathbf{k}(t)$ is the wave energy density in terms of fluctuating electric field $\delta E_\mathbf{k}(t)$. Now using above [generalized (r,q)] distribution function, in Eq. (9), we obtain

$$\frac{dT_{\alpha,\perp}}{dt}=\frac{i8A_1q_\alpha^2}{3k_Bm_\alpha A_2^2\Psi_\alpha^5}\frac{q(r+1)}{q-1}\times\sum_{\mathbf{k}\neq0}\int d\mathbf{v}v^2\left[\frac{\sum_{i=1}^{8}I_i}{k_\parallel v_\parallel-\omega-\Omega_\alpha}-\frac{I_9+I_{10}}{(k_\parallel v_\parallel-\omega-\Omega_\alpha)^2}\right], \tag{10}$$

where

$$I_1=\left(1-\frac{k_\parallel v_\parallel}{\omega}\right)^2\left(\frac{v^2}{\Psi_\alpha^2}\right)^r\left[1+\frac{1}{q-1}\left(\frac{v^2}{\Psi_\alpha^2}\right)^{r+1}\right]^{-q-1}, \tag{11a}$$

$$I_2=\frac{2r}{\Psi_\alpha^2}v_\perp^2\left(\frac{v^2}{\Psi_\alpha^2}\right)^{-1}I_1, \tag{11b}$$

$$I_3=-\frac{2}{\Psi_\alpha^2}\frac{(q+1)(r+1)}{q-1}\left(1-\frac{k_\parallel v_\parallel}{\omega}\right)^2v_\perp^2\left(\frac{v^2}{\Psi_\alpha^2}\right)^{2r}\times\left[1+\frac{1}{q-1}\left(\frac{v^2}{\Psi_\alpha^2}\right)^{r+1}\right]^{-q-2}, \tag{11c}$$

$$I_4=\frac{k_\parallel v_\parallel}{\omega}\left(1-\frac{k_\parallel v_\parallel}{\omega}\right)^{-1}I_1, \tag{11d}$$

$$I_5=\frac{4r}{\Psi_\alpha^2}\frac{k_\parallel v_\parallel}{\omega}v_\perp^2\left(1-\frac{k_\parallel v_\parallel}{\omega}\right)^{-1}\left(\frac{v^2}{\Psi_\alpha^2}\right)^{-1}I_1, \tag{11e}$$

$$I_6=\frac{2k_\parallel v_\parallel}{\omega}\left(1-\frac{k_\parallel v_\parallel}{\omega}\right)^{-1}I_3, \tag{11f}$$

$$I_7=\frac{2r}{\Psi_\alpha^2}\frac{k_\parallel^2v_\parallel^2}{\omega^2}v_\perp^2\left(1-\frac{k_\parallel v_\parallel}{\omega}\right)^{-2}\left(\frac{v^2}{\Psi_\alpha^2}\right)^{-1}I_1, \tag{11g}$$

$$I_8=\frac{k_\parallel^2v_\parallel^2}{\omega^2}\left(1-\frac{k_\parallel v_\parallel}{\omega}\right)^{-2}I_3, \tag{11h}$$

$$I_9=\frac{k_\parallel^2v_\perp^2}{\omega}\left(1-\frac{k_\parallel v_\parallel}{\omega}\right)^{-1}I_1, \tag{11i}$$

$$I_{10}=\frac{k_\parallel^3v_\perp^2v_\parallel}{\omega^2}\left(1-\frac{k_\parallel v_\parallel}{\omega}\right)^{-2}I_1. \tag{11j}$$

Following Qureshi *et al.*,[27] we find that the perpendicular integration yields

$$\frac{dT_{\alpha,\perp}}{dt}=\frac{i8\pi q_\alpha^2 A_1}{3k_B m_\alpha \Psi_\alpha A_2^2}\sum_{\mathbf{k}\neq 0}W_{\mathbf{k}}(t)\int_{-\infty}^{\infty}dv_\parallel\left\{\frac{q(1+r)(q-1)^q}{-1+q+qr}(s^2)^{1-q-qr}\left(2r+1-\frac{k_\parallel v_\parallel}{\omega}\right)\right.$$
$$\times{}_2F_1\left(1+q,q-\frac{1}{1+r},q+\frac{r}{1+r},-\frac{q-1}{s^{2+2r}}\right)+2s^2\left(1+\frac{s^{2+2r}}{q-1}\right)^{-q}\left[1+\frac{q(1+r)}{q-1}\left(1+\frac{s^{2+2r}}{q-1}\right)^{-1}\right]$$
$$\left.-\frac{2q(q+1)(1+r)^2(q-1)^q}{-1+q+qr}(s^2)^{1-q-qr}{}_2F_1\left(2+q,q-\frac{1}{1+r},q+\frac{r}{1+r},-\frac{q-1}{s^{2+2r}}\right)\right\}\frac{1}{k_\parallel v_\parallel-\omega-\Omega_\alpha}$$
$$-\frac{i8\pi q_\alpha^2 A_1}{3k_B m_\alpha \Psi_\alpha A_2^2}\sum_{\mathbf{k}\neq 0}W_{\mathbf{k}}(t)\int_{-\infty}^{\infty}dv_\parallel\left\{\frac{q(1+r)(q-1)^q}{-2+q+qr}(s^4)^{1-q-qr}\frac{k_\parallel^2\Psi_\parallel^2}{\omega}{}_2F_1\left(1+q,q-\frac{2}{1+r},q+\frac{r-1}{1+r},-\frac{q-1}{s^{2+2r}}\right)\right.$$
$$\left.-\frac{q(1+r)(q-1)^q}{-1+q+qr}(s^2)^{1-q-qr}\frac{k_\parallel^2 v_\parallel^2}{\omega}{}_2F_1\left(1+q,q-\frac{1}{1+r},q+\frac{r}{1+r},-\frac{q-1}{s^{2+2r}}\right)\right\}\frac{1}{(k_\parallel v_\parallel-\omega-\Omega_\alpha)^2},\quad(12)$$

where $s=v_\parallel/\Psi_\alpha$ for simplification in Eq. (12) and ${}_2F_1$ is the hypergeometric function. Because, parallel integration cannot be performed exactly due the singularity in the path of the integration. Now, we expand the above expressions for $\xi_\alpha=(\omega+\Omega_\alpha)/(k_\parallel\Psi_\alpha)\leq 1$, the reason being that for small ξ_α the particles resonate with the dust Alfvén velocity and heating in this case will be efficient for these particles. Using the expansion for small ξ_α, we take only the imaginary part of all the expansions and drop the real part of all functions. This allows us to separate between the principal part of the integration and the imaginary part resulting from the poles. We note that to obtain an expression for the heating rate, we need to use only the imaginary parts. Making use of these, we obtain

$$\frac{1}{T_{\alpha,\perp}}\frac{dT_{\alpha,\perp}}{dt}=\frac{-8\pi^2c^2\Omega_\alpha^2A_1}{3B_0^2v_{T\alpha}^3A_2^3}\sum_{\mathbf{k}\neq 0}\frac{W_{\mathbf{k}}(t)}{k_\parallel}\left\{\left(1+2r-A_2\xi_\alpha\frac{\omega+\Omega_\alpha}{\omega}\right)B_1\xi_\alpha^{2(1-q-qr)}{}_2F_1\left(q+1,q-\frac{1}{1+r},q+\frac{r}{1+r},-\frac{q-1}{\xi_\alpha^{2+2r}}\right)\right.$$
$$+2\xi_\alpha^2\left[1+\frac{q(1+r)}{q-1}\xi_\alpha^{2+2r}\left(1+\frac{\xi_\alpha^{2+2r}}{q-1}\right)^{-1}\right]\left(1+\frac{\xi_\alpha^{2+2r}}{q-1}\right)^{-q}-2(1+r)(q+1)B_1\xi_\alpha^{2(1-q-qr)}$$
$$\left.\times{}_2F_1\left(q+2,q-\frac{1}{1+r},q+\frac{r}{1+r},-\frac{q-1}{\xi_\alpha^{2+2r}}\right)\right\},\quad(13)$$

where

$$B_1=\frac{q(1+r)(q-1)^q}{q+qr-1}.$$

It is seen that the resonant heating by dust cyclotron Alfvén wave is explicitly effected by the following parameters: e.g., $n_\alpha, m_\alpha, T_\alpha, \omega, k_\parallel, q_\alpha, B_0$, and $W_{\mathbf{k}}(t)$. If most particle of species α satisfy the resonance condition, $v_{\parallel,\alpha}$ should be about the order of the thermal velocity. In this situation, efficient dust heating through the harmonic resonance is possible. In the limiting case of $r=0$ and $q\to\infty$, the above heating rate expression reduces to the heating rate expression for a Maxwellian distribution function, which is similar to the case discussed by Zhang and Li[22] for positive ions (which, however, we feel is not wholly correct as the electromagnetic nature of the Alfvén waves is not taken into account properly).

In the present work, we have considered parallel propagating dust Alfvén wave along the magnetic field lines. For the cyclotron resonant heating of dust species as stated earlier, we consider only right handed circularly polarized dust Alfvén wave, as the left hand polarized waves do not produce any resonance. We get the following dispersion relation for the right handed circularly polarized EMDAW for any species α using the standard kinetic model

$$\frac{c^2k_\parallel^2}{\omega^2}=1-\sum_\alpha\frac{\omega_{p\alpha}^2}{\omega^2}\frac{\omega}{A_2(\omega+\Omega_\alpha)}\times\left[A_1C_1\xi_\alpha+\xi_\alpha^2\mathbf{Z}_1(\xi_\alpha)-\mathbf{Z}_2(\xi_\alpha)\right],\quad(14)$$

where $\omega_{p\alpha}^2=4\pi n_{0\alpha}e^2/m_\alpha$ is defined as plasma frequency for the electrons, ions, and dust particles, respectively, and $\mathbf{Z}_1(\xi_\alpha)$ and $\mathbf{Z}_2(\xi_\alpha)$ are the generalized plasma dispersion functions defined as

$$\mathbf{Z}_1^{(r,q)}(\xi_\alpha)=A_1\int_{-\infty}^{\infty}\frac{1}{s-\xi_\alpha}\left(1+\frac{s^{2+2r}}{q-1}\right)^{-q}ds\quad(15)$$

and

$$\mathbf{Z}_2^{(r,q)}(\xi_\alpha) = A_1 B_1 \int_{-\infty}^{\infty} \frac{s^{2-2q-2qr}}{s-\xi_\alpha} {}_2F_1 \times \left(q+1, q-\frac{1}{1+r}, q+\frac{r}{1+r}, -\frac{q-1}{s^{2+2r}}\right) ds. \quad (16)$$

We note here that in Eq. (13) only the right handed polarized mode is considered. Since this mode has a resonance at $\omega = \Omega_d$, where Ω_d is the dust cyclotron frequency and is given by $\Omega_d = z_d e B_0/(m_d c)$, this will contribute to the heating of the dust particles.

III. DISCUSSION AND CONCLUSION

In this paper, we have used the quasilinear kinetic model to calculate the heating rate, due to right handed circularly polarized Alfvén waves, in a plasma containing different dust species. We have derived the linear dispersion relation using the kinetic model and the generalized (r, q) distribution function and see that this relationship has a dependence on both the spectral indices r and q. In the limiting case, when $r = 0$ and $q \to \infty$, this dispersion relation reduces to the standard dispersion relation for a Maxwellian plasma, which is given in Stix (in section 11–2).[45] We have then proceeded to derive an expression [given by Eq. (13)] for the heating rate of the dust particles via the quasilinear theory using the (r, q) distribution function. This is a complicated expression, and the results are expressed through hypergeometric functions, thus an analysis of the heating rate has to be carried out numerically.

In this final section, we numerically investigate the expressions for the heating rate and the linear dispersion relation given by Eqs. (13) and (14), respectively. Accordingly, we take some typical plasma parameters that are used to compute the heating rate of the dust particles. Thus, we take $B_0 = 10^4\,\text{G}$, $n_i = 2 \times 10^{11}\,\text{cm}^{-3}$, $n_e = 0.5 \times 10^{10}\,\text{cm}^{-3}$,[22] and $T_0 = 300^\circ\text{K}$.[35] The heating of dust particles by the Alfvén waves with the power law spectra, i.e., $W_k(t) \propto k^{-\gamma_\omega}$,[46] depends on the dust species, where γ_ω is the spectral power index of the Alfvén waves, usually chosen to be $\gamma_\omega \simeq 1.5$–2.[46] For efficient heating, the resonant factor $R_\alpha = (\omega \pm \Omega_\alpha)/(k_\parallel v_{T\alpha}) \approx -1$.[22] A simple analysis of the linear dispersion relation [Eq. (14)] shows that by increasing masses or the number densities or the charge numbers z_d of the dust particles, the corresponding wave number also increases. For graphical investigation, we plot a graph of the heating rate with a normalized temperature versus normalized time. The temperature is normalized by initial equilibrium temperature, and time is normalized by cyclotron frequency of the dust particles. In Fig. 1, we have plotted the heating rate given by Eqs. (13) and (14) of dust particles by increasing its mass, as $m_{d1} = 0.5 \times 10^6 m_i$, $m_{d2} = 10^6 m_i$, and $m_{d3} = 2 \times 10^6 m_i$, while keeping fixed $r = -0.7$, $q = 9$, $n_d = 5n_e/100\,\text{cm}^{-3}$, and $z_d = 1000$. Here, we can substitute different values of r and q subject to the condition $q > 1$ and $q(r+1) > 5/2$. It is seen that the heating rate increases with increasing masses of the different dust species.

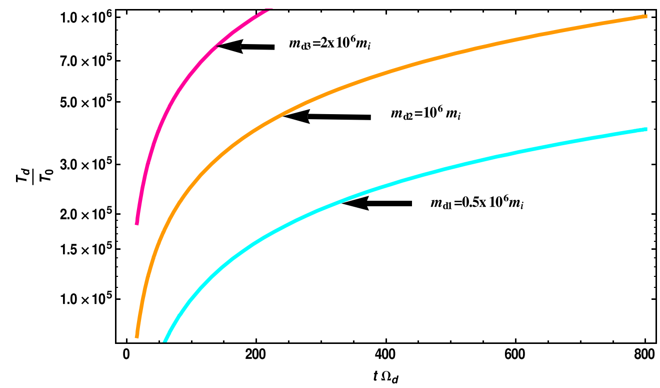

FIG. 1. Heating rate fixed value of r and q, varying mass number m_d.

Similarly, Fig. 2 shows the change in heating rate for different number densities $n_{d1} = 1.2 \times 5n_e/100\,\text{cm}^{-3}$, $n_{d2} = 1.4 \times 5n_e/100\,\text{cm}^{-3}$, $n_{d3} = 4 \times 5n_e/100\,\text{cm}^{-3}$, keeping fixed value of $m_d = 10^6 m_i$, $z_d = 1000$ and with same spectral indices as used in Fig. 1. The heating rate decreases with increasing number density.

In Fig. 3, we have seen the dependence of magnetic field on the heating rate with the same spectral index as used in Figs. 1 and 2, as $B_{01} = 10^4\,\text{G}$, $B_{02} = 5 \times 10^4\,\text{G}$, $B_{03} = 10 \times 10^4\,\text{G}$, $m_d = 10^6 m_i$, $n_d = 5 \times n_e/100\,\text{cm}^{-3}$, and $z_d = 1000$. The graph again shows that heating rate decreases with increasing magnetic field.

We have also checked for the dependence of the heating of dust species on the spectral indices r and q, and in Fig. 4, we have plotted a graph for the dependence of the heating rate on the spectral index q, by keeping the other spectral index $r = -0.7$ fixed along with the other parameters, i.e., $n_d = 5 \times n_e/100\,\text{cm}^{-3}$ and $m_d = 10^6 m_i$. The graph shows that, with increasing the value of q, the heating rate increases. Now, we check the dependence of the heating rate on the other spectral index r while keeping $q = 10$ (fixed along with the other variables of the system as shown in Fig. 3).

Figure 5 shows that when r decreases from some negative values, the heating rate increases slowly and finally heating rate levels off to a constant value.

In our numerical analysis, we have used only negative values of the spectral index r. As was shown in our earlier paper,[27] the distribution function exhibits a pointed or spiked top for negative values of r and the gradient of the

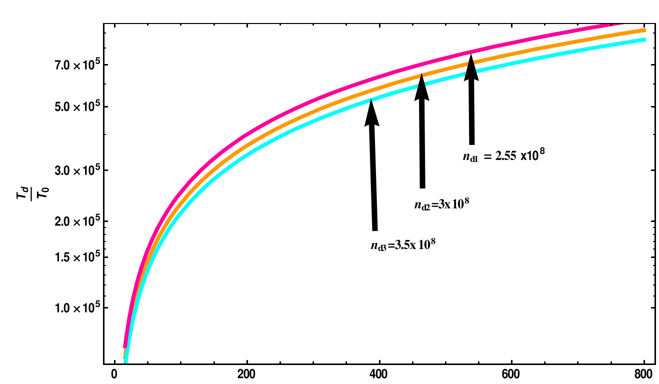

FIG. 2. Heating rate fixed value of r and q, varying number density n_d.

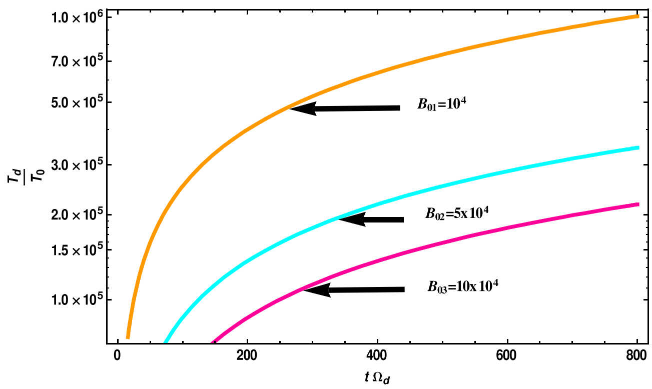

FIG. 3. Heating rate fixed value of r and q, varying magnetic field B.

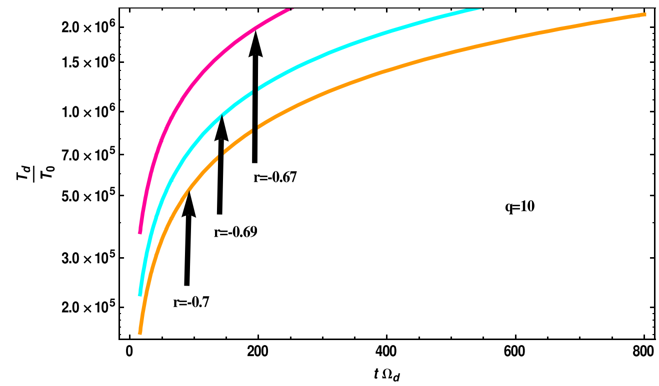

FIG. 5. Heating rate fixed $q=10$ and varying r.

distribution function remains small as compared for positive value of r. Thus, if positive values of r were chosen, the heating rate would increase to unrealistically high values as the distribution function becomes flat topped and will have a sharp gradient. Smaller values of q contribute to the high energy tails, and this is the same as the kappa (κ) distribution as $q=\kappa+1$. For the range of values that we have chosen, we see that in general heating is smaller than the ones obtained for Maxwellian distributions. However, as pointed out above, this is due to the choice of the numerical values of r and q. Finally, we point out that our theoretical considerations are general and would find applications in dusty plasmas in different environments wherever quasilinear effects are important and the background distributions show a deviation from Maxwellian distribution functions.

In conclusion, we state that the quasilinear theory has been used to calculate the resonant heating of dust particles through Alfvén waves by using the generalized (r, q) distribution function. The heating rate depends on mass, magnetic field, and density of the dust species, as well as depends on values of spectral indexes r and q. The heating rate shows a weak dependence on the spectral indices q of the dust distribution function. However, the dependence on the negative value of r is strong and this is shown in our graphical results also. Our results also show that the dust particles heat up very rapidly due to the wave particle interaction between the dust Alfvén waves and thermal motions of the dust particles. We note that although the initial temperature of the dust was very low ($\approx$300 K), it quickly heated and attained temperatures of around 10^{13} K. We feel that our results would have practical applications to both laboratory and space plasmas.

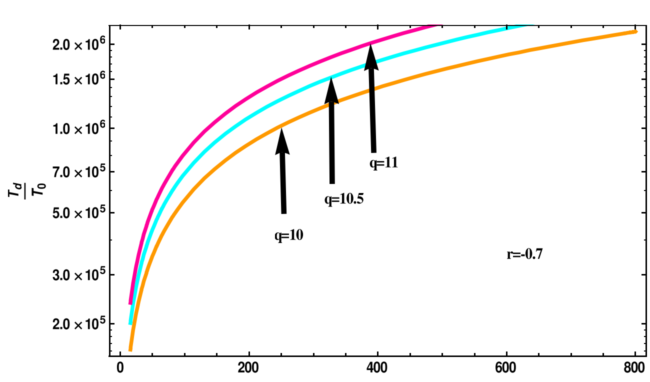

FIG. 4. Heating rate fixed $r=-0.7$ and varying q.

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