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Effect of adiabatic trapping on vortices and solitons in degenerate plasma in the presence of a quantizing magnetic field

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Abstract

The effect of adiabatic trapping as a microscopic phenomenon in an inhomogeneous degenerate plasma is investigated in the presence of a quantizing magnetic field, and a modified Hasegawa Mima equation for the drift ion-acoustic wave is obtained. The linear dispersion relation in the presence of the quantizing magnetic field is investigated. The modified Hasegawa Mima equation is investigated to obtain bounce frequencies of the trapped particles. The Korteweg–de Vries equation is derived for the two-dimensional case and finally the Sagdeev potential approach is used to obtain solitary structures. The theoretically obtained results have been analyzed numerically for different astrophysical plasma and quantizing magnetic field values.

Keywords: degenerate plasmas, adiabatic trapping, quantized magnetic field, quantum plasmas

(Some figures may appear in colour only in the online journal)

1. Introduction

Quantum or degenerate plasmas have become an active area of plasma research for scientists working in both theoretical and experimental fields [1]. Several investigations have been carried out for dense plasmas in astrophysical environments [2, 3], where electrons are dense enough to exhibit quantum behavior [4, 5]. Such behavior is also expected to be present in microelectronic devices [6]; thus it is important to understand the quantum effects on the behavior of linear and nonlinear wave properties of these systems. Linear theory using the quantum fluid model has been extensively used to study different wave modes in degenerate plasmas [7]. Based on the hydrodynamic version of quantum mechanics, Manfredi and Haas [8, 9] formulated the quantum multi-stream model and fluid model for plasmas. Later, this fluid model of plasma was extended to quantum magnetohydrodynamics by Haas [10]. An excellent survey was also presented by Manfredi [11] on the modeling of quantum plasmas. Haque and Mahmood [12] investigated the linear and nonlinear drift waves in inhomogeneous quantum plasmas with neutrals in the background. They found that the properties of drift solitons and shocks are modified by quantum corrections in dense magnetoplasmas. Shukla and Eliasson [13] presented the numerical study of the dark solitons and vortices in quantum electron plasmas.

As an important nonlinear phenomenon, Bernstein, Greene and Kruskal [14] showed that trapped particles have a considerable effect on the nonlinear dynamics of plasmas, where trapping was considered through the wave itself. Adiabatic trapping at the microscopic level was introduced by Gurevich [15], and it was observed that adiabatic trapping introduced a 3/2 power nonlinearity instead of the typical quadratic one when trapping was not present. The presence of trapping as a microscopic phenomenon has been confirmed by computer simulations and theory [16, 17] as well as by experimental work [18]. More recently, propagation characteristics of ion-acoustic solitary waves have been investigated with the trapping effect using non-Maxwellian distribution functions [19] and it was found that the solitary dynamics were modified and that spiky solitons are obtained instead of the usual solitons. The trapping effect on the formation of vortices has also been studied in classical plasma, and a modified Hasegawa Mima (HM) equation was derived by considering shallow and deep potential wells, respectively [20]. Characteristics of solitary structures have been investigated for both fully and partially degenerate plasmas [21] and in a subsequent work fully relativistic effects were included in the investigations [22].

It is well known that in the presence of a magnetic field, electron gas magnetization has two independent parts; (i) the paramagnetic, and (ii) the diamagnetic part. Paramagnetism is caused by the intrinsic or spin magnetic moment of electrons; however, the diamagnetic part is due to the fact that the orbital motion of electrons becomes quantized. This is also known as Landau diamagnetism or Landau quantization [23]. Landau quantization is a quantum mechanical effect as the cyclotron orbits of the electrons are quantized in the magnetic field and it affects the motion of electrons in the direction parallel to the magnetic field itself. Therefore discrete energy levels called Landau levels are occupied by the charged particles. The Landau levels are degenerate and the strength of the magnetic field determines the number of electrons per level. When the energy level separation is greater than the mean thermal energy the Landau level effect is observable for strong magnetic field and low temperature. The field is called quantizing if the Landau quantization of electron motion in a magnetic field is taken into account [24]. By considering Zeeman splitting, each Landau level splits into a pair of levels; one for spin up and one for spin down for electrons. The Zeeman splitting will affect the Landau levels due to them having the same energy scales $2\mu_B B_0 = \hbar\omega_{ce}$, where μ_B is the Bohr's magneton, B_0 is the magnetic field, $\hbar$ is the Plank's constant normalized by 2π and $\omega_{ce} = eB_0/m_e$ is the electron cyclotron frequency. On the other hand, the ground state energy and the Fermi energies remain similar because when summed, the pairs of energy levels cancel each other out. Adiabatic trapping in the presence of a quantizing magnetic field has recently been investigated and the effect of this field was studied both theoretically and numerically [25]. Electron holes and their coupling with Langmuir waves have also been investigated in a quantum regime by considering the Wigner–Poisson model [26, 27].

In the present paper, we continue with our investigations of the influence of trapping for ion-acoustic waves in the presence of Landau quantization for degenerate plasmas when inhomogeneities are present in the number density. Thus in the present work we shall consider the formation of ion-acoustic vortices under the conditions mentioned above. We derive a modified HM equation and present its investigations. In particular situations, we obtain a Korteweg–de Vries (KdV) equation from our modified HM equation and later use the Sagdeev potential approach to study the formation of solitary vortices. The layout of the present work is as follows: In section 1, we give a general overview of the problem. In section 2, we discuss some mathematical preliminaries and give the formulation using the Fermi–Dirac distribution function and finally derive a modified HM equation. In section 3, the linear dispersion relation is derived and the bounce frequency of the electrons trapped in the potential well is calculated. In section 4, the KdV equation is derived and in section 5, solitary structures are investigated by deriving the Sagdeev potential. Finally, numerical results are discussed in section 6.

2. Basic set of equations

In this section we set up the fundamental equations needed for the investigation of ion-acoustic waves in quantum plasma. Our aim is to investigate the formation of solitary vortices associated with these waves, and for this the background number density $n_0(x)$ of the charged particles is taken to be inhomogeneous, which is considered to be weak; i.e., $-\left(\frac{1}{n_0}\right)\left(\frac{dn_0}{dx}\right) = \kappa < 1$ (which means that higher derivatives of $n_0(x)$ and κ^2 type of terms are not taken into account), and the x-direction is chosen perpendicular to the ambient magnetic field. Furthermore, for ion-acoustic waves, electrons are taken to be massless and we need only to consider the total electron density through the distribution function and not the electron dynamics. We note here that a similar treatment was used for the derivation of the HM equation in a classical plasma [28].

Thus, following the method elucidated by Landau and Lifshitz [23], we can obtain the distribution of electrons, which includes the effects of adiabatic trapping. The occupation number for the Fermi–Dirac distribution, which takes into account the effect of the magnetic field via Landau splitting [25], is

$$n_e = \frac{p_{Fe}^2\eta}{2\pi^2\hbar^3}\sqrt{\frac{m_e}{2}}\sum_{\ell=0}^{\infty}\int_{\infty}^{0}\frac{\varepsilon^{-1/2}}{\exp\left\{\frac{\varepsilon - U}{T}\right\}+1}d\varepsilon, \quad (1)$$

where $U = e\varphi + \mu - \ell\hbar\omega_{ce}$, μ is the chemical potential, φ is the trapping potential from the ions, ℓ represents the Landau levels and $p_{Fe} = \sqrt{2m_e\varepsilon_{Fe}}$ is the electron Fermi momentum. Following the method of integration used in [25], the expression for the number density of trapped electrons for fully degenerate plasma is given by

$$n_e = n_0(x)\left[\frac{3}{2}\eta\left(1+\Phi\right)^{1/2} + \left(1+\Phi-\eta\right)^{3/2}\right]. \quad (2)$$

Here, the effect of the quantizing magnetic field appears through the normalized parameter $\eta = \hbar\omega_{ce}/\varepsilon_{Fe}$, $\Phi = \frac{e\varphi}{\varepsilon_{Fe}}$ is the normalized electrostatic potential and $\varepsilon_{Fe} = \frac{\hbar^2}{2m_e}\left(3\pi^2 n_0\right)^{3/2}$ is the electron Fermi energy. In equation (2), the terms containing T^2 are not taken into account as the cold plasma limit is assumed, which is valid for fully degenerate plasma.

The ions on the other hand are considered to be cold and non-degenerate due to their heavy mass. The magnetic field is taken in the z-direction and the density gradient in the x -direction. The ion equations of continuity and motion are respectively:

$$\frac{\partial n_i}{\partial t} + \nabla\cdot(n_i\mathbf{v}_i) = 0 \quad (3)$$

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_i \cdot \nabla\right)\mathbf{v}_i = \frac{e}{m_i}\left(\mathbf{E} + \mathbf{v}_i \times \mathbf{B}\right) - \frac{1}{m_i n_i}\nabla p_i. \quad (4)$$

Here, n_i, $\mathbf{v}_i$, m_i and p_i are the ion density, velocity, mass and pressure, respectively. Plasma is assumed to be quasineutral and only ions are magnetized by the ambient magnetic field. The ion pressure will be neglected in the following sections for simplicity.

By following the method used by Weiland [28], we obtain from equations (3) and (4) the following equation in the absence of baroclinic pressure:

$$\frac{d}{dt}\left[\ln\left(\frac{\Omega_i + \Omega_{ci}}{n_i}\right)\right] = 0. \quad (5)$$

Here, Ω_i is the vorticity, which is defined as $\Omega_i = \nabla \times \mathbf{v}_i$, $\Omega_{ci} = \frac{e\mathbf{B}}{m_i}$ is the ion gyrofrequency and $\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{v}_i \cdot \nabla$ is the total derivative, where $\mathbf{v}_i$ in the drift approximation [28] is taken as $\mathbf{v}_i = \mathbf{v}_e + \mathbf{v}_g$, $\mathbf{v}_e$ and $\mathbf{v}_g$ are the $\mathbf{E} \times \mathbf{B}$ and gravitational drifts, respectively. We consider propagation in the x and y directions and the vorticity only in the z-direction, which is given by

$$\Omega_i = \left(\nabla \times \mathbf{v}_i\right) \cdot \hat{z}. \quad (6)$$

By considering $\Omega_i \ll \Omega_{ci}$, we can write equation (5) as

$$\frac{d}{dt}\left[\ln\frac{\Omega_{ci}}{n_i} + \frac{\Omega_i}{\Omega_{ci}} - \frac{\delta n}{n_0} + \frac{1}{2}\left(\frac{\delta n}{n_0}\right)^2\right] = 0. \quad (7)$$

By assuming the plasma to be quasineutral, i.e. $n_i = n_e = n_0(x)\left(1 + \frac{\delta n}{n_0}\right)$, upon using equation (2), where $\delta n = n_e - n_0$, we obtain

$$\frac{\delta n}{n_0} = \frac{3}{2}\eta\left(1 + \Phi\right)^{1/2} + \left(1 + \Phi - \eta\right)^{3/2} - 1. \quad (8)$$

Using equation (8) in equation (7), we obtain

$$\begin{aligned}
&\left(\partial_t + v_g\partial_y\right)\left(\rho^2\nabla^2\Phi\right) - v_e^*\partial_y\Phi + \left(\partial_t + v_g\partial_y\right) \\
&\times\left[\begin{array}{l} -3\eta\left(1+\Phi\right)^{1/2} - 2\left(1+\Phi-\eta\right)^{3/2} + \frac{1}{2}\left(1+\Phi-\eta\right)^3 \\ + \frac{3}{2}\eta\left(1+\Phi\right)^{1/2}\left(1+\Phi-\eta\right)^{3/2} \end{array}\right] \\
&= \frac{\rho^2}{B_0}\frac{T_F}{e}\left(\nabla\Phi \times \hat{z}\right) \cdot \nabla\left(\nabla^2\Phi\right), \quad (9)
\end{aligned}$$

where $v_e^* = \frac{\kappa T_F}{eB_0}$ is the electron diamagnetic drift, κ is the inverse scale length of the number density inhomogeneity (defined earlier), $\rho = \frac{c_s}{\Omega_{ci}}$ is the ion larmour radius and $c_s = \sqrt{\frac{\varepsilon_{Fe}}{m_i}}$ is the ion sound velocity.

In equation (9), we can note that the presence of trapped particles produces a modified HM equation as the nonlinear term (last term on the left-hand side of equation (9)) differs here from the classical HM equation [20], where the additional terms containing η occur due to the effect of Landau quantization. The fractional power nonlinear terms will make a larger contribution than the quadratic nonlinearity occurring in the original HM equation, and thus the higher order nonlinearities have subsequently been dropped.

$$\begin{aligned}
&\left(\partial_t + v_g\partial_y\right)\left(\rho^2\nabla^2\Phi\right) - v_e^*\partial_y\Phi + \left(\partial_t + v_g\partial_y\right) \\
&\times\left[\begin{array}{l} -3\eta\left(1+\Phi\right)^{1/2} - 2\left(1+\Phi-\eta\right)^{3/2} \\ + \frac{1}{2}\left(1+\Phi-\eta\right)^3 + \frac{3}{2}\eta\left(1+\Phi\right)^{1/2} \\ \times\left(1+\Phi-\eta\right)^{3/2} \end{array}\right] = 0. \quad (10)
\end{aligned}$$

We note here that this is a complicated equation and, in general, analytically exact solutions are not possible to obtain. Thus, in the following sections, we investigate the equation in different limits.

3. Linear dispersion relation and bounce frequency

In the present section we begin by linearizing equation (10) and by using the plane wave solution we obtain the linear dispersion relation for drift ion-acoustic waves in the presence of Landau quantization, as

$$\omega = \frac{2}{3}k_y v_e^*\left(1 - \frac{2}{3}\rho^2 k^2 + \frac{5}{2}\eta\right) + k_y v_g. \quad (11)$$

In the absence of Landau quantization $\eta = 0$, we obtain the same linear dispersion relation as derived in reference [29] except for the last term on the right-hand side. This equation is analyzed graphically in section 6.

We now consider that the trapped particles in the potential well, which can move to and fro in the well itself, remain trapped if their energy is less than the potential energy of the well. We expand Φ around a fixed minimum value Φ_0 of the potential well by taking $\Phi = \Phi_0 + \Phi_1$ and linearizing equation (10), we obtain

$$\omega_b = \frac{k_y v_e^*}{\begin{array}{c}\rho^2 k^2 + \frac{3}{2}\eta\left(1+\Phi_0\right)^{-1/2} + 3\left(1+\Phi_0\right)^{1/2} \\ - \frac{3}{2}\left(1+\Phi_0\right)^2 - 3\eta\left(1+\Phi_0\right) - \frac{9}{4}\eta\end{array}}, \quad (12)$$

where $\omega_b = \omega - k_y v_{gy}$ is the bounce frequency as the particle is reflected off the walls of the potential well.

4. KdV-type solution

In this section, we use the reductive perturbation technique [30] for the long wavelength solution of equation (10) and derive a KdV-type equation which is valid for large-scale motions [31]. Stretched variables are introduced in the following manner: $\xi = \varepsilon^{\frac{1}{2}}(y - ut)$, $\tau = \varepsilon^{\frac{3}{2}}t$, $x = x$ and the perturbations in potential are $\Phi = \varepsilon\Phi_1 + \varepsilon^2\Phi_2$. Here, u is the speed of perturbation in the co-moving frame of reference. By using these perturbations in equation (10), and collecting the lowest order terms in ε (i.e. $\varepsilon^{\frac{3}{2}}$), we obtain

$$(v_g - u)\left(\rho^2\partial_{xx}\right)\partial_\xi\Phi_1 - v_e^*\partial_\xi\Phi_1 = \frac{3}{2}(v_g - u)\,\partial_\xi\Phi_1. \tag{13}$$

Equation (13) corresponds to the linear regime and integration will yield the linear dispersion relation derived in the preceding section. By collecting the terms in the next order, i.e. $\varepsilon^{\frac{5}{2}}$, we get

$$\begin{aligned}&\partial_\tau\left(\rho^2\partial_{xx}\right)\Phi_1 + (v_g - u)\rho^2\,\partial_{\xi\xi\xi}\Phi_1\\ &\quad + (v_g - u)\left(\rho^2\partial_{xx}\right)\partial_\xi\Phi_2 - v_e^*\partial_\xi\Phi_2\\ &\quad = \frac{3}{2}(v_g - u)\,\partial_\xi\Phi_2 - \frac{3}{4}(v_g - u)\,\partial_\xi{\Phi_1}^2\\ &\quad + \frac{3}{2}\,\partial_\tau\Phi_1 - \frac{15}{8}\,\eta\,(v_g - u)\,\partial_\xi{\Phi_1}^2.\end{aligned} \tag{14}$$

Following the method used by Dodd *et al* [30], we do a separation of variables as: $\Phi_1 = A(\eta, \tau)\ Y(x)$ and by using equation (13) in equation (14), we obtain

$$\begin{aligned}&\partial_\tau\left(\rho^2 Y^{\cdot\cdot}\right)A - \frac{3}{2}Y\partial_\tau A + \rho^2(v_g - u)\,Y\partial_{\xi\xi\xi}A\\ &\quad + \frac{3}{4}(v_g - u)\,Y^2\partial_\xi A^2 + \frac{15}{8}\,\eta\,(v_g - u)\,Y^2\partial_\xi A^2 = 0.\end{aligned}$$

Multiplying by ‘y’ on both sides and integrating by taking boundary conditions such that when $x \to \pm\infty$, $y\frac{dy}{dx} \to 0$, we get

$$\partial_\tau A + a\partial_{\xi\xi\xi}A + b\,\partial_\xi A^2 = 0. \tag{15}$$

where a and b are the coefficients given by

$$\begin{aligned}a &= \frac{\int_{-\infty}^{+\infty}\rho^2(v_g - u)y^2dx}{\int_{-\infty}^{+\infty}\left(\rho^2 Y^{\cdot\cdot} - \frac{3}{2}y\right)ydx}\\ b &= \frac{3}{4}(v_g - u)\frac{\int_{-\infty}^{+\infty}y^3dx}{\int_{-\infty}^{+\infty}\left(\rho^2 Y^{\cdot\cdot} - \frac{3}{2}y\right)ydx}\\ &\quad + \frac{15}{8}\eta(v_g - u)\frac{\int_{-\infty}^{+\infty}y^3dx}{\int_{-\infty}^{+\infty}\left(\rho^2 Y^{\cdot\cdot} - \frac{3}{2}y\right)ydx}\end{aligned}$$

The solution of equation (15) is

$$A = \frac{3}{2}\frac{\lambda}{b}sech^2\left(\frac{1}{2}\sqrt{\frac{\lambda}{a}}\,(\xi - \lambda t)\right). \tag{16}$$

Here, $\frac{3}{2}\frac{\lambda}{b}$ is the amplitude and λ is the velocity of the co-moving frame of reference. Equation (16) is the standard solution of the KdV equation. However, we note that x dependence on y can be evaluated if a specific form of the x dependence is given. However, we have left the result general here.

5. Sagdeev potential

In order to proceed further in the analysis of our modified HM equation, i.e. Equation (10), we derive the Sagdeev potential to investigate the formation of solitary waves and to this end we shift to the co-moving frame of reference, i.e. $\xi = \alpha x + \beta y - ut$. Here, α and β are the direction cosines.

Thus equation (10) can now be reset into the following form

$$\begin{aligned}\frac{d^2\Phi}{d\xi^2} = -\frac{v_e^*}{u} - &\begin{bmatrix}-3\,\eta\left(1 + \Phi\right)^{1/2} - 2\left(1 + \Phi - \eta\right)^{3/2}\\ +\frac{3}{2}\eta\left(1 + \Phi\right)^{1/2}\left(1 + \Phi - \eta\right)^{3/2}\\ +\frac{1}{2}\left(1 + \Phi - \eta\right)^3\end{bmatrix}\\ + &\begin{bmatrix}-3\,\eta - 2\left(1 - \eta\right)^{3/2}\\ +\ \frac{3}{2}\eta\left(1 - \eta\right)^{3/2} + \frac{1}{2}\left(1 - \eta\right)^3\end{bmatrix}.\end{aligned} \tag{17}$$

By standard manipulation [32], equation (17) can be expressed in the form of an energy integral

$$\frac{1}{2}\left(\frac{d\Phi}{d\xi}\right)^2 + V(\Phi) = 0, \tag{18}$$

where $V(\Phi)$ is the Sagdeev or pseudo potential and is given below

$$\begin{aligned}V(\Phi) = &\frac{v_e^*}{u}\frac{\Phi^2}{2} + 3\,\eta\Phi + 2\left(1 - \eta\right)^{3/2}\Phi - \frac{3}{2}\\ &\times\eta\left(1 - \eta\right)^{3/2}\Phi - \frac{1}{2}\left(1 - \eta\right)^3\Phi - 2\eta\ \left(1 + \Phi\right)^{3/2}\\ &+ 2\eta - \frac{4}{5}\left(1 + \Phi - \eta\right)^{5/2} + \frac{4}{5}\left(1 - \eta\right)^{5/2}\\ &+ \frac{1}{8}\left(1 + \Phi - \eta\right)^4 - \frac{1}{8}\left(1 - \eta\right)^4\\ &+ \frac{1}{16}\eta\ \left(1 + \Phi\right)^{1/2}\left(1 + \Phi - \eta\right)^{1/2}\begin{pmatrix}8\ \left(1 + \Phi\right)^2\\ -14\eta\ \left(1 + \Phi\right)\end{pmatrix}\\ &- \frac{1}{16}\eta\ \left(1 - \eta\right)^{1/2}\left(8{-}14\eta\right).\end{aligned} \tag{19}$$

In order to obtain solitary waves, the conditions which must be fulfilled are: $V(\Phi) < 0$ when $0 > \Phi > \Phi_{min}$ for refractive solitary waves and for compressive solitary waves $V(\Phi) < 0$ when $0 < \Phi < \Phi_{max}$, where Φ_{min} and Φ_{max} are the maximum and minimum values of potential for which $V(\Phi) = 0$ [33]. However, we can see that the Sagdeev potential given by equation (19) has a complicated dependence on the different parameters, i.e. η and v_e^*, as well as having whole power nonlinearities and fractional power nonlinearities. As an exact solution of equation (19) is not possible, the behavior of the Sagdeev potential is investigated numerically in the next section.

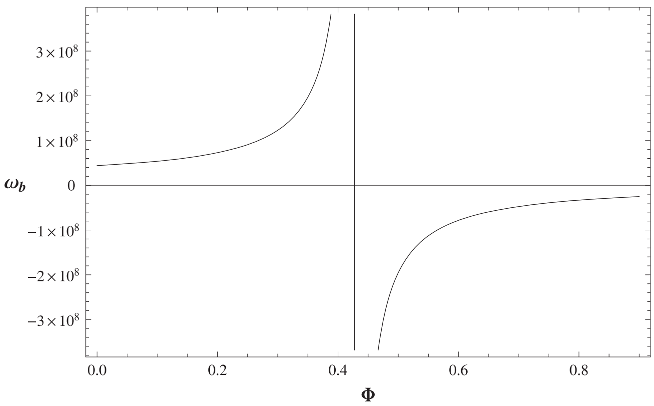

Figure 1. Bounce frequency versus potential for fixed values of $n_0 = 10^{32}$ m^{-3}, $B_0 = 10^6$ T and $\kappa = 10^8$ m^{-1}.

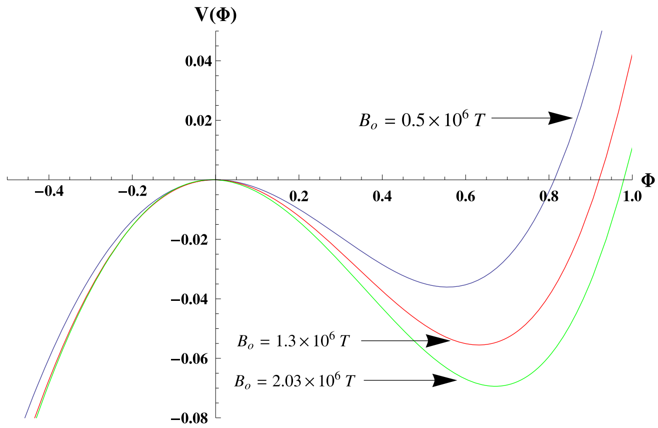

Figure 2. Sagdeev potential $V(\Phi)$ versus Φ for different values of B_0 when $n_0 = 10^{32}$ m^{-3} and $\kappa = 4 \times 10^8$ m^{-1}.

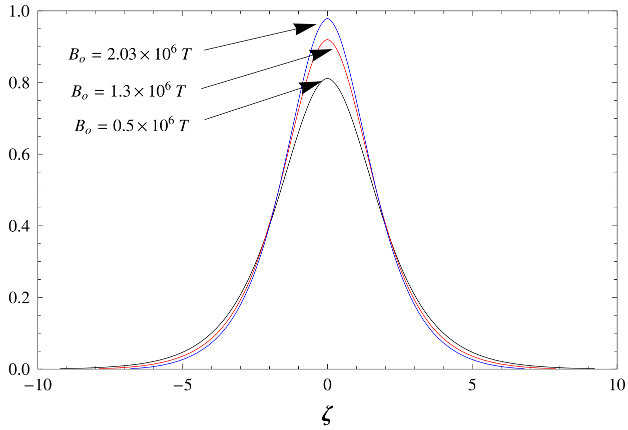

Figure 3. Solitary structures corresponding to the Sagdeev potential $V(\Phi)$ shown in figure 2.

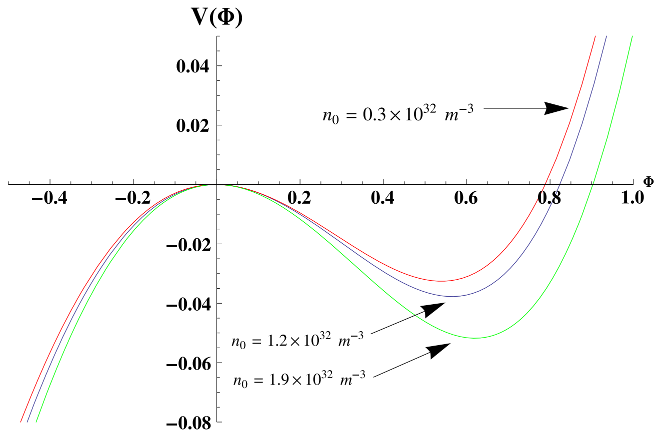

Figure 4. Sagdeev potential $V(\Phi)$ versus Φ for different values of n_0 when $B_0 = 0.5 \times 10^6$ T and $\kappa = 4 \times 10^8$ m^{-1}.

6. Results and discussion

In this section, we present the numerical solution of our theoretical results. For numerical values, we use the parameters of dense astrophysical objects like the white dwarf star, where the values of number density and magnetic field are of the order of 10^{32} m^{-3} and 10^6 T, respectively [34, 35].

We start by considering the linear dispersion relation (equation (11)) and investigate the effect of number density and quantizing magnetic field. Numerical investigations show that an increase in number density enhances the frequency. Higher frequencies are observed with a decrease in magnetic field (expressed through an increase of η).

Figure 1 depicts the relation between the bounce frequency and the potential using equation (12). We can note that as the potential reaches a value of 0.425, the bounce frequency becomes infinity, which means that no particle remains trapped in the potential well. If we compare our result of bounce frequency with the results given in reference [29], in which bounce frequency is derived for a classical plasma without taking into account the Landau quantization η, the bounce frequency in the present case approaches infinity at a much lower value of potential; i.e., the range of trapping potential is reduced.

Graphical investigations of the Sagdeev potential (equation (19)) and corresponding solitary structures are numerically obtained and are shown in figures 2–5. In these plots the dependence of the Sagdeev potential on the magnetic field, number density (through η) and inverse of scale length κ of number density inhomogeneity are investigated. From figure 2, it is observed that the depth and width of the Sagdeev potential increases with an increase in the magnetic field, when density is taken as $n_0 = 10^{32}$ m^{-3}. Corresponding solitary structures are shown in figure 3, where we can see that by increasing the magnetic field (as η increases) the amplitude of solitons increases but the width decreases slightly.

In figure 4, the Sagdeev potential is plotted for different values of number densities by keeping the magnetic field fixed. In this figure, we notice that by increasing the number density there is an enhancement in the width and depth of the Sagdeev potential. The corresponding solitary structures are plotted in figure 5. It is found that the width of soliton decreases but the amplitude increases with the increase of number density. Similarly, by increasing the strength of the inhomogeneity κ, the Sagdeev potential becomes deeper and

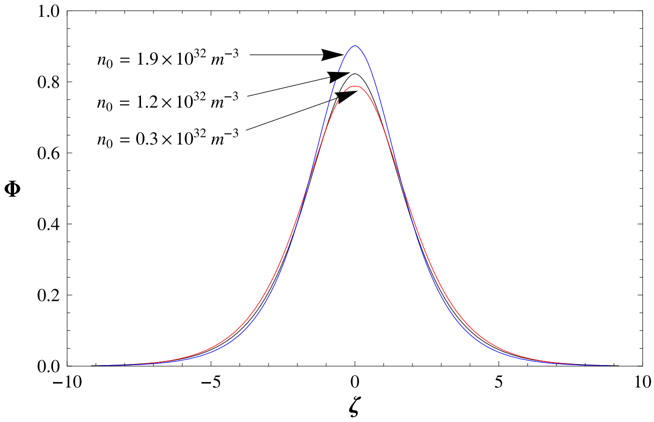

Figure 5. Solitary structures corresponding to the Sagdeev potential $V(\Phi)$ shown in figure 4.

the value of the potential increases as well. The same trend can also be noted in the solitary structures.

To conclude, in this paper, we have investigated the formation of solitary structures in an inhomogeneous degenerate quantum plasma in the presence of a quantizing magnetic field. We have derived the modified HM equation for an ion-acoustic wave. Large scale structures have been investigated via the KdV equation. We have investigated our theoretical results numerically for different parameters such as magnetic field, density and inverse of inhomogeneity scale length. These results have been presented graphically showing the formation of solitary structures. The present study can be useful in understanding the propagation characteristics of nonlinear drift waves in dense astrophysical plasmas such as white dwarf stars where quantum effects are expected to play an important role.

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