AIP | Physics of Plasmas
#1 JOURNAL IN 2014 GOOGLE SCHOLAR METRICS FOR THE PLASMA & FUSION CATEGORY

Properties of solitary ion acoustic waves in a quantized degenerate magnetoplasma with trapped electrons

N. L. Tsintsadze, H. A. Shah, M. N. S. Qureshi, and M. N. Tagviashvili

Citation: Physics of Plasmas **22**, 022303 (2015); doi: 10.1063/1.4907222
View online: http://dx.doi.org/10.1063/1.4907222
View Table of Contents: http://scitation.aip.org/content/aip/journal/pop/22/2?ver=pdfcov
Published by the AIP Publishing

PHYSICS OF PLASMAS 22, 022303 (2015)

Properties of solitary ion acoustic waves in a quantized degenerate magnetoplasma with trapped electrons

N. L. Tsintsadze,[1] H. A. Shah,[2] M. N. S. Qureshi,[3] and M. N. Tagviashvili[1]
[1]Faculty of Exact and Natural Sciences, Andronikashvili Institute of Physics, Tbilisi State University, Tbilisi 0128, Georgia
[2]Department of Physics, Forman Christian College, Ferozepur Road, Lahore 54600, Pakistan
[3]Department of Physics, GC University, Katchery Road, Lahore 54000, Pakistan

(Received 4 November 2014; accepted 19 January 2015; published online 5 February 2015)

We have undertaken the investigation of ion acoustic solitary waves in both weakly and strongly quantized degenerate magnetoplasmas. It is seen that a singular point clearly demarcates the regions of weak and strong quantization due to the ambient magnetic field. The effect of the magnetic field is taken into account via the parameter $\eta_0 = \hbar\omega_{ce}/\varepsilon_{Fe}$ and the Mach number, and their effect on the formation of solitary structures is investigated in both cases and some results are presented graphically. © *2015 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4907222]

I. INTRODUCTION

The nonlinear dynamics of plasma waves in isotropic as well as magnetoactive plasmas has been fairly well studied in the last four decades.[1–13] More recently, a significant volume of literature has emerged which has been devoted to the investigation of collective behavior in degenerate plasmas (for a review of works, see Ref. 14 and the references therein). Quantum or degenerate plasmas are a subject of increasing interest due to their potential applications in modern emerging technologies, e.g., metallic and semiconductor nano-structures which include metallic nano particles, metal clusters, thin films spintronics, nanotubes, quantum wells and quantum dots, nanoplasmonic devices, quantum x-ray free electron lasers, etc.

The properties of linear electron oscillations in a degenerate Fermi plasma have been investigated in the recent past.[15–17] Tsintsadze and Tsintsadze[18,19] developed new type of quantum kinetic equations for Fermi particles of various species and subsequently obtained a set of fluid equations describing a quantum plasma. The dispersion properties of electrostatic oscillations were discussed later in Refs. 20 and 21. Based on these studies, the investigation of linear and nonlinear ion acoustic waves in quantum plasmas has attracted substantial attention. In particular, the investigation of ion acoustic solitary structures has attracted special attention.[22–24] The effects of quantization of the electron orbital motion and electron spin on the propagation of longitudinal waves in a Fermi gas have been reported recently,[25] where a detailed investigation of the effects of a quantizing magnetic field was presented. The cases of a weakly quantizing magnetic field as well as the case for a strongly quantizing magnetic field were presented and the effect of adiabatic trapping in a quantizing magnetic field was investigated in Ref. 26.

In the present work, we wish to undertake the investigation of ion acoustic solitary waves in a degenerate electron-ion plasma by considering the effect of Landau quantization. The layout of the current work is as follows: In Sec. II, we consider the effect of a quantizing magnetic field both in the instances of a weak field and strong field (Landau quantization). In Sec. III, we consider the effects of the quantizing magnetic field on the linear dispersion relation and in Sec. IV, the nonlinear evolution equations are developed and various limiting cases are studied. Finally in Sec. V, we present a brief conclusion of our results.

II. LANDAU QUANTIZATION

In an earlier paper it was shown by Tsintsadze[25] that when Landau quantization is taken into account in the non-relativistic limit, the electron number density and Fermi energy are determined by

$$n_e = \frac{p_F^3}{3\pi^2\hbar^3}\left\{\frac{3}{2}\eta + (1-\eta)^{3/2}\right\} \tag{1}$$

and

$$\varepsilon_{Fe} = \frac{(3\pi^2)^{2/3}\hbar^2 n^{2/3}}{2m_e\left(\frac{3}{2}\eta + (1-\eta)^{3/2}\right)^{2/3}}, \tag{2}$$

where $\eta = \hbar\omega_{ce}/\varepsilon_{Fe}$ and $\omega_{ce} = |e|H/m_e c$ is the electron cyclotron frequency, n_e is the electron number density and p_F and ε_{Fe} are the electron Fermi momentum and energy, respectively, when the electrons are quantized by the magnetic field.

As was shown by Landau, in a constant magnetic field $\vec{H}(0,0,H_0)$, electrons, under the action of it, rotate in circular orbits in a plane perpendicular the field $\vec{H}_0$. Therefore, the motion of the electrons can be resolved into two parts: one along magnetic field, in which the longitudinal component of energy is not quantized $E_z = \frac{p_z^2}{2m_e}$, and the second in a plane perpendicular to $\vec{H}_0$ (the transverse component) in which energy is quantized.[21,22] Thus, in the non-relativistic case, the net energy of electron in a magnetic field without taking in to account its spin is $E(p_z, l) = \frac{p_z^2}{2m_e} + \hbar\omega_c\left(l + \frac{1}{2}\right)$, where $\omega_c = \frac{|e|H_0}{m_e c}$ is the cyclotron frequency of the electron, $\hbar$ is the Planck constant divided by 2π, and m_e is the electron rest mass.

If the particle has a spin, the intrinsic magnetic moment of the particle interacts directly with the magnetic field. The correct expression for the energy is obtained by adding an extra term $\vec{\mu}\vec{H}_0$ corresponding to the energy of the magnetic moment $\vec{\mu}$ in the field $\vec{H}_0$. Hence, the electron energy levels $\varepsilon_e^{l,\delta}$ are determined in the nonrelativistic limit by the expression

$$\varepsilon_e^{l,\delta} = \frac{p_z^2}{2m_e} + (2l + 1 + \delta)\mu_B, \tag{3}$$

where l is the orbital quantum number ($l = 0, 1, 2, 3, \ldots$), δ is the operator of the z-component which describes the spin orientation $\vec{s} = \frac{1}{2}\vec{\delta}(\delta = \pm 1)$, and $\mu_B = \frac{|e|\hbar}{2m_e c}$ is the Bohr magneton.

From Eq. (1), one can see that the energy spectrum of electrons consists of the lowest Landau level $l = 0,\ \delta = -1$, and pairs of degenerate levels with opposite polarization $\delta = 1$. Thus, each value with $l \neq 0$ occurs twice, and that with $l = 0$ once. Therefore, in the non relativistic limit, $\varepsilon_e^{l,\delta}$ can be rewritten as

$$\varepsilon_e^{l,\delta} = \varepsilon_e^l = \frac{p_z^2}{2m_0} + \hbar\omega_c l. \tag{4}$$

The number of quantum states[27,28] of a particle moving in a volume V and interval dp_z for any value of l is

$$\frac{2V|e|dp_z}{(2\pi\hbar)^2 c} = \frac{V\varepsilon_F \cdot \eta m_0 dp_z}{2\pi^2\hbar^3}, \tag{5}$$

where $\eta = \frac{\hbar\omega_c}{\varepsilon_F}$ and ε_F is the electron Fermi energy. The equilibrium density of electrons is defined as

$$n_e = \frac{m_0\varepsilon_F\eta}{2\pi^2\hbar^3}\sum_{l=0}^{\infty}\int_{-\infty}^{\infty} dp_z \cdot f(p_z, l), \tag{6}$$

where

$$f(p_z, l) = \frac{1}{\exp\left(\dfrac{p_z^2/2m_0 + \hbar\omega_c l - \mu}{T}\right) + 1},$$

and μ is the electron chemical potential.

Assuming the Fermi degeneracy temperature $T_F = \frac{\varepsilon_F}{k_B}$ (k_B is the Boltzmann coefficient) much higher than the Fermi gas temperature, the Fermi distribution function is in a good approximation described by the Heaviside step function $H(\mu - \varepsilon_e^l)$, which follows $\mu = \varepsilon_F = \varepsilon_e^l = p_z^2/2m_0 + \hbar\omega_c l$. This allows us to integrate Eq. (4) by $p_z = \sqrt{2m_0(\varepsilon_F - \hbar\omega_c l)}$. The last expression reads that the summation along l is limited by the condition $\varepsilon_F > \hbar\omega_c$, so that $l_{max} = \frac{\varepsilon_F}{\hbar\omega_c} = \frac{1}{\eta}$.

We see that it is more useful to introduce new parameters $\eta_0 = \hbar\omega_{ce}/\varepsilon_{F0}$ and $\gamma = \varepsilon_{Fe}/\varepsilon_{F0}$, where $\varepsilon_{F0} = \frac{(3\pi^2)^{2/3}\hbar^2 n_0^{2/3}}{2m_e}$ is the Fermi energy and n_0 is the equilibrium number density in the absence of the magnetic field. We now note that η_0 explicitly depends on the magnetic field only via the electron cyclotron frequency ω_{ce}. The relationship between η_0 and η is given by $\eta = \eta_0/\gamma$ and Eq. (2) can be rewritten in the following manner:

$$\frac{3}{2}\eta_0\gamma^{1/2} + (\gamma - \eta_0)^{3/2} = \sigma = \frac{n_e}{n_0}. \tag{7}$$

We note here that if $\sigma = 1$ (as may be the case in metals or semiconductors), then Eq. (3) reduces to

$$\frac{3}{2}\eta_0\gamma^{1/2} + (\gamma - \eta_0)^{3/2} = 1. \tag{8}$$

From Eq. (8), it can be seen that the singular point occurs when $\hbar\omega_{ce} = \varepsilon_{Fe}$ ($\eta = 1$) or when

$$\gamma^* = \eta_0^* = \left(\frac{2}{3}\right)^{2/3}. \tag{9}$$

The singular point is clearly elucidated in Ref. 25 and this clearly marks the separation of the regions of weak $\eta < 1$ and strong magnetic fields $\eta > 1$, the former being dependent on the orbital quantum number and the latter being independent of it and corresponding to the case of Landau ground state or quantization.

For the case $\gamma < \eta_0$, the Fermi energy depends on the magnetic field in the following manner:[25]

$$\gamma = \left(\frac{2}{3\,\eta_0}\right)^2. \tag{10}$$

Figure 1 shows the dependence of γ on the magnetic field via η_o and the * on the curve indicates the singular point, separating the weak and strong field regions.

III. PHASE VELOCITY AND MACH NUMBER

Quite recently in Ref. 25, the spectra of longitudinal waves were considered for the case of a strongly magnetized

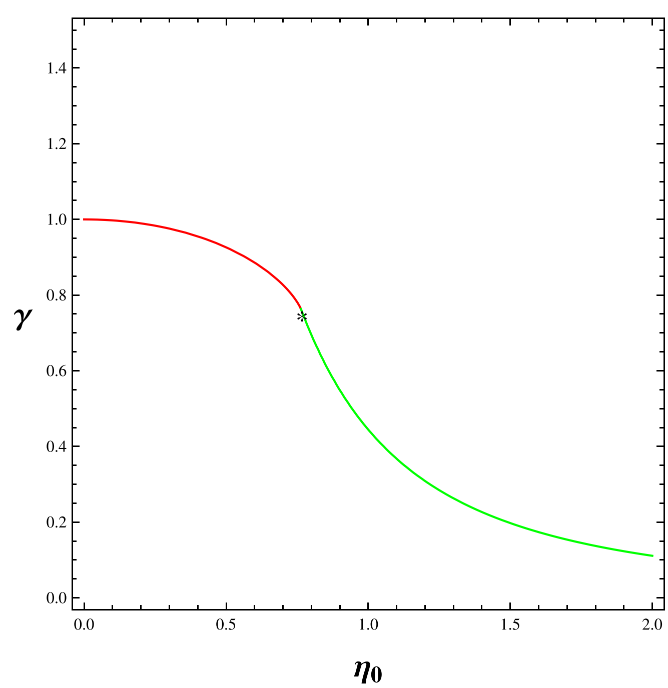

FIG. 1. Plot of $\gamma(=\frac{\varepsilon_{Fe}}{\varepsilon_{F0}})$ against magnetic field η_0. The * on the figure indicates the singular point for $\gamma^* = \eta_0^*$.

spatially homogeneous collisionless plasma with degenerate electrons and classical ions. It was seen that in the intermediate frequency range where the phase velocity $v_\phi = \omega/k$ satisfies the inequality

$$v_{Ti} \ll v_\phi \ll v_{Fe}\sqrt{1-\eta}, \tag{11}$$

where v_{Fe} is the Fermi velocity in the presence of the magnetic field (which can be obtained from Eq. (2)) and $v_{Ti} = \sqrt{T_i/m_i}$ is the ion thermal velocity. Furthermore, under the assumption that the quasineutrality condition is satisfied, i.e., $n_e \approx n_i$, for the real part of the spectra $\omega' = Re(\omega)$, is given by[25]

$$v_\phi^2 = \left(\frac{\omega'}{k}\right)^2 = \frac{m_e}{3m_i} v_{Fe}^2 \frac{3\eta + 2(1-\eta)^{3/2}}{\eta + 2\sqrt{1-\eta}}. \tag{12}$$

If $\eta = 0$, i.e., in the absence of the magnetic field, we obtain the standard expression for the phase velocity $v_{\phi 0} = \sqrt{\frac{m_e}{3m_i}} v_{F0.}$, where v_{F0} is the Fermi velocity in the absence of the magnetic field. However, in the case of a strong magnetic field $\eta > 1$, we obtain a new type of a phase velocity $v_\phi = \sqrt{3}\, v_{\phi 0}$.

Using Eq. (12), we can obtain an expression for the Mach number

$$M^2 = \left(\frac{u}{v_\phi}\right)^2 = \left(\frac{u}{v_{\phi 0}}\right)^2 \frac{\eta + 2\sqrt{1-\eta}}{3\eta + 2(1-\eta)^{3/2}}$$
$$= M_0^2 \frac{\eta + 2\sqrt{1-\eta}}{3\eta + 2(1-\eta)^{3/2}}, \tag{13}$$

where $M_0 = u/v_{\phi 0}$ is the Mach number without the magnetic field.

Let us further explore the possibilities about recasting certain quantities through the new variables η_0 and γ ignoring as above the variation in the number density $n_e \approx n_0$, thus allowing us to write down the Fermi momenta $p_F(H)$ and $p_{F0}(H=0)$ in the following way:

$$\frac{P_F}{P_{F0}} = \frac{1}{\left(\frac{3}{2}\eta + (1-\eta)^{3/2}\right)^{\frac{1}{3}}} = \gamma^{1/2}.$$

Thus, we obtain the following expression for the Mach number:

$$M^2 = \frac{M_0^2(\eta_0 + 2\gamma^{\frac{1}{2}}\sqrt{\gamma - \eta_0})}{2\,\gamma^{1/2}}. \tag{14}$$

We note here that we must use Eqs. (8) and (14) for the definition of the Mach number.

Further we see that if the magnetic field is strong enough so that inequality $\eta_0 > \gamma$ is satisfied, then the Mach number becomes

$$M^2 = \frac{3}{4} M_0^2 \eta_0^2. \tag{15}$$

In Fig. 2, we have plotted M^2/M_0^2 versus η_0, here as in Fig. 1 the * (where the red and green lines meet) on the curve

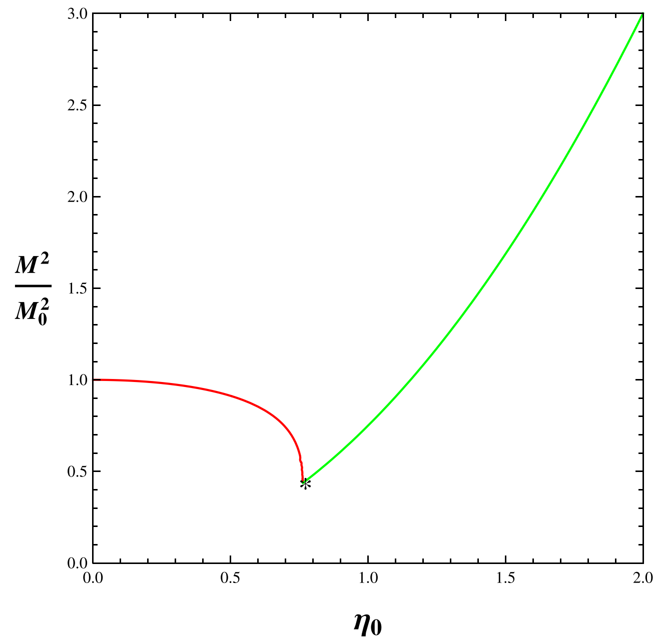

FIG. 2. Mach number $\frac{M^2}{M_0^2}$ plotted against magnetic field η_0. The * on the figure indicates the singular point for $\gamma^* = \eta_0^*$.

denotes the singular point. In fact in Fig. 2, the singular point clearly demarcates the weak and strong field regions.

IV. BASIC EQUATIONS

In this section, we give the basic mathematical formulation for considering the propagation of one dimensional nonlinear ion sound waves in a plasma with degenerate electrons and classical ions, in the presence of an ambient quantizing magnetic field. The electron motion is quantized via the Landau quantization in the direction parallel to the ambient magnetic field which is also the direction of propagation of the ion acoustic wave. The ions by comparison are considered classical, due to their relatively heavy mass. We begin by recalling Poisson's equation which is

$$\frac{\partial^2}{\partial x^2}\varphi = 4\pi e(n_e - n_i), \tag{16}$$

where φ is the electrostatic potential.

The ion number density is obtained from the ion equations of motion and continuity under the quasistationary assumption,[23,25] i.e., shifting to a co-moving frame of reference which in normalized form is given by $X = \frac{\sqrt{2}\,\omega_{pi}}{u}(x - ut)$. Such solution describes wave's propagating with speed u and without change of profile. From equation of continuity $n_i v_i = n_{oi} u$ and from energy conservation $\frac{1}{2} m_i v_i^2 + e\varphi = \frac{m_i u^2}{2}$ (ions are cold) for the ion density, we get the following expression:

$$n_i = \frac{n_0}{\sqrt{1 - \dfrac{2e\varphi}{m_i u^2}}} = n_0(1-\Phi)^{-1/2}, \tag{17}$$

where $\Phi = \frac{2e\varphi}{m_i u^2}$, u is the ion velocity at $\varphi = 0$, and ω_{pi} is the ion plasma frequency. On the other hand, we have for the electron number density,[25] in the weak magnetic field limit $\eta < 1$

$$n_e = \left\{ \frac{3}{2}\eta \left(1 + \frac{e\varphi}{\varepsilon_{Fe}}\right)^{\frac{1}{2}} + \left(1 - \eta + \frac{e\varphi}{\varepsilon_{Fe}}\right)^{\frac{3}{2}} \right\}. \tag{18}$$

Using the expression for Φ given above, we can rewrite Eq. (18) in normalized form as

$$n_e = \left\{ \frac{3}{2}\frac{\eta_0}{\gamma} \left(1 + \frac{M_0^2}{3\gamma}\Phi\right)^{\frac{1}{2}} + \left(1 - \frac{\eta_0}{\gamma} + \frac{M_0^2}{3\gamma}\Phi\right)^{\frac{3}{2}} \right\}. \tag{19}$$

This is the expression of the electron number density taking trapping of the electron distribution into account for the weak magnetic field case. We now use Eqs. (16), (17), and (19) and casting them into dimensionless form, we obtain the Poisson equation

$$\frac{d^2}{dX^2}\Phi = \left\{ \frac{3}{2}\eta_0 \left(\gamma + \frac{M_0^2}{3}\Phi\right)^{\frac{1}{2}} + \left(\gamma - \eta_0 + \frac{M_0^2}{3}\Phi\right)^{\frac{3}{2}} \right\} - (1-\Phi)^{-1/2}. \tag{20}$$

In Eq. (20) above, we have introduced the dimensionless coordinate $X = \frac{\sqrt{2}\omega_{pi}}{u}(x - ut)$ and this along with Eq. (4) forms a close set of equations which define the potential field Φ. We note here that a similar expression can be obtained for the strong magnetic field $\eta_0 > \gamma$ limit by using Eq. (10).

Small Amplitude Limit: In order to illustrate that a singular point exits which demarcates the weak and strong regions of magnetic field quantization, we consider here for the case of the weak magnetic field region, the small amplitude limit, i.e., when $\Phi < 1$. We expand Eq.(20) and retain terms up to Φ^2 to obtain the standard KdV equation which reads as

$$\frac{d^2}{dX^2}\Phi - \frac{1}{2}(M^2 - 1)\Phi + \frac{3}{8}\beta\Phi^2 = 0. \tag{21}$$

Here, M is given by expression (14) and $\beta = 1 + \frac{M_0^4}{9\gamma^{3/2}}\left(\frac{\eta_0}{2} - \frac{\gamma^{3/2}}{\sqrt{\gamma - \eta_0}}\right)$ and we see from here that we formally have a singularity when $\gamma = \eta_0$. The above equation has a standard solution given by

$$\Phi = 2\frac{(M^2-1)}{\beta}\,\mathrm{sec\,h}^2\left(\frac{\sqrt{M^2-1}}{2\sqrt{2}}X\right). \tag{22}$$

V. SAGDEEV POTENTIAL

In order to obtain solitary wave solutions in the arbitrary amplitude case, we follow the Sagdeev potential approach [e.g., Ref. 25] and note that for such potentials obtainable from Poisson's equation, certain conditions must be fulfilled. These conditions are[29] that at $\Phi = 0$, the Sagdeev potential $V(\Phi)_{\Phi=0} = 0$ and at this point the Sagdeev potential must also have a maximum, i.e., the fixed point is unstable at the origin if $\frac{d^2V(\Phi)}{d\Phi^2} < 0$. Thus, expressions for the Sagdeev potentials in the weak and strong field limits are given by $V_W(\Phi)$ and $V_S(\Phi)$, respectively.

Thus for the weak magnetic field case (see, e.g., Ref. 25), we obtain from Eq. (20)

$$\begin{aligned} V_W(\Phi) = {} & \frac{3\eta_0}{M_0^2}\left[\gamma^{\frac{3}{2}} - \left(\gamma + \frac{M_0^2}{3}\Phi\right)^{\frac{3}{2}}\right] \\ & + \frac{6}{5}M_0^2\left\{(\gamma - \eta_0)^{\frac{5}{2}} - \left(\gamma - \eta_0 + \frac{M_0^2}{3}\Phi\right)^{\frac{3}{2}}\right\} \\ & - 2\left[1 - (1-\Phi)^{1/2}\right]. \end{aligned} \tag{23}$$

Equation (23) is complete only when read with Eq. (8) and $\eta_0 > \left(\frac{2}{3}\right)^{2/3}$.

In order to obtain the expression for the Sagdeev potential in the strong magnetic field limit, we obtain from Eqs. (9) and (20)

$$\begin{aligned} V_S(\Phi) = {} & 3\eta_0\left[-\left(\frac{3}{2\eta_0}\right)^3 - \left(\left(\frac{3}{2\eta_0}\right)^2 + \frac{M_0^2}{3}\Phi\right)^{\frac{3}{2}}\right] \\ & - 2\left[1 - (1-\Phi)^{1/2}\right]. \end{aligned} \tag{24}$$

Further we note here that Eq. (24) is valid only when $\eta_0 > \left(\frac{2}{3}\right)^{2/3}$ which is obtained from Eq. (9).

We have investigated the dependence of the Sagdeev potentials in both the weak and strong magnetic field regions and these are shown in Figs. 3 and 4, respectively. In both cases, the depth and the maximum value of the potential increase with increasing Mach number M, but the relative value of the Sagdeev potential is larger for the weak field case and so is the maximum value of the potential for the same Mach number M. A similar trend can be seen in Fig. 5 where the maximum value of the potential Φ_{max} is plotted against the magnetic field strength η_0 for different values of M_0. Here, we see that in the weak field region, Φ_{max} decreases with increasing η_0 (i.e., with increasing magnetic field strength), at first slowly then rather rapidly as it approaches the singular point. However, in the strong field limit when $\eta_0 > \gamma$, the maximum value of the potential Φ_{max} increases with increasing magnetic field strength, i.e., with

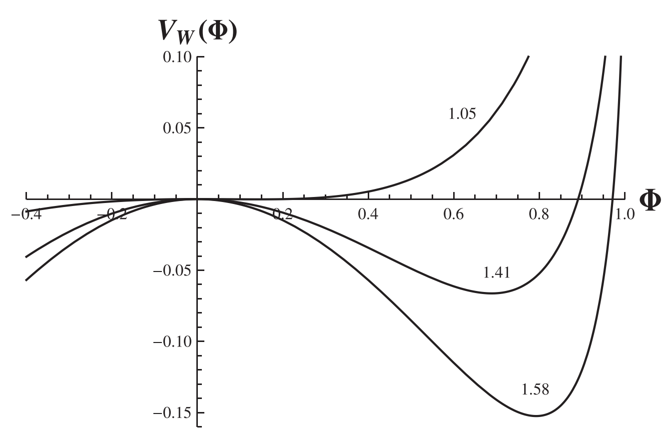

FIG. 3. Sagdeev potentials for the case of weak magnetic field $\eta_0 < \gamma$ when $M = 1.05,\ 1.41,$ and 1.58.

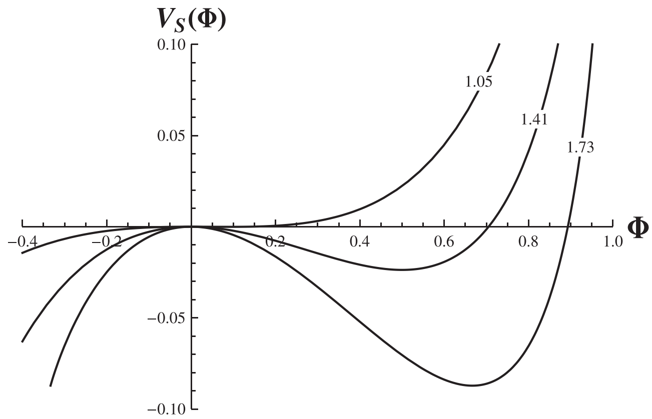

FIG. 4. Sagdeev potentials for the case of strong magnetic field $\eta_0 > \gamma$ when $M = 1.05, \ 1.41$, and 1.73.

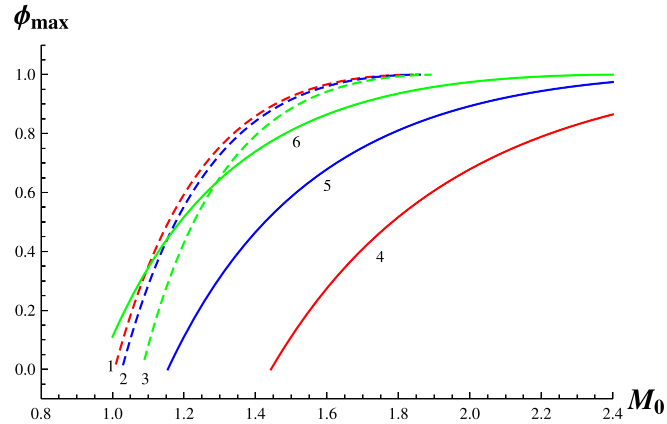

FIG. 6. The amplitude of solitary waves Φ_{max} as a function of Mach number M_0 for different values of magnetic fields 1, $\eta_0 = 0.2$; 2, $\eta_0 = 0.4$; 3, $\eta_0 = 0.6$; 4, $\eta_0 = 0.8$; 5, $\eta_0 = 1.0$; 6, $\eta_0 = 1.2$. The dashed and solid lines represent weak and strong magnetic fields, respectively.

increasing η_0. This increase is quite fast as it increases from the singular point but then the increase becomes more gradual. We also note the maximum value of the potential Φ_{max} increases with increasing M_0 as shown in Fig. 6. The trend was observed in Figs. 3 and 4 also where Sagdeev potential plots were shown for different values of the Mach number for the weak and strong field regions, respectively.

VI. SUMMARY

We have investigated the effect of Landau quantization in a dense degenerate plasma in the presence of trapped electrons. Both the weak and strong magnetic field regions are considered. In the nonlinear regime, it is seen that solitary structures are formed in both regimes. It is also seen that the Sagdeev potential in both cases depends dramatically on the quantizing magnetic field strength. In our novel approach, we see that a singular point separating the weak and strong field regimes is present. We feel that the results presented in this work can find applications in astrophysical and laser plasma interactions when high powered lasers are employed. Our results are specifically applicable to neutron stars where surface magnetic fields can be of the order of $H \sim 10^{11} - 10^{13}$ G and internal fields can be higher still due to the rotation of neutron stars, which can produce additional effect of the order of $H \sim 10^5$ G.[19,29–32] Here effects of Landau quantization would be expected to play a very important role. Also the effects of adiabatic trapping make the inclusion of nonlinearities more realistic.

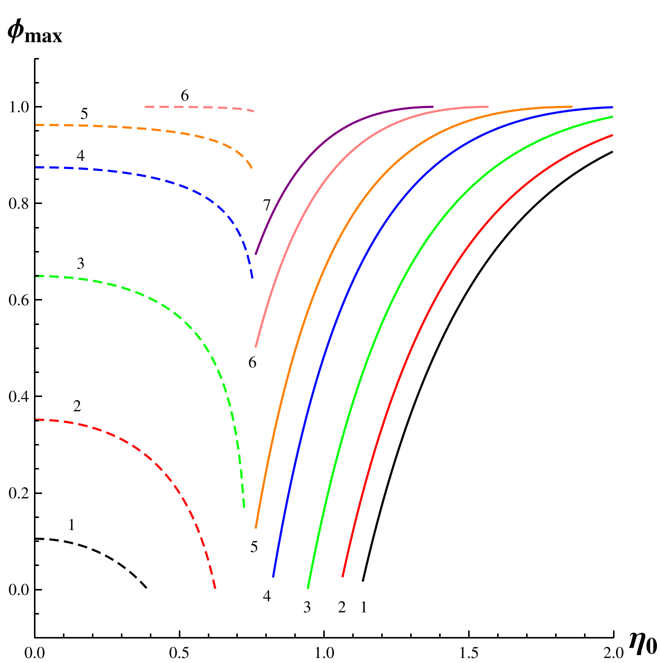

FIG. 5. The amplitude of solitary waves Φ_{max} as a function of magnetic field η_0 for different values of Mach number 1, $M_0 = 1.02$; 2, $M_0 = 1.10$; 3, $M_0 = 1.22$; 4, $M_0 = 1.41$; 5, $M_0 = 1.58$; 6, $M_0 = 1.87$; 7, $M_0 = 2.12$.

ACKNOWLEDGMENTS

N.L.T. would like to acknowledge the partial support of GNSF Grant Project No. FR/101/6-140/13 and H.A.S. and M.N.S.Q would gratefully like to acknowledge the partial support of Grant Project No. 20-2595/NRPU/R&D/HEC/13.

[1]V. E. Zakharov, Sov. JETP **35**, 908 (1972).
[2]R. Z. Sagdeev, in *Reviews of Plasma Physics 4*, edited by M. A. Leontovich (Consultants Bureau, New York, 1966).
[3]P. K. Shukla, M. N. Rao, M. Y. Yu, and N. L. Tsintsadze, Phys. Rep. **138**, 1 (1986).
[4]P. K. Shukla, N. L. Tsintsadze, and L. N. Tsintsadze, Phys. Fluids **B5**, 233 (1993).
[5]N. L. Tsintsadze, Astrophys. Space Sci. **191**, 151 (1992).
[6]V. I. Berezhiani, N. L. Tsintsadze, and P. K. Shukla, J. Plasma Phys. **48**, 139 (1992).
[7]N. L. Tsintsadze and P. K. Shukla, Phys. Lett. A **187**, 67 (1994).
[8]L. N. Tsintsadze and V. I. Berezhiani, Sov. Plasma Phys. Rep. **19**, 132 (1993).
[9]L. N. Tsintsadze, Phys. Scr. **50**, 413 (1994).
[10]S. Kartal, L. N. Tsintsadze, and V. I. Berezhiani, Phys. Rev. E **35**, 4225 (1996).
[11]M. Lontano, S. Bulanov, J. Koga, and M. Passoni, Phys. Plasmas **9**, 2562 (2002).
[12]L. N. Tsintsadze, Phys. Plasmas **2**, 4462 (1995).
[13]L. N. Tsintsadze, K. Nishikawa, T. Tajima, and J. J. Mendonca, Phys. Rev. E **60**, 7435 (1999).
[14]P. K. Shukla and B. Elliasson, Phys. Usp. **53**, 51 (2010).
[15]I. I. Goldman, Zh. Eks. Teor. Fiz. **17**, 681 (1947).
[16]Y. L. Klimontovich and V. P. Silin, Zh. Eks. Teor. Fiz. **23**, 151 (1952).
[17]D. Bohm and D. Pines, Phys. Rev. **92**, 609 (1953).
[18]N. L. Tsintsadze and L. Tsintsadze, Eur. Phys. Lett. **88**, 3500 (2009).
[19]N. L. Tsintsadze and L. N. Tsintsadze, in *From Leonardo to ITER: Nonlinear and Coherence Aspects*, edited by J. Weiland (New York, 2009), e-print arXiv:physics/0903.5368v1.

[20]B. Eliasson and P. K. Shukla, J. Plasma Phys. **76**, 7 (2010).
[21]L. N. Tsintsadze and N. L. Tsintsadze, J. Plasma Phys. **76**, 403 (2010).
[22]A. Rasheed, G. Murtaza, and N. L. Tsintsadze, Phys. Rev E **82**, 016403 (2010).
[23]H. A. Shah, W. Masood, M. N. S. Qureshi, and N. L. Tsintsadze, Phys. Plasmas **18**, 102306 (2011).
[24]N. L. Tsintsadze, L. N. Tsintsadze, A. Hussain, and G. Murtaza, Eur. Phys. D **64**, 447 (2011).
[25]L. N. Tsintsadze, AIP Conf. Proc. **1306**, 89 (2010).
[26]H. A. Shah, M. J. Iqbal, N. L. Tsintsadze, W. Masood, and M. N. S. Qureshi, Phys. Plasmas **19**, 092304 (2012).
[27]L. D. Landau and E. M. Lifshitz, *Statistical Physics, Part 1* (Butterworth-Heinemann, Oxford, 1998).
[28]L. D. Landau and E. M. Lifshitz, *Quantum Mechanics* (TTL, Moscow, 1948).
[29]J. Landstreet, Phys. Rev. **153**, 1372 (1967).
[30]S. L. Shapiro and S. A. Teukolsky, *Black Holes, White Dwarfs and Neutron Stars* (John Wiley and Sons, New York, 1983).
[31]V. M. Lipunov, *Astrophysics of Neutron Star* (Nauka, Moscow, 1987).
[32]G. S. Bisnovati-Kogan, Astron. Zh. **47**, 813 (1970).