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Solitary structures in a spatially nonuniform degenerate plasma in the presence of quantizing magnetic field

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In the present investigation, linear and nonlinear propagation of low frequency ($\omega \ll \Omega_{ci}$) electrostatic waves have been studied in a spatially inhomogeneous degenerate plasma with one dimensional electron trapping in the presence of a quantizing magnetic field and finite temperature effects. Using the drift approximation, formation of 1 and 2D drift ion solitary structures have been studied both for fully and partially degenerate plasmas. The theoretical results obtained have been analyzed numerically for the parameters typically found in white dwarfs for illustrative purpose. It is observed that the inclusion of Landau quantization significantly changes the expression of the electron number density of a dense degenerate plasma which affects the linear and nonlinear propagation of drift acoustic solitary waves in such a system. The present work may be beneficial to understand the propagation of drift solitary structures with weak transverse perturbation in a variety of physical situations, such as white dwarfs and laser-induced plasmas, where the quantum effects are expected to dominate. © *2015 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4914859]

I. INTRODUCTION

The field of quantum plasma physics has decades long diverse tradition,[1–3] with interest having arisen recently[4,5] owing to its wide ranging potential applications in modern technology (metallic and semiconductor nanostructures such as metallic nanoparticles, metal clusters, thin metal films, spintronics, nanotubes, quantum wells and quantum dots, nanoplasmonic devices, quantum X-ray free-electron lasers, etc.). The recent advances in the ultrafast spectroscopy techniques have enabled us to monitor the femto-second dynamics of an electron gas confined in metallic plasmas. In dense quantum plasmas, the number densities of degenerate electrons and/or positrons are extremely high, and the plasma particles (mainly electrons and positrons) obey the Fermi-Dirac statistics.

It was recognized many decades ago that the underlying physics of nonlinear quantum-like equations can be better understood by rewriting them in the form of hydrodynamical (or Euler) equations, which essentially represent the evolution of quantum particle densities and momenta. This was elegantly done by Bohm[2] and Madelung[6] by introducing an eikonal representation for the wave function evolution in the nonstationary Schrodinger equation.

Recently, there has been growing and vibrant interest in investigating new aspects of quantum plasma physics by developing nonrelativistic quantum hydrodynamical (QHD) equations[5,7] that include the quantum statistical electron pressure and the quantum force involving tunneling of degenerate electrons through the Bohm potential.[8] The electrostatic QHD equations are useful for studying collective interactions (e.g., different types of waves, instabilities, quantum fluid turbulence, and nonlinear structures).[5,9–14] For instance, Mamun and Shukla[15] investigated the propagation of solitary waves in a relativistically degenerate plasmas and applied their results to compact stars like white dwarfs. It was observed that the plasma parameters affected the propagation characteristics of the nonlinear electrostatic excitations in the vicinity of compact astrophysical objects like white dwarfs. The authors also studied the shock wave formation in planar and nonplanar geometries and showed that plasma number density and ion viscosity, which was introduced by considering the strong correlation among ions, significantly changed the shock structure.[16] The investigation of numerous collective interactions in dense plasmas is relevant in the context of intense laser-solid density plasma experiments;[17–23] the cores of giant planets and the crusts of old stars;[24,25] superdense astrophysical objects[26–28] (e.g., interiors of white dwarfs and magnetospheres of neutron stars and magnetars); micro and nano-scale objects (e.g., quantum diodes,[29–31] quantum dots and nanowires,[32] nanophotonics,[33,34] plasmonics,[35] ultra-small electronic devices,[36–38] and metallic nanostructures[39]); and microplasmas[40] and quantum X-ray free-electron lasers.[41,42] Furthermore, a Fermi-degenerate dense plasma may also arise when a pellet of hydrogen is compressed to many times the solid density in the fast ignition scenario for inertial confinement fusion.[42–44] Due to the fast paced development in the field of short-pulse petawatt laser technology, it is highly likely that such plasma conditions are achieved by intense laser-pulse compression using powerful X-ray pulses. An excellent review on quantum plasmas is written by Shukla and Eliasson[45] which not only introduces the subject to the reader but also explores the areas of applicability of quantum plasmas in its full glory. Much of the linear and nonlinear work in quantum plasmas has effectively been recapitulated in this review.

The effect of strong magnetic fields has not been the main focus of attention in degenerate plasmas. The presence of a strong ambient magnetic field qualitatively changes the properties of atoms, molecules, and condensed matter when the electron cyclotron energy $\hbar\omega_{ce}$ is larger than the typical Coulomb energy. As is well known, electron gas magnetization in a weak magnetic field has two independent parts, namely, paramagnetic and diamagnetic. The intrinsic or spin magnetic moment of electrons gives rise to Pauli paramagnetism. The diamagnetic part owes its existence to the quantized orbital motion of electrons in the magnetic field. This is also called Landau diamagnetism or Landau quantization.[46] The gas is degenerate if the system temperature T (in eV) $\ll \varepsilon_F$ (ε_F is the Fermi energy). If the Landau quantization of electron motion in a magnetic field is taken into account, then the field is called quantizing[47] and the condition $k_BT \ll \hbar\omega_{ce}$ must be fulfilled. The details of Landau quantization are given in Ref. 48.

Many investigations have also been carried out to study the linear and nonlinear wave propagation in inhomogeneous degenerate quantum plasmas using the QHD model. Masood *et al*.[49,50] studied nonlinear drift ion acoustic waves in inhomogeneous quantum magnetoplasma and showed the variation of shock strength with quantum diffraction term and positron concentration. Tariq *et al*.[51] studied the electrostatic drift instability in a nonuniform quantum magnetoplasma with shear flows and found that sheared ion flow parallel to the external magnetic field could drive the quantum drift-ion acoustic wave unstable. Masood *et al*.[52] revisited the coupled Shukla Varma and convective cell mode in dense plasmas and showed theoretically how quantum effects could be incorporated in it. Kendl and Shukla[53] studied drift wave turbulence for a degenerate inhomogeneous magnetoplasma and investigated the growth rate of the collisional drift wave instability. It was observed that the quantum effects enhanced the growth rate of the collisional drift wave instability. It was shown in the pioneering work of Bernstein *et al*.[54] that trapped particles have a prominent effect on the nonlinear dynamics of plasma. The trapping in this work was considered by the wave itself. An alternative approach considered the effect of adiabatic trapping at the microscopic level was introduced a decade later by Gurevich[55] and it was observed that the adiabatic trapping produced a 3/2 power nonlinearity instead of the usual quadratic one when trapping was absent. Experimental investigations[56] and computer simulations[57] confirmed the presence of trapping as a microscopic phenomenon.

Demeio[58] investigated the effects of trapping on Bernstein, Greene, and Kruskal equilibria and solved the Wigner-Poisson system employing the perturbative technique in order to study the effect of trapping in quantum phase space. However, the statistical nature of trapping in Ref. 58 was not investigated as the Wigner-Poisson equation was used, which showed only the quantum diffraction effects. Trapping in quantum plasma has been considered by Shah *et al*.[59] recently using the Gurevich approach where the formation of one dimensional ion acoustic solitary structures in both partially and fully degenerate plasma with small temperature effects were investigated. This work was later extended to the case of partially relativistic degenerate plasma including both trapping and finite temperature effects.[60] In the present work, we consider the propagation of one and two dimensional solitary ion drift waves in a quantum degenerate plasma taking into account the effect of trapped particles and finite temperature in the presence of a quantizing magnetic field via Landau quantization. The Fermi-Dirac distribution function is used to describe the massless electrons, whereas the ions are considered to be classical owing to their three orders of magnitude heavier mass than electrons.

The layout of the present work is as follows: In Sec. II, we present the basic set of governing equations and derive the linear dispersion relation for a two dimensional drift ion acoustic waves in a dense degenerate plasma for trapped electrons with finite temperature in the presence of a quantizing magnetic field. In Sec. III, we derive the Korteweg de Vries equation for a one dimensional ion drift wave for the trapped electrons for a partially degenerate plasma in the presence of the quantizing magnetic field. In Sec. IV, we consider the coupling of the drift wave with the ion acoustic wave and derive Kadomtsev-Petviashvili (KP) equation. In Sec. V, we give an analysis of our results. Finally, the summary and conclusion of the current investigation is presented in Sec. VI.

II. GOVERNING EQUATIONS

We study an inhomogeneous quantum magnetoplasma composed of ions and electrons. In order to derive the nonlinear drift-ion acoustic waves for adiabatic trapping of electrons in a degenerate plasma in the presence of a quantizing magnetic field, the ions are considered to be classical whereas the electrons are assumed to follow the Fermi–Dirac distribution. The equilibrium magnetic field is considered to be in the z-direction, whereas the density gradients are assumed to be in the x-direction. The motion of a particle in the plane perpendicular to the magnetic field is quantized.[61] The quantized electron energy levels ε_e^l in the presence of potential field ϕ in the non-relativistic limit are given by $\varepsilon_e^l = l\hbar\omega_{ce} + \frac{p_z^2}{2m_e} - e\varphi$, where $\omega_{ce} = eB_0/m_ec$ is the electron cyclotron frequency, p_z is the parallel momentum associated with the electron, and $-e\varphi$ is the potential energy of the well in which the electrons are trapped. Trapping occurs when the condition $\varepsilon_e^l = 0$ is satisfied. Electrons with energy $\varepsilon_e^l < 0$ are trapped and with $\varepsilon_e^l > 0$ are free electrons. We follow Landau and Lifshitz[46] to obtain the expression for the number densities of the trapped and free electrons, where the trapping of the electrons occurs in the potential of the ions.

The total occupation number for the Fermi-Dirac distribution after integration over the polar coordinates and change of variables from momentum p to energy ε is given by

$$n_e = \frac{p_{Fe}^2\eta}{2\pi^2\hbar^3}\sqrt{\frac{m_e}{2}}\sum_{l=0}^{\infty}\int_0^{\infty}\frac{\varepsilon^{-1/2}}{\exp\left\{\frac{\varepsilon-U}{T}\right\}+1}d\varepsilon, \tag{1}$$

where $U = e\varphi + \mu - l\hbar\omega_{ce}$ and μ is the chemical potential. The summation above is over the Landau levels and we note here $l=0$ refers to the case without a quantizing magnetic field. Following the general treatment for Fermi integrals and the method outlined by Landau and Lifshitz[46] and Shah *et al.*,[48] the total electron number density for a partially degenerate plasma can be expressed as

$$n_e = N_0\left[\frac{3}{2}\eta(1+\phi)^{1/2} + (1+\phi-\eta)^{3/2} - \frac{\eta T^2}{2}(1+\phi)^{-3/2} + T^2(1+\phi-\eta)^{-1/2}\right]. \tag{2}$$

The effect of the quantizing magnetic field appears through $\eta = \hbar\omega_{ce}/\varepsilon_{Fe}$, $N_0 = p_{Fe}^3/3\pi^2\hbar^3$ is the equilibrium number density for fully degenerate plasma (i.e., for $T=0$), $\varepsilon_{Fe} = (\hbar^2/2m_e)(3\pi^2 N_0)^{2/3}$ is the electron Fermi energy, and m_e is the mass of the electron. The Landau quantization parameter plays a role similar to that of the small finite temperature T in modifying the electron occupation number density n_e given by Eq. (2). It is worth mentioning here that Eq. (2) has been obtained by assuming that electrons travel very fast owing to their tenuous mass by comparison with ions and hence their perpendicular motion can be ignored while considering the wave on an ion time scale. The potential φ and temperature T have been normalized in the following manner: $T = \pi T/2\sqrt{2}\varepsilon_{Fe}$ and $\phi = e\varphi/\varepsilon_{Fe}$. Using Binomial series expansion, Eq. (2) can be written as

$$\begin{aligned}\frac{n_e}{N_0} = &\left[\frac{\eta}{2}(3-T^2) + T^2(1-\eta)^{-1/2} + (1-\eta)^{3/2}\right] \\ &+\frac{3}{2}\left[\frac{\eta}{2}(1+T^2) + (1-\eta)^{1/2} - \frac{T^2}{3}(1-\eta)^{-3/2}\right]\phi \\ &+\frac{3}{8}\left[-\frac{\eta}{2}(1+5T^2) + (1-\eta)^{-1/2} + T^2(1-\eta)^{-5/2}\right]\phi^2.\end{aligned} \tag{3}$$

We now draw our attention to the ions; the ions are taken to be cold and non-degenerate due to their heavy mass by comparison with the electrons. The equation of motion for ions is

$$m_i n_i\left(\frac{\partial}{\partial t} + \mathbf{v}_i.\nabla\right)\mathbf{v}_i = q_i n_i\left(E + \frac{1}{c}\mathbf{v}_i\times\mathbf{B}\right), \tag{4}$$

where $E = -\nabla\phi$. The perpendicular component of velocity from Eq. (4) can be written as

$$\mathbf{v}_{i\perp} = \frac{c}{B_0}(\hat{z}\times\nabla\phi) - \frac{c}{B_0\Omega_{ci}}\partial_t\nabla_\perp\phi, \tag{5}$$

where $(\perp)$ mean perpendicular to the magnetic field B_0, $\Omega_{ci} = eB_0/m_i c$ is the ion cyclotron frequency and the limit $\partial_t \ll \Omega_{ci}$ for low frequency drift waves has been used in deriving the above expression.

The parallel component of velocity from Eq. (4) can be written as

$$\hat{A}\,\mathbf{v}_{iz} = -c_s^2\partial_z\phi, \tag{6}$$

where $\hat{A}$ is an operator defined as

$$\hat{A} = \partial_t + v_E.\nabla_\perp + \mathbf{v}_{iz}\partial_z$$

and $c_s = \sqrt{\varepsilon_{Fe}/m_i}$ is the quantum ion acoustic speed. The Poisson's equation is

$$\nabla^2\phi = n_e - n_i. \tag{7}$$

The perturbed ion number density by using Eqs. (7) and (3) can be written as

$$\begin{aligned}\frac{n_i}{N_0} = &\left[\frac{\eta}{2}(3-T^2) + T^2(1-\eta)^{-1/2} + (1-\eta)^{3/2}\right] \\ &+\frac{3}{2}\left[\frac{\eta}{2}(1+T^2) + (1-\eta)^{1/2} - \frac{T^2}{3}(1-\eta)^{-3/2}\right]\phi \\ &+\frac{3}{8}\left[-\frac{\eta}{2}(1+5T^2) + (1-\eta)^{-1/2} + T^2(1-\eta)^{-5/2}\right]\phi^2 \\ &-\lambda_{Fe}^2\nabla^2\phi,\end{aligned} \tag{8}$$

where $\lambda_{Fe} = \sqrt{\varepsilon_{Fe}/4\pi e^2 N_0}$ is the Thomas Fermi Length and $n_{i0} = n_{e0} = N_0$ is the equilibrium plasma density.

The ion continuity equation is

$$\frac{\partial n_i}{\partial t} + \nabla.(n_i v_i) = 0. \tag{9}$$

Using Eqs. (5) and (8) in Eq. (9) and assuming $\partial_x < \partial_z < \partial_y$, we get

$$\begin{aligned}&\partial_t[\alpha_1\partial_t\phi + \alpha_2\partial_t\phi^2 - (\lambda_{Fe}^2+\rho_s^2)\partial_t\partial_y^2\phi + v_*\partial_y\phi + \alpha_1 v_*\partial_y\phi^2] \\ &+\partial_t(\partial_z v_{iz}) = 0,\end{aligned} \tag{10}$$

where

$$\alpha_1 = \frac{3}{2}\left[\frac{\eta}{2}(1+T^2) + (1-\eta)^{1/2} - \frac{T^2}{3}(1-\eta)^{-3/2}\right],$$
$$\alpha_2 = \frac{3}{8}\left[-\frac{\eta}{2}(1+5T^2) + (1-\eta)^{-1/2} + T^2(1-\eta)^{-5/2}\right],$$

$v_* = (-c\varepsilon_{Fe}/eB_0)\kappa_n$ is the drift velocity, $\kappa_n = |d_x \ln N_0|$ is the equilibrium density inhomogeneity and $\rho_s^2 = c_s^2/\Omega_{ci}^2$ is the ion Larmor radius. It is pertinent to mention here that while deriving Eq. (10), terms containing higher than order 2 in ϕ as well as nabla operator ∇ have been ignored as they become very small for weakly nonlinear systems such as the one considered here. Moreover, the ordering of the propagation vectors $(\partial_x < \partial_z < \partial_y)$ is justified because diamagnetic drift velocity is much smaller than the acoustic velocity $(i.e., v_* \ll c_s)$ and therefore to allow for the coupling of drift and acoustic waves ∂_z ought to smaller than ∂_y. Since the gradients in density are weaker than the background number densities (using WKB approximation), therefore the condition $\partial_x < \partial_z, \partial_y$ is readily justified.

Equation (10) can be written by using Eq. (6) as

$$\begin{aligned}&\alpha_1\partial_t^2\phi + \alpha_2\partial_t^2\phi^2 - (\lambda_{Fe}^2+\rho_s^2)\partial_t^2\partial_y^2\phi + v_*\partial_t\partial_y\phi \\ &+\alpha_1 v_*\partial_t\partial_y^2\phi - c_s^2\partial_z^2\phi = 0.\end{aligned} \tag{11}$$

Equation (11) can be transformed into a form analogous to the KP equation derived for a homogeneous quantum plasma.[62]

A. Linear analysis

On linearizing Eq. (11) and assuming the perturbation $\propto [ik_y y + ik_z z - i\omega t]$, the dispersion relation for the coupled quantum drift ion acoustic wave in a 2-D quantum magnetoplasma reads as

$$\omega = \frac{\omega_* \pm \sqrt{\omega_*^2 + 4c_s^2 k_z^2 \left(\alpha_1 + \left(\lambda_{Fe}^2 + \rho_s^2\right) k_y^2\right)}}{2\left(\alpha_1 + \left(\lambda_{Fe}^2 + \rho_s^2\right) k_y^2\right)}, \tag{12}$$

where $\omega_* = v_* k_y$ is the drift frequency, ω is the wave frequency and k_y, k_z are the wave numbers. Note that if we ignore the parallel contribution, then Eq. (12) becomes the dispersion relation for drift wave in 1-D. Moreover, the linear dispersion relation reduces to an oscillation if we ignore the parallel and transverse perturbations and goes to zero if we ignore the background density gradient. Figure 1 explores the dependence of the wave frequency on the Landau quantization parameter, η, for a range of the values of propagation vectors k_y and k_z. It is observed that the wave frequency increases with the increase in η.

III. DERIVATION OF KdV EQUATION

In order to derive the KdV equation for a drift wave, we drop the weak parallel perturbation from Eq. (10) to obtain

$$\alpha_1 \partial_t \phi + \alpha_2 \partial_t \phi^2 - (\lambda_{Fe}^2 + \rho_s^2)\partial_t \partial_y^2 \phi + v_* \partial_y \phi + \alpha_1 v_* \partial_y \phi^2 = 0. \tag{13}$$

It is pertinent to mention here that dropping the parallel equation of motion implies the decoupling of the drift and the acoustic ion modes. Let us choose a coordinate ξ in the moving frame such that $\xi = y - ut$ to find the localized solution of the KdV equation, where u is the velocity of the nonlinear structure moving with the frame. Equation (13) can be written in the transformed frame as

$$-U\partial_\xi \phi + A\partial_\xi \phi^2 + B\partial_\xi^3 \phi = 0, \tag{14}$$

where $U = \alpha_1 - v_*/u$, $A = \alpha_1 v_*/u - \alpha_2$, and $B = (1 + \lambda_{Fe}^2/\rho_s^2)$. Equation (14), as is well known, has a stationary soliton solution given by

$$\phi(y,t) = \frac{12B}{A} \sec h^2(y - ut). \tag{15}$$

IV. DERIVATION OF KP EQUATION

In this section, we derive the KP equation for a drift ion acoustic soliton in an inhomogeneous quantum magnetoplasma. In order to find the localized solution, let us choose a coordinate ξ in the moving frame such that $\xi = (y + \delta z - ut)$, where δ represents the obliqueness. Equation (11) in the transformed frame can be written as

$$\partial_\xi[a_1 \partial_\xi \phi + a_2 \partial_\xi \phi^2 + a_3 \partial_\xi^3 \phi] + \delta^2 a_4 \partial_\xi^2 \phi = 0, \tag{16}$$

where $a_1 = \alpha_1 - v_*/u, a_2 = \alpha_2 - \alpha_1 v_*/2u, a_3 = -(1 + \lambda_{Fe}^2/\rho_s^2)$, and $a_4 = -c_s^2/u^2$. Equation (16) can be simplified further such as

$$\partial_\xi[\partial_\xi \phi + A\partial_\xi \phi^2 + B\partial_\xi^3 \phi] + \delta^2 G \partial_\xi^2 \phi = 0, \tag{17}$$

where $A = a_2/a_1$, $B = a_3/a_1$, and $C = a_4/a_1$. Equation (17) is analogous to the KP equation derived for the homogeneous plasmas. The above equation admits soliton solutions. It is interesting to note that contrary to its homogeneous counterpart, the KP equation in an inhomogeneous plasma is predominantly in the transverse direction and weak in the parallel direction. This difference arises due to the drift approximation used in solving the inhomogeneous plasmas that assumes a stronger perturbation in the perpendicular direction by comparison with the parallel motion along the ambient magnetic field.

There are a number of methods to solve the nonlinear partial differential equations (NPDEs), for instance, inverse scattering method,[63] Hirota bilinear formalism,[64] Backlund transformation,[65] tanh,[66] etc. We employ here the tangent

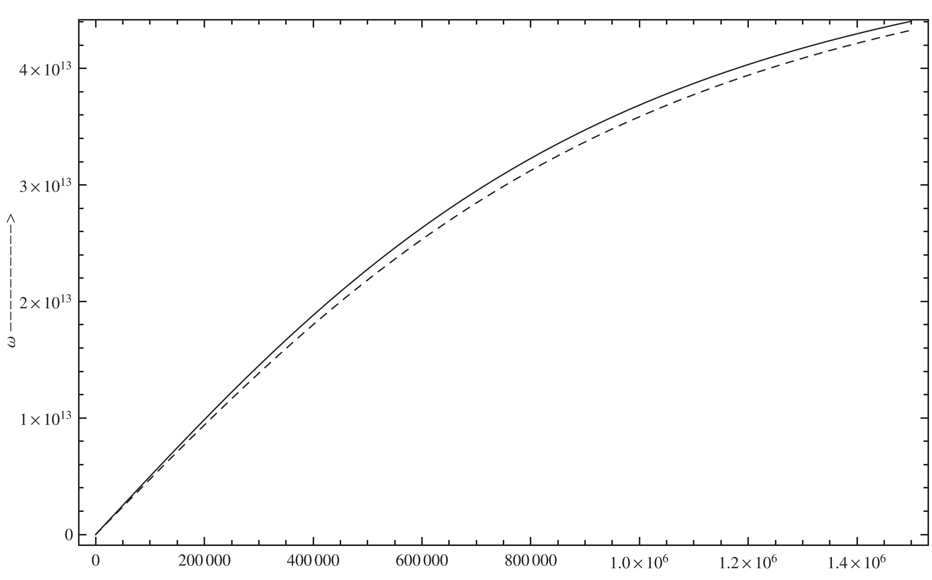

FIG. 1. Dispersion relation for the 2D coupled drift ion acoustic wave with electron trapping in the presence of quantizing magnetic field and finite temperature effects for different values of η. Dashed line is for $\eta = 0.1$ whereas thin line is for $\eta = 0.6$. Other parameters are $N_0 = 10^{27} cm^{-3}, B_0 = 10^9 G$ and $T = 0.2$.

hyperbolic method[67] to arrive at the following solution of Eq. (17)

$$\phi(y,z,t) = \frac{12B}{A}\sec h^2(y + \delta z - ut), \tag{18}$$

where $\delta = \sqrt{(1+4B)/C}$.

V. RESULTS AND DISCUSSION

In this section, we numerically investigate the propagation of one and two dimensional drift solitary waves in a dense magnetoplasma with trapped electrons in the presence of a quantizing magnetic field. It is pertinent to mention here that in 2-D, the drift wave couples with the acoustic wave and give rise to drift acoustic solitary structures in the nonlinear regime whereas in 1-D only drift solitary structures are obtained. It is found that the system under consideration admits compressive solitary structures for both the drift and drift-acoustic solitary structures. Most importantly, it is observed that the solitary structures are formed only for sub drift and ion acoustic velocity cases, i.e., $u < v_* < c_s$ both for the 1 and 2-D solitary structures.

In dense astrophysical objects such as neutron stars and white dwarfs, the plasma densities are very high and the quantum effects can no longer be ignored. Recently, it has been conjectured that electrostatic structures could be excited in extreme events, such as supernova explosions at the outer shells of the star or during collisions of the white dwarf with other astrophysical bodies.[68] We, therefore, choose here the parameters that are typically found in the pulsating white dwarfs (which have been described in detail in Sec. I), i.e., $n_0 \sim 10^{26}$–$10^{28}\,\mathrm{cm}^{-3}$and $B_0 \sim 10^9$–10^{11} G.[69,70]

Figure 2 explores the effect of the Landau quantization parameter η, on the structure of 1-D ion drift solitary waves. It is observed that the increase in Landau quantization parameter increases the amplitude of the drift solitary wave. Figure 3 investigates the variation of the 2-D drift-ion acoustic solitary structure with the increasing Landau quantization parameter, η. It is found that the increasing η increases the amplitude of the solitary wave under consideration. Figure 4 depicts the behavior of the nonlinearity and dispersive coefficients with the change in Landau quantization parameter. It is observed that the nonlinearity and dispersive coefficients enervate with the increase in the Landau quantization parameter.

Finally, Figure 5 explores the effect of the increasing degeneracy temperature on the drift-ion acoustic solitary

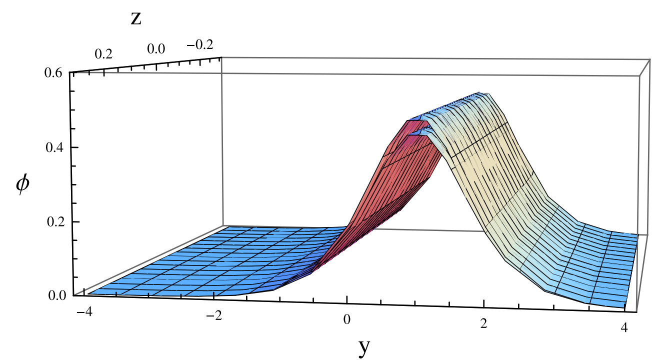

FIG. 3. Variation of electrostatic potential ϕ versus y and z for drift KP equation with $\eta = 0.1$ and $\eta = 0.6$. Other parameters are the same as in Fig. 1.

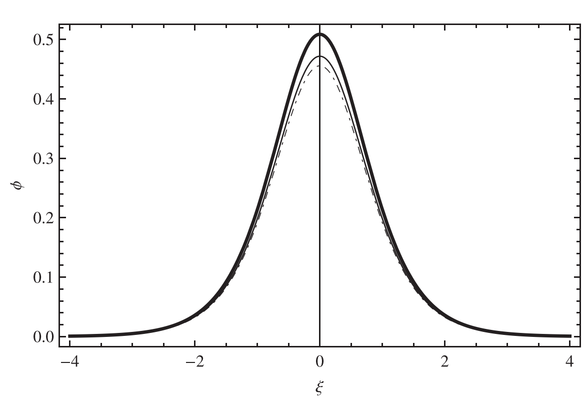

FIG. 2. Effect of η on the solitary wave solution of drift KdV equation. Dotted-dashed line is for $\eta = 0.1$, thin line is for $\eta = 0.4$, and thick line is for $\eta = 0.6$. Other parameters are the same as in Fig. 1.

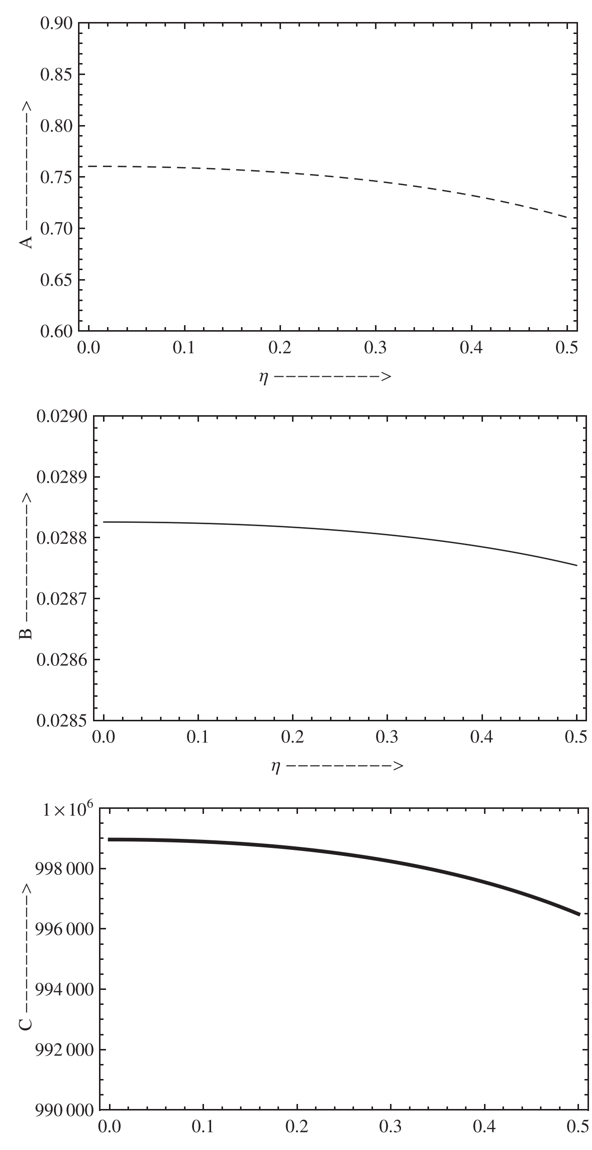

FIG. 4. Variation of coefficients A, B, and C of drift KP equation versus η. Other parameters are same as in Fig. 1.

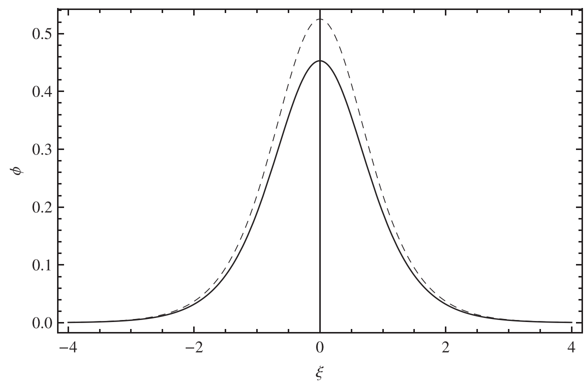

FIG. 5. Effect of temperature on the solitary wave solution of drift KdV equation. Solid line is for $T = 0.1$, whereas dashed line is for $T = 0.6$. Other parameters are same as in Fig. 1.

structure. It is pertinent to mention here that the inclusion of finite degeneracy temperature modifies the shape of the Fermi-Dirac distribution and it falls less rapidly. It is observed that increasing the degeneracy temperature enhances the amplitude of the solitary structure.

We would like to mention here that linear and nonlinear investigations of the electrostatic waves with classically trapped electrons have been made in the past.[71,72] The difference here is that the electrons are quantum mechanically trapped and follow the Fermi-Dirac distribution. For instance, the third and fourth terms in Eq. (2) are possible only in a partially degenerate quantum plasma and have no classical equivalent. Another important thing is that due to enormous differences in densities and temperatures between classical and quantum plasmas, the spatio-temporal scales over which the nonlinear structures form in dense plasmas are far shorter than their classical counterparts (∼eight orders of magnitude shorter in dense plasmas from that of classical plasma).[73]

VI. CONCLUSION

To summarize, we have investigated the linear and nonlinear propagation characteristics of 1 and 2D ion drift acoustic solitary structures in an inhomogeneous quantum plasma for trapped electrons in the presence of quantizing magnetic field and finite temperature effects. Using the drift approximation, we have derived drift KdV and KP equations for ion drift and coupled drift-ion acoustic solitary structures. We have numerically investigated our theoretical results for different parameter values such as degeneracy temperature and Landau quantization parameter using the values that are typically found in the outer shells of white dwarfs as an illustration. It is observed that the one and two dimensional drift ion equations for the system under consideration admits compressive solitary structures. The present investigation may be beneficial to understand the multi-dimensional solitary structures in dense plasmas such as those found in the white dwarf stars, fast ignition scenario for inertial confinement fusion and in short pulsed petawatt laser technology.

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