Obliquely propagating electromagnetic excitations in dissipative plasmas with relativistically degenerate electrons

M. A. Rehman, R. Jahangir, W. Masood, and H. A. Shah

Citation: Physics of Plasmas **24**, 062310 (2017); doi: 10.1063/1.4986112
View online: http://dx.doi.org/10.1063/1.4986112
View Table of Contents: http://aip.scitation.org/toc/php/24/6
Published by the American Institute of Physics

PHYSICS OF PLASMAS **24**, 062310 (2017)

Obliquely propagating electromagnetic excitations in dissipative plasmas with relativistically degenerate electrons

M. A. Rehman,[1] R. Jahangir,[2,3] W. Masood,[2,4] and H. A. Shah[1]
[1]*Department of Physics, Government College University, Lahore 54000, Pakistan*
[2]*National Centre for Physics (NCP), Shahdra Valley Road, Islamabad 44000, Pakistan*
[3]*Department of Physics, Quaid-i-Azam University, Islamabad 44000, Pakistan*
[4]*COMSATS Institute of Information Technology, Park Road, Chak Shahzad, Islamabad 44000, Pakistan*

(Received 24 October 2016; accepted 31 May 2017; published online 23 June 2017)

In this paper, we have investigated the formation of obliquely propagating magnetoacoustic shock structures in dense dissipative plasmas with relativistically degenerate electrons. Using the reductive perturbation technique, we have derived the nonlinear Kadomtsev-Petviashvilli-Burgers (KPB) equation for both fast and slow magnetoacoustic modes. We have explored the non-relativistic and ultrarelativistic limits for degenerate electrons for both the modes and highlighted the differences in propagation characteristics of their respective shock structures. We have also studied the limiting cases of KPB in one dimension for both the fast and slow modes. Interestingly, it has been found that unlike the other cases, the one dimensional Burgers equation for the fast mode changes the nature of the shock waves. It has been explained in the paper that this happens owing to the change of sign of the nonlinearity coefficient. *Published by AIP Publishing.*
[http://dx.doi.org/10.1063/1.4986112]

I. INTRODUCTION

Quantum plasmas have engendered enormous interest in the past decade or so on account of its applications in a variety of physical situations of interest such as in ultra cold plasmas,[1,2] in plasmonic devices,[3] in next generation high intensity light-sources experiments,[4] in metal nanostructures, and in stellar environments.[5] Plasmas with degenerate electrons having number densities comparable with solids and temperatures of several electron volts fall under the category of warm dense matter.[6,7] Such plasmas have been conjectured to exist in the core of giant planets[8,9] and the crusts of old stars.[10] High-energy density physics[11] has gained interest due to its applications in astrophysical and cosmological environments[12–15] and in inertial fusion science that involves intense laser-solid density plasma interaction experiments[3,16–21] and inertial confinement fusion.[22] Collective interactions in non-relativistic, dense quantum plasmas are frequently investigated by means of quantum hydrodynamic (QHD)[5,23,24] and quantum kinetic[25–27] models.

The QHD equations have been frequently used to investigate linear and nonlinear plasma waves and the stability of quantum plasmas[5,24,28–30] at nanoscales involving the quantum tunneling effects[23,31] and the quantum statistical pressure law for an unmagnetized quantum plasma with degenerate electrons. Linear and nonlinear electromagnetic waves have also been extensively studied in quantum plasmas employing the QHD model. Spin-1/2 quantum magneto hydrodynamics models for hydrodynamic waves, respectively, were proposed[32,33] for applications to solid state plasmas and dense astrophysical environments. Several other investigations have been carried out to study the linear and nonlinear structure formation for obliquely propagating magnetoacoustic waves in both non-dissipative and dissipative cases.[34–36]

Magnetoacoustic waves propagate perpendicular to the ambient magnetic field and are usually termed as fast hydromagnetic waves having phase speed always greater than the Alfven wave. The magnetic field and density compressions are responsible for the formation of this mode. Oblique propagation of the magnetoacoustic wave to the ambient magnetic field gives rise to fast and slow magnetoacoustic (SM) modes. Fast mode arises when magnetic field and plasmas density oscillations are in phase whereas slow mode exists when these two oscillations are out of phase. These modes have both transverse and longitudinal components and also form nonlinear wave structures such as solitons, shocks, and double layers. The nonlinear propagation characteristics of magnetoacoustic waves have been explored by many authors due to their applications in space and fusion plasmas where they are used in particle acceleration and heating experiments.[37–41]

The self-steepening and shock formation in relativistically degenerate plasmas have been investigated using simple wave solutions.[42] Pulsations observed in white dwarf stars are thought to be originating from gravity (g-mode) waves in the inhomogeneous density of the star,[43,44] while compressional (p-mode) waves are still to be observed.[45] It was also suggested that investigation can be readily generalized to include an ambient magnetic field, in which we have the possibility of magnetosonic solitons[46] and shocks[47] in dense plasmas. Motivated by these observations, we examine here the linear and nonlinear oblique propagation of magnetoacoustic shocks in non-relativistic and relativistic plasmas.

This paper is organized in the following manner. In Sec. II, we write down the basic set of equations for the system under consideration. In Sec. III, the Kadomtsev Petviashvili Burgers (KPB) equation is derived and its solution is obtained using the tangent hyperbolic method. In Sec. IV, we present and discuss the results. Finally, in Sec. V, we recapitulate the main findings of the paper.

II. MODEL EQUATIONS

We consider an electron-ion plasma embedded in a magnetic field in which the ions are singly charged and fully ionized and considered to be cold. The electrons are taken to be degenerate and are assumed to obey the Fermi Dirac statistics. We consider an oblique background magnetic field in the (x, y) plane making a small angle θ with the x-axis and the propagation is considered in the (x, z) plane. The derivation of the Kadomtsev-Petviashvili-Burger (KPB) equation in the relativistic plasma assumes a predominant propagation direction along the x-axis and a weakly transverse propagation along the z-axis. The continuity and momentum equations describing the dynamics of ions are

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{v}_i) = 0, \tag{1}$$

$$m_i \frac{d\mathbf{v}_i}{dt} = Ze\mathbf{E} + Z\frac{e}{c}\mathbf{v}_i \times \mathbf{B} + \acute{\mu}\nabla^2 \mathbf{v}_i. \tag{2}$$

Here, n_i is the perturbed ion density, m_i is the mass of ion, $\mathbf{v}_i$ is the ion velocity, e is the electron charge, $\mu = \acute{\mu}/m_i$ is the kinematic viscosity, $\mathbf{E}$ and $\mathbf{B}$ are the applied electric and magnetic fields, respectively, and $d/dt = \partial/\partial t + (\mathbf{v}_i.\nabla)$ is the convective derivative. Here, we have ignored the ion quantum effects due to large ion inertia as compared to electrons. The ion pressure is ignored because the main restoring force comes from electrons; however, this does not imply $T_i = 0$ owing to the fact that the viscosity of ions requires a finite ion temperature. The momentum equation governing the dynamics of the degenerate relativistic inertialess electrons is given by

$$0 = -e\mathbf{E} - \frac{e}{c}(\mathbf{v}_e \times \mathbf{B}) - \frac{\nabla P_{eR}}{n_e}. \tag{3}$$

Here, n_e is the perturbed electron density and $\mathbf{v}_e$ is the electron velocity. The quantum diffraction effect due to wave nature of particles in the quantum plasma is ignored because its contribution is negligible compared to the relativistic quantum pressure term. The relativistic quantum pressure term is obtained by considering the quantum statistics of an electron, where P_{eR} is the relativistic degenerate pressure and the expression for this pressure is given later.

The Faraday's law and Ampère's law are

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \tag{4}$$

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{j}, \tag{5}$$

where the current density $\mathbf{j}$ is

$$\mathbf{j} = e(Zn_i v_i - n_e v_e). \tag{6}$$

We express the two fluid equations in the form of one fluid equation by eliminating the electric field. We substitute the value of v_e from Eq. (6) to Eq. (3) to obtain

$$\mathbf{E} = -\frac{\mathbf{v}_i}{c} \times \mathbf{B} + \frac{1}{4\pi e Z n_i}\nabla \times \mathbf{B} \times \mathbf{B} - \frac{\nabla P_{eR}}{Zn_i e}. \tag{7}$$

We use the quasineutrality condition $n_e \simeq Zn_i$ and substitute the above equation in Eq. (2) to obtain the following one fluid momentum equation

$$\frac{d\mathbf{v}_i}{dt} = \frac{1}{4\pi m_i n_i}(\nabla \times \mathbf{B}) \times \mathbf{B} - \frac{\nabla P_{eR}}{nm_i} + \boldsymbol{\mu}\nabla^2 \mathbf{v}_i. \tag{8}$$

The relativistic degenerate pressure P_{eR} for the fully degenerate electrons is given as[48]

$$P_{eR} = P_{e0} + \frac{2}{3\gamma_{e0}}\varepsilon_{Fe}\delta n_e + \frac{(\gamma_e^2 + 2)}{9\gamma_{e0}^3 n_{e0}}\varepsilon_{Fe}\delta n_e^2, \tag{9}$$

where γ_{e0} is the relativistic gamma factor of an electron, defined as $\gamma_{e0} = \sqrt{(1+\gamma_e^2)}$ with $\gamma_e = P_{Fe}/m_e c$, where $P_{Fe} = \left(\frac{3h^3 n_e}{8\pi}\right)^{\frac{1}{3}}$ is the momentum of the electron on the Fermi surface and electron Fermi energy is given as $\varepsilon_{Fe} = \left(\frac{\hbar^2}{2m_e}\right)(3\pi^2 n_e)^{2/3}$. Now, we have the following normalized effective one fluid momentum equation

$$\frac{d\mathbf{v}_i}{dt} = \frac{1}{n_i}(\nabla \times \mathbf{B}) \times \mathbf{B} - \frac{\alpha_e \beta}{n_i}\nabla \delta n_i + \frac{\beta_e \beta}{n_i}\nabla \delta n_i^2 + \eta \nabla^2 \mathbf{v}_i, \tag{10}$$

where $\beta = C_s^2/v_A^2$, $C_s = \sqrt{Z\varepsilon_{Fe}/m_i}$ is the ion acoustic speed, $v_A = B_0/\sqrt{4\pi m_i n_i}$ is the Alfvén speed, $n_e \simeq Zn_i$, $\alpha_e = 2/3\gamma_{e0}$, $\beta_e = (\gamma_e^2 + 2)/9\gamma_{e0}^3$, and η is the normalized kinematic viscosity of ions defined as $\eta = \mu\omega_{ci}/v_A^2$ with $\omega_{ci} = ZeB_0/m_i$ is the ion cyclotron frequency.

The following normalization scheme is used for the above equations

$$\mathbf{v}_i \to \frac{\mathbf{v}_i}{v_A}, \quad \mathbf{B} \to \frac{\mathbf{B}}{B_0}, \quad t \to \omega_{ci} t, \quad r \to r\frac{\omega_{ci}}{v_A}, \quad n_i \to \frac{n_i}{n_0}. \tag{11}$$

Now substituting E from Eq. (2) to Faraday's law and normalizing the resultant equation, we get magnetic field induction equation as

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v}_i \times \mathbf{B}) - \nabla \times \frac{d\mathbf{v}_i}{dt}. \tag{12}$$

Equations (10) and (12) along with the continuity equation (1) serve as the basic governing equations for the one fluid relativistic quantum model of electron-ion plasma. The one fluid equations are linearized by assuming $k_z \ll k_x$ and $B_z \ll B_y$ and considering the perturbations $\propto \exp[i(kx - \omega t)]$, we obtain the following linear dispersion relation

$$\frac{\omega^2}{k^2} = \frac{1}{2}(1 + \alpha_e\beta)\left[1 \pm \sqrt{1 - \frac{4\alpha_e\beta\cos^2\theta}{(1+\alpha_e\beta)^2}}\right]. \tag{13}$$

This linear normalized dispersion relation for the relativistic quantum plasma describes two modes of obliquely propagating magnetoacoustic waves. "+" sign is for fast magnetoacoustic (FM) whereas "−" sign is for slow magnetoacoustic (SM) waves. These modes show variations with the number density and ambient magnetic field via β and the angle of propagation θ.

A. Special cases

1. Non-relativistic limit

In the non-relativistic limit $\gamma_e \ll 1$, so that $\gamma_{e0} = 1$, then the linear dispersion relation becomes

$$\frac{\omega^2}{k^2} = \frac{1}{2}\left(1 + \frac{2\beta}{3}\right)\left[1 \pm \sqrt{1 - \frac{\frac{8}{3}\beta\cos^2\theta}{\left(1 + \frac{2}{3}\beta\right)^2}}\right]. \tag{14}$$

This dispersion relation is similar to the already known results.[35]

2. Parallel propagation

When $\theta = 0°$, then FM waves become the pure Alfvén waves and the SM become the acoustic waves, for Alfvén speed greater than the acoustic speed and their normalized dispersion relations are $\omega^2/k^2 = 1$ and $\omega^2/k^2 = \alpha_e\beta$, respectively. However, for acoustic speed greater than Alfvén speed, FM waves become the pure acoustic wave and SM waves become the pure Alfvén waves.

3. Perpendicular propagation

When $\theta = 90°$, the SM vanishes and only fast relativistic quantum magnetoacoustic mode exists. The dispersion relation of FM is then

$$\frac{\omega^2}{k^2} = \frac{1}{2}(1 + \alpha_e\beta). \tag{15}$$

III. KADOMTSEV-PETVIASHVILI-BURGER (KPB) EQUATION

To study the magnetoacoustic perturbations propagating in a relativistic degenerate dense plasma, we construct a weakly nonlinear theory which leads to the scaling of the independent variables,[49] through the standard stretching as

$$\begin{aligned} \xi &= \epsilon^{1/2}(x - \lambda t), \\ \chi &= \epsilon z, \\ \tau &= \epsilon^{3/2} t, \end{aligned} \tag{16}$$

where ϵ is a small parameter measuring the weakness of the nonlinearity and λ is the wave phase velocity normalized by v_A. The perturbed quantities n, v_{ix}, v_{iy}, v_{iz}, B_x, B_y, B_z, and η can be expanded in terms of power series of ϵ about their equilibrium values as

$$\begin{aligned} n &= 1 + \epsilon n_1 + \epsilon^2 n_2 + \cdots\cdots, \\ v_{ix} &= \epsilon u_1 + \epsilon^2 u_2 + \epsilon^3 u_3 + \cdots\cdots, \\ v_{iy} &= \epsilon v_1 + \epsilon^2 v_2 + \epsilon^3 v_3 + \cdots\cdots, \\ v_{iz} &= \epsilon^{3/2} w_1 + \epsilon^{5/2} w_2 + \cdots\cdots, \\ B_x &= \cos\theta, \\ B_y &= \sin\theta + \epsilon B_{y1} + \epsilon^2 B_{y2} + \cdots\cdots, \\ B_z &= \epsilon^{3/2} B_{z1} + \epsilon^{5/2} B_{z2} + \cdots\cdots, \\ \eta &= \epsilon^{\frac{1}{2}} \eta_0. \end{aligned} \tag{17}$$

Using the stretched variables for Eqs. (1), (10), and (12), we develop equations in various powers of ϵ. Eliminating the higher order terms and doing some tedious algebraic manipulations lead to the Kadomtsev-Petviashvili-Burger (KPB) equation for fast and slow magnetoacoustic shock waves in the relativistic quantum plasma (see Appendix for the details)

$$\frac{\partial}{\partial \xi}\left[\frac{\partial B_{y1}}{\partial \tau} + lB_{y1}\frac{\partial B_{y1}}{\partial \xi} + m\frac{\partial^3 B_{y1}}{\partial \xi^3} - n\frac{\partial^2 B_{y1}}{\partial \xi^2}\right] + p\frac{\partial^2 B_{y1}}{\partial \chi^2} = 0, \tag{18}$$

where $l = C/A$, $m = D/A$, $n = F/A$, and $p = G/A$. Here, A, C, D, F, and G are defined in terms of plasma parameters as

$$\begin{aligned} A &= \frac{\cos^2\theta + \lambda^2}{\lambda^2} + \frac{\sin^2\theta(\alpha_e\beta + \lambda^2)}{(\alpha_e\beta - \lambda^2)^2}, \\ C &= \frac{\lambda\sin\theta}{(\lambda^2 - \alpha_e\beta)}\left[3 + \frac{2\beta\sin^2\theta(\alpha_e + \beta_e)}{(\alpha_e\beta - \lambda^2)^2}\right], \\ D &= \frac{-\lambda\cos^2\theta}{\lambda^2 - \cos^2\theta}, \\ G &= \frac{\lambda^5\sin^2\theta}{(\alpha_e\beta - \lambda^2)^2(\lambda^2 - \cos^2\theta)}, \\ F &= \eta_0\left[\frac{\lambda^2\sin^2\theta}{(\alpha_e\beta - \lambda^2)^2} + \frac{\cos^2\theta}{\lambda^2}\right]. \end{aligned} \tag{19}$$

Note that all the coefficients of the shock wave are modified by relativistic quantum statistical effects. Now, we switch to a co-moving frame to solve the nonlinear differential equation (NLDE) (18) by using the transformation $\zeta = k(\xi + \chi - U\tau)$, where k is the wave number. There are many methods to solve this NLDE like inverse scattering method,[50] Hirota bilinear formalism,[51] Backlund transformation,[52] and tanh.[53] To solve such partial differential equations which contains the effects of dispersion and dissipation, the tanh method is most suitable.[54] We obtain the following solution with the help of the tanh method

$$\begin{aligned} B_{y1}(\xi,\chi,\tau) &= \frac{6n^2}{25\mathrm{ml}}\left[1 - \tanh\frac{n}{10\mathrm{m}}(\xi + \chi - U\tau)\right] \\ &\quad + \frac{3n^2}{25\mathrm{ml}}\left[\mathrm{sech}^2\frac{n}{10\mathrm{m}}(\xi + \chi - U\tau)\right], \end{aligned} \tag{20}$$

where $U = \frac{6n^2}{25\mathrm{m}} + p$ is the velocity in commoving frame for nonlinear structure.

A. Stability analysis of the KPB equation

In order to check the stability of the shock structures, the KPB equation is transformed into a new coordinate frame $\zeta = k(\xi + \chi - U\tau)$, where k is the dimensionless wave number and U is the velocity of co-moving frame. Integrating the resultant equation twice gives

$$mk^2\frac{\partial^2 B_{y1}}{\partial \zeta^2} - nk\frac{\partial B_{y1}}{\partial \zeta} - (U - p)B_{y1} + \frac{l}{2}B_{y1}^2 = 0. \tag{21}$$

The boundary conditions $B_{y1} \to B_0, \frac{\partial B_{y1}}{\partial \zeta} \to 0, \frac{\partial^2 B_{y1}}{\partial \zeta^2} \to 0$ as $\zeta \to -\infty$ are applied to investigate the asymptotic behavior of above equation by linearizing it with respect to B_{y1}.[55] Simplifying the equation gives

$$mk^2 \frac{\partial^2 B_{y1}}{\partial \zeta^2} - nk \frac{\partial B_{y1}}{\partial \zeta} + (U-p)B_{y1} = 0. \tag{22}$$

The asymptotic solution of the above equation is proportional to $\exp(W\zeta)$, where

$$W = \frac{n}{2mk}\left[1 \pm \sqrt{1 - \frac{4m(U-p)}{n^2}}\right]. \tag{23}$$

This shows that the shock structure would be stable when $\frac{4m(U-p)}{n^2} \le 1$, else it would be an oscillatory shock. It can easily be seen that the weak transverse perturbation plays a crucial role in the stability of the shock.

B. Limiting cases

We can deduce the one dimensional nonlinear partial equation Korteweg–de Vries Burgers equation from Eq. (18) by ignoring the weak transverse perturbation (i.e., $p = 0$) for obliquely propagating magnetoacoustic wave in the relativistic degenerate quantum plasma and the solution is given as

$$B_{y1}(\xi, \tau) = \frac{6n^2}{25\text{ml}}[1 - \tanh k(\xi - U\tau)] + \frac{3n^2}{25\text{ml}}\left[\text{sech}^2 k(\xi - U\tau)\right], \tag{24}$$

where $k = \frac{n}{10\text{m}}$ and $U = \frac{6n^2}{25\text{m}}$. The exact solution of this planar KdVB equation contains the contribution from both dispersive and dissipative effects which changes the eventual shape of the wave potential. If we ignore both predominant and weak dispersions in Eq. (18), then we end up with the one dimensional Burgers equation, which has the following solution:

$$B_{y1}(\xi, \chi, \tau) = \frac{2n}{l}[1 - \tanh\xi]. \tag{25}$$

IV. NUMERICAL RESULTS

In this section, we will see the effects of number densities, magnetic field intensities, and the kinematic viscosity on the dense relativistic magnetoacoustic shock structures. In high density plasmas found in dense astrophysical objects like white dwarfs and neutron stars, the properties of shocks are studied. The plots of shock waves are obtained and compared for different plasma parameters in nonrelativistic and the ultrarelativistic regimes for both the fast and slow magnetoacoustic modes.

A wide range of values exist for the number densities and the magnetic field in white dwarfs.[56,57] Typical values of densities are $n_0 = 10^{32} - 10^{35}\text{m}^{-3}$ and those of magnetic field are $B_0 = 10^5 - 10^8$T for the nonrelativistic electron region ($k_B T_{Fe} \ll m_e c^2$). The system temperature is taken to be $T = 10^5$ K which is less than the electron Fermi temperature (10^7–10^8 K) and hence it is justified to consider degenerate electrons. The ion Fermi temperature (10^3–10^4 K) comes out to be less than the system temperature, so ions are taken to be non-degenerate. These ranges of densities and magnetic field are permissible for the validity of quantum fluid model, i.e., $\lambda_{Fe} > d$, where the electron Fermi length is defined as $\lambda_{Fe} = (2k_B T_{Fe}/4\pi n_{e0} e^2)^{1/2}$ and the mean distance between particles is $d = (3/4\pi n_{e0})^{1/3}$. The spin effects for these densities can also be ignored since the condition $\mu_B B_0 \ll k_B T_{Fe}$ is met, along with the condition $\mu_B B_0 \ll m_i v_A^2$,[58] where the Bohr magneton is $\mu_B = e\hbar/2m_e$.

For the ultra-relativistic limit, the Fermi energy of electrons is much greater than rest mass energy of electrons, i.e., $k_B T_{Fe} \gg m_e c^2$ or $\gamma_e \gg 1$. Typical values of densities and magnetic fields in this regime are $n_{e0} = 10^{36} - 10^{40}\text{m}^{-3}$ and $B_0 = 10^8$–10^{10} T in the literature.[59] The system temperature is again less than the electron Fermi temperature and greater than the ion Fermi temperature, so that electrons are taken to be degenerate and ions are considered as non degenerate. Interestingly, it is observed that spin effects can be ignored for relatively larger values of ambient magnetic fields (i.e., $B_0 = 10^8 - 10^{10}$T), since for high densities of the ultrarelativistic region the electron Fermi energy is also increased and condition $\mu_B B_0 \ll k_B T_{Fe}$ still holds.

A. Plots of KPB equation

Before we discuss anything else, it is pertinent to mention here that the system under consideration admits rarefactive shock structures for both the fast and slow modes for the KPB equation. Figure 1 shows the dependence of magnetic field on the magnetoacoustic shock potential for fast mode. It is found that increasing the magnetic field strength mitigates the strength of magnetoacoustic shock potential in terms of magnitude for both the non relativistic and the ultra relativistic shock waves for the fast mode. However, the reverse happens for the case of the slow magnetoacoustic shock wave as shown in Fig. 2. It is worthwhile to note here that the direction of propagation of the shock wave is found to be opposite for the case of slow mode as compared to the fast mode. This is due to the fact that the dissipation coefficient along the z-direction changes its sign for the case of slow magnetoacoustic wave.

Figure 3 explores the dependence of the non relativistic and ultra relativistic magnetoacoustic shock waves for the

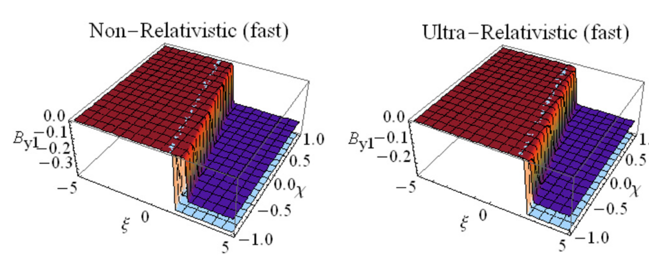

FIG. 1. Effect of the magnetic field on the fast mode of the KPB shock structure. The bold curve is for $B_0 = 2 \times 10^5$ T, dotted curve is for $B_0 = 2.3 \times 10^5$ T, and $n_o = 10^{33}\,\text{m}^{-3}$ for the non-relativistic case. For the ultra relativistic case, $n_o = 10^{36}\,\text{m}^{-3}$, bold curve is for $B_0 = 6 \times 10^7$ T, and dotted curve is for $B_0 = 7 \times 10^7$ T. Other parameters are $Z = 6$, $t = 1$, $\eta_0 = 0.1$, and $\theta = 30°$.

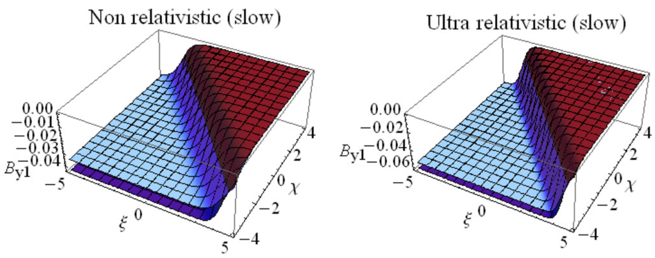

FIG. 2. Effect of magnetic field on the slow mode of the KPB shock structure. The bold curve is for $B_0 = 5 \times 10^6$ T, dotted curve is for $B_0 = 5.5 \times 10^6$ T, and $n_o = 10^{33}\ \mathrm{m}^{-3}$ for the non relativistic case. For the ultra relativistic case $n_o = 5 \times 10^{35}\ \mathrm{m}^{-3}$, bold curve is for $B_0 = 9 \times 10^8$ T, and dotted curve is for $B_0 = 9.5 \times 10^8$ T. Other parameters are $Z = 6$, $t = 1$, $\eta_0 = 0.1$, and $\theta = 30°$.

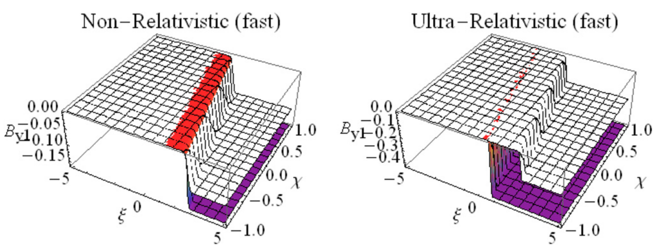

FIG. 3. Effect of number density on the fast mode of the KPB shock structure. The bold curve is for $n_o = 1.2 \times 10^{33}\ \mathrm{m}^{-3}$, dotted curve is for $n_o = 1.5 \times 10^{33}\ \mathrm{m}^{-3}$, and $B_0 = 4 \times 10^5$ T for the non relativistic case. For the ultra relativistic case, $B_0 = 4 \times 10^7$ T, bold curve is for $n_o = 5 \times 10^{35}\ \mathrm{m}^{-3}$, and dotted curve is for $n_o = 8 \times 10^{35}\ \mathrm{m}^{-3}$. Other parameters are $Z = 6$, $t = 1$, $\eta_0 = 0.1$, and $\theta = 30°$.

fast mode on number density. It is found that the increasing number density enhances the strength of the magnetoacoustic shock in terms of magnitude for the nonrelativistic as well as the ultra-relativistic shock. The reverse of the above happens for the slow magnetoacoustic shocks as shown in Fig. 4. It is pertinent to mention here that not only the behavior of fast and slow shocks is different for magnetic field and number density but the relative strength of the non relativistic and ultra relativistic shocks also varies for both the parameters as can be seen from the close observation of Figs. 1–4.

Figure 5 manifests the effect of kinematic viscosity only for the ultra relativistic shock potential for both the fast and slow magnetoacoustic modes. It is seen that, in terms of magnitude, the increase in resistivity enhances the shock strength for both the fast and slow modes.

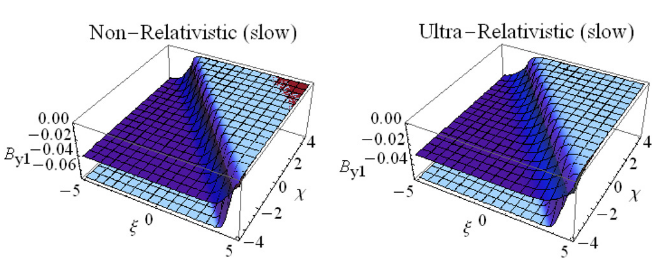

FIG. 4. Effect of number density on the slow mode of the KPB shock structure. The bold curve is for $n_o = 1.2 \times 10^{33}\ \mathrm{m}^{-3}$, dotted curve is for $n_o = 1.6 \times 10^{33}\ \mathrm{m}^{-3}$, and $B_0 = 8 \times 10^6$ T for the non relativistic case. For the ultra relativistic case $B_0 = 9 \times 10^8$ T, bold curve is for $n_o = 5 \times 10^{35}\ \mathrm{m}^{-3}$, and dotted curve is for $n_o = 7 \times 10^{35}\ \mathrm{m}^{-3}$. Other parameters are $Z = 6$, $t = 1$, $\eta_0 = 0.1$, and $\theta = 30°$.

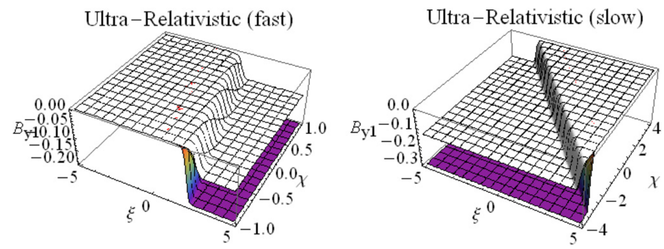

FIG. 5. Effect of resistivity on the KPB shock structure for the ultra relativistic case. The bold curve is for $\eta_0 = 0.1$, dotted curve is for $\eta_0 = 0.15$ and $n_o = 10^{36}\ \mathrm{m}^{-3}$, and $B_0 = 10^8$ T is for the fast mode. For slow magnetoacoustic mode $n_o = 10^{35}\ \mathrm{m}^{-3}$, $B_0 = 4 \times 10^8$ T, bold curve is for $\eta_0 = 0.1$, and dotted curve is for $\eta_0 = 0.15$. Other parameters are $Z = 6$, $t = 1$, and $\theta = 30°$.

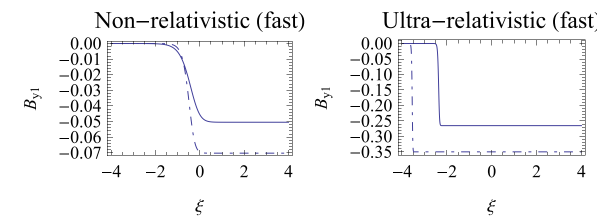

FIG. 6. Effect of number density on the fast mode of the KdVB shock structure. The bold curve is for $n_o = 10^{33}\ \mathrm{m}^{-3}$, dotted curve is for $n_o = 1.2 \times 10^{33}\ \mathrm{m}^{-3}$, and $B_0 = 5 \times 10^5$ T for the non relativistic case. For the ultra relativistic case $B_0 = 6 \times 10^7$ T, bold curve is for $n_o = 10^{36}\ \mathrm{m}^{-3}$, and dotted curve is for $n_o = 1.2 \times 10^{36}\ \mathrm{m}^{-3}$. Other parameters are $Z = 6$, $t = 1$, $\eta_0 = 0.1$, and $\theta = 30°$.

B. Plots of KdVB equation

The magnetoacoustic waves in this case admit the rarefactive shock structures for all the non relativistic and ultra relativistic cases of fast and the slow modes. We discuss here the dependence of magnetoacoustic shock waves only on the number density for both fast and slow modes. Figures 6 and 7 exhibit the behavior of fast and slow modes for both non relativistic and ultra relativistic cases. It is found that the increasing number density enhances the strength of the fast magnetoacoustic shock in terms of magnitude for the nonrelativistic as well as the ultra-relativistic shock (see Fig. 6). The reverse of the above happens for the slow magnetoacoustic shocks as shown in Fig. 7.

C. Plots of Burgers equation

Before we go on and discuss the results in this case, we take a brief digression to reflect on the behavior of fast and

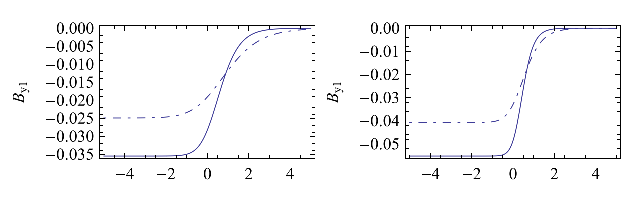

FIG. 7. Effect of number density on the slow mode of the KdVB shock structure. The bold curve is for $n_o = 10^{33}\ \mathrm{m}^{-3}$, dotted curve is for $n_o = 1.2 \times 10^{33}\ \mathrm{m}^{-3}$, and $B_0 = 5 \times 10^6$ T for the non relativistic case. For ultra relativistic case $B_0 = 9 \times 10^8$ T, bold curve is for $n_o = 5 \times 10^{35}\ \mathrm{m}^{-3}$, and dotted curve is for $n_o = 6 \times 10^{35}\ \mathrm{m}^{-3}$. Other parameters are $Z = 6$, $t = 1$, $\eta_0 = 0.1$, and $\theta = 30°$.

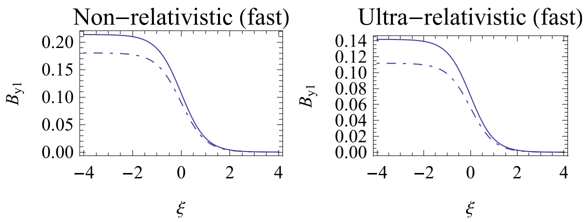

FIG. 8. Effect of number density on the fast mode of the Burgers shock structure. The bold curve is for $n_o = 10^{32}\,\mathrm{m}^{-3}$, dotted curve is for $n_o = 1.4 \times 10^{32}\,\mathrm{m}^{-3}$, and $B_0 = 5\times 10^5$ T for the non relativistic case. For ultra relativistic case $B_0 = 5 \times 10^8$ T, bold curve is for $n_o = 10^{36}\,\mathrm{m}^{-3}$, and dotted curve is for $n_o = 1.3\times 10^{36}\,\mathrm{m}^{-3}$. Other parameters are $Z=6$, $t=1$, $\eta_0 = 0.1$, and $\theta = 30°$.

slow magnetoacoustic waves for the Burgers equation. Interestingly, the fast magnetoacoustic mode for the Burgers equation admits compressive shock structures, whereas the slow magnetoacoustic waves allow the formation of rarefactive shock structures. The reason for compressive and rarefactive shocks is the sign of nonlinearity that changes for slow and fast modes and that is due to the difference in physics of the two modes. Figure 8 investigates the dependence of nonlinear fast magnetoacoustic shock potential on the number density for both the non relativistic and ultra relativistic cases for the Burgers equation. As mentioned earlier, the fast acoustic mode admits compressive shock structures for the Burgers equation. It is found that the increasing number density reduces the strength of the magnetoacoustic shock for the nonrelativistic as well as the ultra-relativistic shock. The shock strength is found to be greater for the non-relativistic shock by comparison with its ultra-relativistic counterpart. The same behavior is observed for the slow magnetoacoustic shocks as shown in Fig. 9; however, rarefactive shocks are formed in this case.

V. CONCLUSION

In this paper, we have investigated the formation of obliquely propagating non-relativistic and ultra-relativistic cases of magnetoacoustic shock structures in dense dissipative plasmas consisting of cold non-degenerate ions and degenerate inertialess electrons by means of the Kadomtsev-Petviashvili-Burger (KPB) equation. The assumptions under which the KPB equation has been obtained have been discussed. The solution of the KPB equation has been obtained by using the tanh method. It has been pointed out that limiting cases of the KPB equation yield the one-dimensional KdVB and Burgers equation which also admits shock solutions. Stability analysis of the KPB equation has also been presented and it has been shown that the weak transverse perturbation plays a vital role in the stability of the shock wave. The variation in the propagation characteristics of fast and slow magnetoacoustic modes for non-relativistic and relativistic cases has been discussed in detail. The system has been found to admit rarefactive shock structures for KdVB and KPB equations. Interestingly, it has been found that the nature of the shock wave changes for the Burgers equation which allows the formation of both compressive (for fast mode) and rarefactive (for slow mode) shock structures. It has been found to be due to the change in the sign of the nonlinearity coefficient for the two modes which further highlights the difference in the fundamental character of fast and slow modes. The results presented here may be beneficial to understand the formation and propagation of nonlinear dissipative structures in extremely dense environments such as those found in the vicinity of white dwarfs or neutron stars.

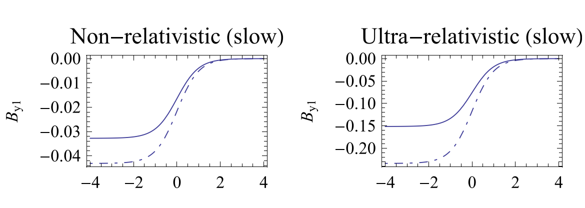

FIG. 9. Effect of number density on the slow mode of the Burgers shock structure. The bold curve is for $n_o = 10^{33}\,\mathrm{m}^{-3}$, dotted curve is for $n_o = 1.3 \times 10^{33}\,\mathrm{m}^{-3}$, and $B_0 = 5 \times 10^6$ T for the non relativistic case. For the ultra relativistic case $B_0 = 5\times 10^8$ T, bold curve is for $n_o = 10^{36}\,\mathrm{m}^{-3}$ and dotted curve is for $n_o = 1.3\times 10^{36}\,\mathrm{m}^{-3}$. Other parameters are $Z=6$, $t=1$, $\eta_0=0.1$, and $\theta = 30°$.

APPENDIX: DERIVATION OF THE KPB EQUATION

In this section, the KPB equation is derived using the small amplitude perturbation expansion method. For the lowest order of ϵ, (O-$\epsilon^{3/2}$) we obtain the following equations:

$$\begin{aligned}
\lambda\partial_\xi n_1 &= \partial_\xi u_1,\\
-\lambda\partial_\xi u_1 &= -\sin\theta\partial_\xi B_{y1} - \alpha_e\beta\partial_\xi n_1,\\
-\lambda\partial_\xi v_1 &= \cos\theta\partial_\xi B_{y1},\\
-\lambda\partial_\xi B_{y1} &= -\sin\theta\partial_\xi u_1 + \cos\theta\partial_\xi v_1.
\end{aligned} \tag{A1}$$

Solving the system (A1) yields the dispersion relation as obtained in Eq. (13),

$$\lambda = \left[\frac{1}{2}(1+\alpha_e\beta)\left[1\pm\sqrt{1-\frac{4\alpha_e\beta\cos^2\theta}{(1+\alpha_e\beta)^2}}\right]\right]^{1/2}. \tag{A2}$$

Collecting the terms in next higher order of ϵ (O-ϵ^2), we get

$$\begin{aligned}
-\lambda\partial_\xi B_{z1} &= \cos\theta\partial_\xi w_1 + \lambda\partial^2_{\xi,\xi} v_1,\\
-\lambda\partial_\xi w_1 &= -\sin\theta\partial_\chi B_{y1} + \cos\theta\partial_\xi B_{z1} - \alpha_e\beta\partial_\chi n_1.
\end{aligned} \tag{A3}$$

Now using relations (A3) along with the equations for the lowest order in ϵ, i.e., (A1) we obtain the following expression between w_1 and B_{y1}:

$$(\cos^2\theta - \lambda^2)\partial_\xi w_1 = \cos^2\theta\partial^2_{\xi,\xi}B_{y1} + \frac{\lambda^3\sin\theta}{\alpha_e\beta - \lambda^2}\partial_\chi B_{y1}. \tag{A4}$$

From the set of equations of the next higher order of ϵ, we obtain the following set of equations:

$$
\begin{aligned}
&-\lambda\partial_\xi n_2 + \partial_\xi u_2 \\
&\quad = -\partial_\tau n_1 - \partial_\xi(n_1 u_1) - \partial_\chi w_1 \\
&\quad\quad -\lambda\partial_\xi B_{y2} - \cos\theta\partial_\xi v_2 + \sin\theta\partial_\xi u_2 \\
&\quad = -\partial_\tau B_{y1} - \sin\theta\partial_\chi w_1 - \partial_\xi(B_{y1}u_1) \\
&\quad\quad +\lambda\partial^2_{\xi,\chi}u_1 - \lambda\partial^2_{\xi,\xi}w_1, \\
&-\lambda\partial_\xi u_2 + \sin\theta\partial_\xi B_{y2} + \alpha_e\beta\partial_\xi n_2 \\
&\quad = -\partial_\tau u_1 - B_{y1}\partial_\xi B_{y1} - u_1\partial_\xi u_1 \\
&\quad\quad +\sin\theta n_1\partial_\xi B_{y1} + \beta(\alpha_e - 2\beta_e)n_1\partial_\xi n_1 + \eta_0\partial^2_{\xi,\xi}u_1, \\
&-\lambda\partial_\xi v_2 - \cos\theta\partial_\xi B_{y2} \\
&\quad = -\partial_\tau v_1 - u_1\partial_\xi v_1 - \cos\theta n_1\partial_\xi B_{y1} + \eta_0\partial^2_{\xi,\xi}v_1.
\end{aligned} \tag{A5}
$$

Solving this set of equations leads to the Kadomtsev-Petviashvili-Burger (KPB) equation as given in Eq. (18).

[1]M. P. Robinson, B. L. Tolra, M. W. Noel, T. F. Gallagher, and P. Pillet, Phys. Rev. Lett. **85**, 4466 (2000).
[2]R. S. Fletcher, X. L. Zhang, and S. L. Rolston, Phys. Rev. Lett. **96**, 105003 (2006).
[3]S. H. Glenzer, O. L. Landen, P. Neumayer, R. W. Lee, K. Widmann, S. W. Pollaine, R. J. Wallace, G. Gregori, A. Höll, T. Bornath, R. Thiele, V. Schwarz, W.-D. Kraeft, and R. Redmer, Phys. Rev. Lett. **98**, 065002 (2007).
[4]G. A. Mourou, T. Tajima, and S. V. Bulanov, Rev. Mod. Phys. **78**, 309 (2006).
[5]G. Manfredi, Fields Inst. Commun. **46**, 263 (2005).
[6]S. Ichimaru, Rev. Mod. Phys. **54**, 1017 (1982).
[7]V. E. Fortov, Phys. Usp. **52**, 615 (2009).
[8]H. M. Horn, Science **252**, 384 (1991).
[9]G. Chabrier, Plasma Phys. Controlled Fusion **51**, 124014 (2009).
[10]T. Guillot, Science **286**, 72 (1999).
[11]R. P. Drake, Phys. Plasmas **16**, 055501 (2009).
[12]D. Lai, Rev. Mod. Phys. **73**, 629 (2001).
[13]M. Opher, L. O. Silva, D. E. Dauger, V. K. Decyk, and J. M. Dawson, Phys. Plasmas **8**, 2454 (2001).
[14]O. G. Benvenuto and M. A. De Vito, Mon. Not. R. Astron. Soc. **362**, 891 (2005).
[15]A. K. Harding and D. Lai, Rep. Prog. Phys. **69**, 2631 (2006).
[16]S. X. Hu and C. H. Keitel, Phys. Rev. Lett. **83**, 4709 (1999).
[17]A. V. Andreev, JETP Lett. **72**, 238 (2000).
[18]J. T. Mendonça, *Theory of Photon Acceleration* (Institute of Physics, Bristol, 2001).
[19]A. L. Kritcher *et al.*, Science **322**, 69 (2008).
[20]P. Neumayer *et al.*, Phys. Rev. Lett. **105**, 075003 (2010).
[21]D. H. Froula, S. H. Glenzer, N. C. Luhmann, Jr., and J. Sheffield, *Plasma Scattering of Electromagnetic Radiation: Theory and Measurement Techniques*, 2nd ed. (Academic Press, New York, 2011).
[22]H. Azechi and FIREX Project. Plasma Phys. Controlled Fusion **48**, B267 (2006).
[23]C. L. Gardner and C. Ringhofer, Phys. Rev. E **53**, 157 (1996).
[24]P. K. Shukla and B. Eliasson, Phys. Usp. **53**, 51 (2010).
[25]M. Bonitz, *Quantum Kinetic Theory* (Teubner, Stuttgart, 1998).
[26]D. Kremp, Th. Bornath, M. Bonitz, and M. Schlanges, Phys. Rev. E **60**, 4725 (1999).
[27]N. L. Tsintsadze and L. N. Tsintsadze, Europhys. Lett. **88**, 35001 (2009).
[28]F. Haas, L. G. Garcia, J. Goedert, and G. Manfredi, Phys. Plasmas **10**, 3858 (2003).
[29]F. Haas, Phys. Plasmas **12**, 062117 (2005).
[30]P. K. Shukla and B. Eliasson, Phys. Rev. Lett. **96**, 245001 (2006).
[31]H. E. Wilhelm, Z. Phys. **241**, 1 (1971).
[32]G. Brodin and M. Marklund, New J. Phys. **9**, 277 (2007).
[33]M. Marklund and G. Brodin, Phys. Rev. Lett. **98**, 025001 (2007).
[34]W. Masood and A. Mushtaq, Phys. Lett. A. **372**, 4283 (2008).
[35]W. Masood, Phys. Plasmas. **16**, 042314 (2009).
[36]W. Masood, B. Eliasson, and P. K. Shukla, Phys. Rev. E. **81**, 066401 (2010).
[37]J. H. Adlam and J. E. Allen, Philos. Mag. **3**, 448 (1958).
[38]L. Davis, R. Lüst, and A. Schulüter, Z. Naturforsch. **13a**, 916 (1958).
[39]C. S. Gardner and G. K. Morikawa, Commun. Pure Appl. Math. **18**, 35 (1965).
[40]Y. A. Berezin and V. I. Karpman, Sov. Phys. JETP **19**, 1265 (1964).
[41]T. Kakutani and H. Ono, J. Phys. Soc. Jpn. **26**, 1305 (1969).
[42]B. Eliasson and P. K. Shukla, Europhys. Lett. **97**, 15001 (2012).
[43]D. E. Winget and S. O. Kepler, Annu. Rev. Astron. Astrophys. **46**, 157 (2008).
[44]G. Fontaine and P. Brassard, Publ. Astron. Soc. Pac. **120**, 1043 (2008).
[45]R. Silvotti, G. Fontaine, M. Pavlov, T. R. Marsh, V. S. Dhillon, S. P. Littlefair, and F. Getman, Astron. Astrophys. **525**, A64 (2011).
[46]M. Marklund, B. Eliasson, and P. K. Shukla, Phys. Rev. E **76**, 067401 (2007).
[47]P. K. Shukla, B. Eliasson, and L. Stenflo, EPL **95**, 045001 (2011).
[48]S. Chandrasekhar, Mon. Not. R. Astron. Soc. **170**, 405 (1935).
[49]H. Washimi and T. Tanuiti, Phys. Rev. Lett. **17**, 996 (1966).
[50]M. J. Ablowitz and P. A. Clarkson, *Solitons, Nonlinear Evolution Equations and Inverse Scattering* (Cambridge University Press, Cambridge, 1991).
[51]R. Hirota, Phys. Rev. Lett. **27**, 1192 (1971).
[52]M. R. Miura, *Backlund Transformation* (Springer-Verlag, Berlin, 1978).
[53]W. Malfliet, Am. J. Phys. **60**, 650 (1992).
[54]W. Malfliet, J. Comput. Appl. Math. **164**, 529 (2004).
[55]V. I. Karpman, *Nonlinear Waves in Dispersive Media* (Pergamon, Oxford, 1975).
[56]T. Padmanabhan, *Theoretical Astrophysics, Volume II: Stars and Stellar Systems* (Cambridge University Press, London, 2001).
[57]W. M. Moslem, S. Ali, P. K. Shukla, X. Y. Tang, and G. Rowlands, Phys. Plasmas **14**, 082308 (2007).
[58]G. Brodin, M. Marklund, and G. Manfredi, Phys. Rev. Lett. **100**, 175001 (2008).
[59]R. Sabry, W. M. Moslem, and P. K. Shukla, Phys. Plasmas **19**, 122903 (2012).