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Electrons in regions of high phase density exhibit uncanny traits in the study of linear and nonlinear drift waves in spatially non-uniform magnetoplasmas

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Linear and nonlinear waves are examined on the ion time scale in a spatially inhomogeneous plasma having electrons that follow product bi (r,q) distribution. It has been shown that the linear dispersion relation for product bi (r,q) distribution undergoes appreciable changes as opposed to the one for the Maxwellian electrons. It has been found that the drift wave frequency is highest for flat-topped distribution, whereas it is lowest for the spiky distribution. It has been found that the drift solitary wave with flat-topped distribution (i.e., $r > 0$) is one of a kind and exhibits peculiar characteristics. It has been shown that Maxwellian and kappa-like electrons cannot alter the nature of the electrostatic drift waves under consideration; however, the spiky electrons can. The results obtained here are general and can be applied to many regions of space plasmas where the satellite missions have reported the presence of electron distribution functions that show deviation from the Gaussian behavior. *Published by AIP Publishing.* <https://doi.org/10.1063/1.5052220>

I. INTRODUCTION

The importance of drift waves with regard to particle and energy transport is very well established.¹ Most of the plasma systems are spatially non uniform and give rise to drift motions and the allied drift waves.^{2–5} Drift waves on the ion time scale have frequencies much smaller than the ion cyclotron frequency and they propagate in the direction at right angles to the ambient magnetic field. The basic assumption that is employed to obtain the dispersion relation for drift waves is that $E \times B$ drift is the most prominent drift. Unlike the ion acoustic waves where space charge effects are responsible for the wave dispersion, the dispersion of the drift waves appears through the ion polarization drift. Ignoring the polarization drift will yield the dispersionless drift wave. The study of the nonlinear behavior of the drift waves leads to the emergence of structures like solitons, vortices, and shocks in plasmas. The literature is replete with the investigation of these structures in a variety of physical situations of interest and the study of their properties.^{6–14}

Nonlinear drift waves in inhomogeneous plasmas have been investigated with the inclusion of a non-zero ion temperature gradient. It has been shown that the presence of a nonzero ion temperature gradient induces a corresponding perpendicular thermal flux that significantly alters the transverse stress tensor and, therefore, the perpendicular ion polarization drift must incorporate the corrections in the magnetic viscosity due to the nonzero thermal flux.¹⁵ The decay instability and Kolmogorov spectra of ion-drift waves were studied in low- β dusty plasmas. In this regard, the matrix elements of the three-wave interactions in an inhomogeneous plasma with uniform ion temperature were derived. The growth rate of decay instability and the weakly turbulent plasma wave spectra were also calculated and analyzed.¹⁶ Shukla *et al.*¹⁷ revisited the coupling between

various low-frequency modes in a nonuniform magnetoplasma and explained the way to obtain electrostatic drift waves from a general set of equations. It was shown that the parallel electron current associated with the electron drift waves is coupled with the ion polarization current. Nonlinear interactions between drift waves and zonal flows were also investigated and it was shown that finite amplitude drift waves can parametrically excite zonal flows in a nonuniform magnetoplasma.¹⁸

The modified tails of the distribution functions that are usually termed suprathermal or non-Maxwellian plasmas have been observed in the magnetospheres of many planets of our solar system and in the solar wind;^{19–21} however, the reason for the development of these distributions is still not fully understood.^{22–28} The observed distributions of charged particles can be adequately represented by a generalized Lorentzian distribution and have been found to concur both with the thermal and the suprathermal parts of the observed energy velocity spectra.^{29–32} Besides the suprathermal tails, many satellite missions have observed electron distribution functions in space plasmas not only with the modified tails but also low energy electrons that exhibit departure from the Gaussian behavior.^{33–40} For instance, cluster PEACE data have shown flat-topped electron distribution functions downstream of the terrestrial bow shock.^{38,39} The flat-topped distributions are usually observed downstream of the quasi-perpendicular terrestrial bow shock. The reason for the development of these distribution functions has been shown to be adiabatic heating of the electrons in the shock layer.^{35,37} Non adiabatic heating processes have also been proposed to account for the heating of electrons in the regions of high phase space density.^{41–43} Flat-topped electron distributions have also been found around the magnetic reconnection region in the magnetotail.⁴⁴ It has been shown that various features observed in space plasmas can be satisfactorily explained by (r,q) distribution.^{40,45}

In this paper, we have studied the ion drift waves in the linear and nonlinear regimes in the presence of product bi (r,q) distribution. The layout of the paper is as follows. In Sec. II, we write down the model equations for our system and also write down the functional form of the product bi (r,q) distribution. Next, we derive the linear dispersion relation of ion drift waves in a magnetoplasma with non-Gaussian electrons and also obtain the nonlinear KdV-like equation for drift waves. In Sec. III, we present and discuss the results and finally in Sec. IV, we recapitulate the main findings of the paper.

II. MATHEMATICAL MODEL

We assume an electron-ion (ei) plasma that is immersed in a constant ambient magnetic field. The direction of the background magnetic field is considered to be in the z-direction, i.e., $B_0 = B_0 \hat{z}$ and the wave propagation is taken to be in the y-direction. It is supposed that the equilibrium density gradient is in the negative x-direction, i.e., $\nabla n_0 = -\hat{x} |dn_0/dx|$. Note that the drift wave cannot propagate in one dimension as electron thermalization requires a non-zero component along the direction of magnetic field. We use the ordering $\partial_x < \partial_z < \partial_y$ and neglect the propagation vector along the magnetic field for the pure drift wave though it is non-zero.

The dynamics of singly charged ions can be expressed by the following momentum balance equation:

$$m_i n_i \left(\frac{\partial}{\partial t} + \mathbf{v}_i \cdot \nabla \right) \mathbf{v}_i = e n_i \left(\mathbf{E} + \frac{1}{c} \mathbf{v}_i \times \mathbf{B}_0 \right), \quad (1)$$

where the symbols n_i , \mathbf{v}_i , m_i , and e represent number density, fluid velocity, mass and charge of ions.

The ion continuity equation is given by

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{v}_i) = 0. \quad (2)$$

We employ the drift approximation, i.e., $\partial/\partial t \ll \Omega_i$, which enables us to write the perpendicular ion fluid velocity as

$$v_{i\perp} \simeq -\frac{c}{B_0} \nabla \varphi \times \hat{z} - \frac{c}{B_0 \Omega_i} \left(\frac{\partial}{\partial t} + \mathbf{v}_E \cdot \nabla \right) \nabla_{\perp} \varphi = \mathbf{v}_E + \mathbf{v}_p, \quad (3)$$

where $\Omega_i = eB_0/m_i c$ is the cyclotron frequency for ions, $\mathbf{v}_E = -c/B_0 (\nabla \varphi \times \hat{z})$ is the $E \times B$ drift, and $\mathbf{v}_p = -c/B_0 \Omega_i (\partial/\partial t + \mathbf{v}_E \cdot \nabla) \nabla_{\perp} \varphi$ is the ion polarization drift. $\mathbf{E} = -\nabla \varphi$, where φ is the electrostatic potential. This expression implies that we are dealing with electrostatic waves and it is assumed that the propagation vector and the perturbed electric field are parallel to each other. Note that this is determined from the geometry of the electrostatic drift waves given below the start of this section and the assumption that $\partial_x < \partial_z < \partial_y$.

The ion continuity equation can then be written as

$$\begin{aligned} & \left(\frac{\partial}{\partial t} + \mathbf{v}_E \cdot \nabla \right) \tilde{n}_i + \frac{c}{B_0} \hat{z} \times \nabla \varphi \cdot \nabla n_{i0} \\ & + \frac{c n_{i0}}{B_0 \Omega_i} \left(\frac{\partial}{\partial t} + \mathbf{v}_E \cdot \nabla \right) \nabla_{\perp}^2 \varphi = 0, \end{aligned} \quad (4)$$

where \tilde{n}_i is the number density of the perturbed ions.

We use product bi (r,q) distribution for electrons that closely resembles the distribution functions for many satellites observed in different space plasma environments. The functional form of the product bi (r,q) distribution is given by⁴⁶

$$f = A \left[1 + \frac{1}{q-1} \left(\frac{p_{\parallel}^2}{b^2 m T_{\parallel}} + \frac{2U}{b^2 T_{\parallel}} \right)^{r+1} \right]^{-q} \times \left[1 + \frac{1}{q-1} \left(\frac{p_{\perp}^2}{d^2 m T_{\perp}} \right)^{r+1} \right]^{-q}, \quad (5)$$

where $U = -e\varphi$,

$$\begin{aligned} A &= (\Gamma(q))^2 \left/ \left(2\pi(q-1)^{3/2+2r} \sqrt{mb^2 T_{\parallel}} (md^2 T_{\perp}) \right) \right. \\ &\quad \times \Gamma\left(q - \frac{1}{2+2r}\right) \Gamma\left(1 + \frac{1}{2+2r}\right) \Gamma\left(q - \frac{1}{1+r}\right) \\ &\quad \times \Gamma\left(1 + \frac{1}{1+r}\right), \\ b &= (q-1)^{-1/2+2r} \left(3\Gamma\left(q - \frac{1}{2+2r}\right) \Gamma\left(1 + \frac{1}{2+2r}\right) \right) / \\ &\quad \Gamma\left(q - \frac{3}{2+2r}\right) \Gamma\left(1 + \frac{3}{2+2r}\right) \right)^{1/2}, \end{aligned}$$

and

$$d = 2(q-1)^{-1/2+2r} \left(\Gamma\left(q - \frac{1}{1+r}\right) \Gamma\left(1 + \frac{1}{1+r}\right) \right) / \Gamma\left(q - \frac{2}{1+r}\right) \Gamma\left(1 + \frac{2}{1+r}\right) \right)^{1/2}.$$

T_{\parallel} and T_{\perp} denote the electron temperatures along and perpendicular directions and Γ is the standard Gamma function. We normalize the distribution function to obtain coefficient A and come up with the restrictions on the values of r and q such that that $q > 1$ and $q(1+r) > 3/2$. Note that the product bi (r,q) distribution is a generalization of kappa and the Davydov-Druyvestien distribution function. It gives us bi-kappa distribution at $r=0$ and $q = \kappa + 1$. For $r=0$ and $q \rightarrow \infty$, we retrieve the standard bi-Maxwellian distribution function from Eq. (5). Integrating the product bi (r,q) distribution function [see Eq. (5)] over the velocity space yields the following expression for the total number density of electrons under the ordering $\partial_x < \partial_z < \partial_y$:

$$n_e = n_{e0} (1 + \gamma \phi + \delta \phi^2), \quad (6)$$

where $\phi = e\varphi/T_e$,

$$\begin{aligned} \gamma &= \left(2(1+r)(q-1)^{-1/1+r}/b^2 \right) \left(\Gamma\left(\frac{1+2r}{2+2r}\right) \right. \\ &\quad \times \Gamma\left(q + \frac{1}{2+2r}\right) / \Gamma\left(q - \frac{1}{2+2r}\right) \Gamma\left(\frac{1}{2+2r}\right) \right), \end{aligned}$$

and

$$\delta = \left(-2(q-1)^{-2/1+r}(1+r)^2/(-1+2r)b^4 \right) \\ \times \left(\Gamma\left(\frac{1+4r}{2+2r}\right)\Gamma\left(q+\frac{3}{2+2r}\right) / \right. \\ \left. \Gamma\left(q-\frac{1}{2+2r}\right)\Gamma\left(\frac{1}{2+2r}\right) \right).$$

It is pertinent to mention here that after performing the tedious integrations, we are left with just the parallel temperature and we denote that by T_e . Note that in order to obtain Eq. (6), we have expanded the terms containing the potential up to the square order. It is appropriate to mention here that the index r modifies the behavior of the electrons at low energies, whereas the index q alters the tail or the electron behavior at high energies.

Finally, the quasi-neutrality condition is given by

$$n_e \simeq n_i. \quad (7)$$

A. Linear analysis

Assuming the sinusoidal variation of the perturbations, i.e., $\exp[i(k_y y - \omega t)]$ and linearizing Eqs. (4), (6), and (7) give us the following expression for linear ion drift waves in the presence of product bi (r,q) distributed electrons:

$$\omega = \frac{v_d^* k_y}{\gamma + \rho_i^2 k_y^2}, \quad (8)$$

where $\rho_i = c_s/\Omega_i$ is the ion acoustic Larmor radius, $c_s = (T_e/m_i)^{1/2}$ is the ion acoustic speed, and v_d^* is the ion velocity due to background density gradient commonly termed the diamagnetic drift velocity and is given by $(cT_e/eB_0)\kappa_{ni}$ and $\kappa_{ni} = |d \ln N_{i0}/dx|$ is the inverse gradient scalelength. Note that unlike the Maxwellian plasmas, the expression of ion drift wave gets modified in the presence of product bi (r,q) distribution. Since γ contains both r and q , therefore, it can easily be seen that the dispersion relation for the drift wave gets affected by electrons both in the regions of low and high phase densities. Figure 1 exhibits the behavior of ion drift frequency for different r values which essentially means changing the percentage of electrons in the

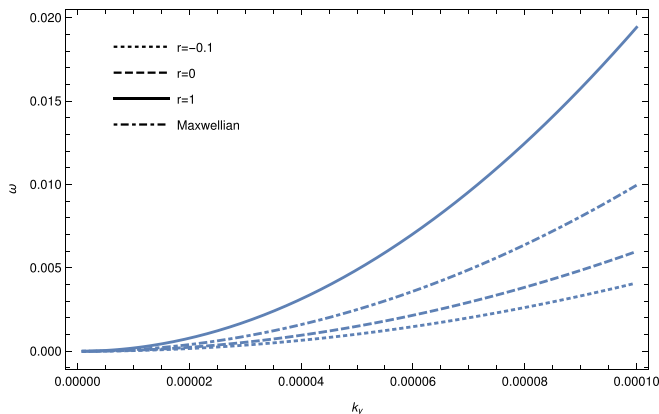


FIG. 1. Comparison of the dispersion relation of ion drift waves for different electron distribution functions for $q = 3$.

regions of high phase space density or behavior of low energy electrons for a fixed value of q . It is found that the drift wave frequency is highest for flat-topped distribution, whereas it is lowest for the spiky distribution. The kappa-like distribution turns out to have a lower ion drift frequency than the flat-topped counterpart but higher than the spiky distribution. It can be seen that the frequency of ion drift waves is second highest for Maxwellian electrons. Owing to the paucity of space, suffice it to say that increasing the number of electrons in the region of low phase density or enhancing q augments the frequency of the ion drift wave for the kappa-like, flat-topped, and spiky distribution cases. The change in drift frequency for flat-topped distribution for the increasing q values is less pronounced by comparison with kappa and spiky electron distribution functions. Note that unlike the Maxwellian distribution, the product bi (r,q) distribution gives us a dispersion relation that contains the coefficient γ which radically alters the behavior of the drift wave frequency with the change of spectral indices r and q .

B. Nonlinear analysis

We focus our attention now to obtain nonlinear structures for the drift waves on the ion time scale. Using Eqs. (6) and (7) in the ion continuity equation i.e., Eq. (4), we obtain

$$\gamma \partial_t \phi + 2\delta \phi \partial_t \phi - \rho_i^2 \partial_t \partial_y^2 \phi + v_d^* \partial_y \phi + \gamma v_d^* \phi \partial_y \phi = 0. \quad (9)$$

The above equation has been obtained under the assumption that $\partial_x < \partial_z < \partial_y$ and we have retained the nonlinear terms containing the potential up to square order since we are dealing with weak nonlinearity i.e., $\phi \ll 1$. The nonlinear terms in the above equation arise from the convective derivative and the perturbed number density terms. In order to arrive at the stationary solution, we use the variable transformation $\xi = y - ut$ and convert the above nonlinear partial differential equation (NLPDE) into a nonlinear ordinary differential equation (NLODE) which reads as

$$-U \partial_\xi \phi + A \phi \partial_\xi \phi + B \partial_{\xi\xi\xi} \phi = 0, \quad (10)$$

where $U = (1 - v_d^*/\gamma u)$, $A = -2\delta(1 - \gamma v_d^*/2\delta u)/\gamma$, and $B = 1/\gamma$. A is the coefficient of quadratic nonlinearity and B is the coefficient of dispersion. Note that although the untransformed equation i.e., Eq. (9) does not resemble any standard nonlinear equation, its transformed counterpart closely resembles the ordinary Korteweg de Vries (KdV) equation. The solution of this equation is given by

$$\phi = \frac{3U}{A} \operatorname{sech}^2 \left[\frac{\xi}{\sqrt{4B/U}} \right]. \quad (11)$$

III. RESULTS AND DISCUSSION

In this section, we will investigate the effect of changing the percentage of electrons in low and high phase space density regions (i.e., changing the values of spectral indices r and q of the product bi (r,q) electron distribution function, respectively) on the solitary structures propagating in a

spatially non-uniform magnetoplasma. We have selected the plasma parameters that are representative of the ionospheric F-layer.^{47,48} Before we go on and discuss the results, we would like to mention that the values of number density and magnetic field determine (which are different for different space plasma environments) the ion acoustic Larmor radius which eventually dictates the spatio-temporal regime and also the values of the wave vector and the inverse scale-length for the study of drift waves. However, the qualitative behavior stays the same and, therefore, the present study is qualitatively valid for the propagation of linear and nonlinear propagation of drift waves in different environments in space plasmas.

Figure 2 examines the behavior of ion drift solitary waves with the increasing number of energetic electrons in the tail of the distribution function i.e., increasing q values and Maxwellian top i.e., $r=0$ or kappa-like electrons. It is observed that such an arrangement produces rarefactive solitary waves in the system. It is found that, in terms of magnitude, the amplitude and width of the drift solitary wave experience an increase with the increasing values of q keeping the value of ratio of diamagnetic drift to nonlinear structure velocity i.e., v_d^*/u fixed. It is, however, noticed that we cannot increase the percentage of energetic electrons too much as for higher values of q , the normalized electrostatic drift potential exceeds one which indicates that those values are not permissible for the weakly nonlinear study. Figure 3 exhibits the behavior of solitary ion drift waves with the variation in the values of v_d^*/u keeping the value of q fixed for the kappa-like electrons. It is found that the amplitude of drift solitary structures, in terms of magnitude, experiences a decrement, whereas the width increases for the increasing values of v_d^*/u for kappa-like electrons. It is observed that the drift solitary structures exist both for the values of diamagnetic drift velocity less than and greater than the velocity of the nonlinear structure.

Figure 4 depicts the behavior of the ion drift solitary waves with the increasing number of electrons in the region of low phase space density for flat-topped electron distribution function ($r=1$ in this case). Unlike the kappa-like electrons, compressive drift ion solitary structures are observed

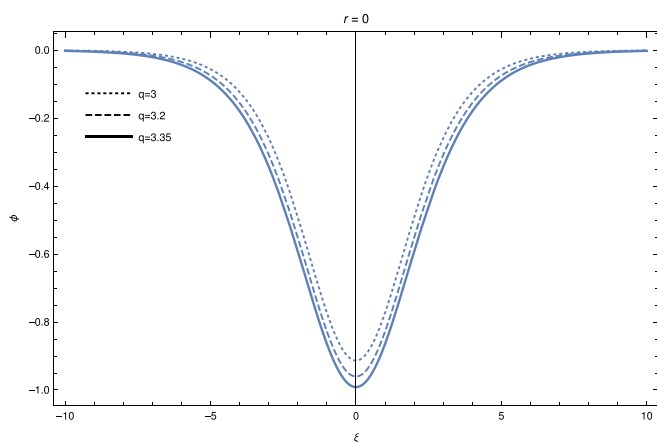


FIG. 2. Solitary drift wave for the kappa-like electron distribution function ($r=0$) for different values of q and $v_d^*/u = 0.98$.

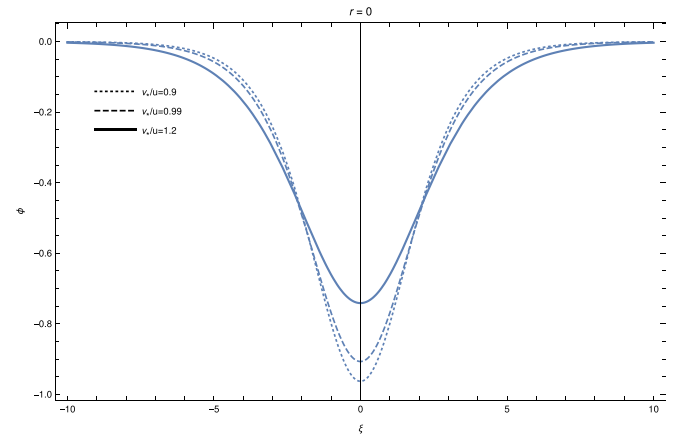


FIG. 3. Solitary drift wave for the kappa-like electron distribution function ($r=0$) for different values of v_d^*/u with $q=3$.

for this case. It is found that the amplitude of drift solitary wave does not alter significantly; however, there is an increase in the width of the wave with the increasing q values. Moreover, it is found that the drift solitary structures form only for $v_d^*/u < 0$ and soliton formation for the $v_d^*/u > 0$ case is forbidden for the flat-topped distribution function. This type of behavior, as we shall see later, is a peculiar trait of the flat-topped distribution function. Figure 5 shows the behavior of the ion drift solitary wave for increasing q values with flat-topped distribution but for $r=3$. Interestingly, in this case, the amplitude of the drift solitary wave decreases with increasing percentage of energetic electrons in regions of low phase density in this case. It is observed that this shift in behavior of the drift solitary wave for different values of r is a complex interplay of the nonlinear and dispersive coefficients. Figure 6 examines the behavior of the drift solitary wave for flat-topped distribution with increasing values of v_d^*/u . It is observed that the increasing values of v_d^*/u mitigate the amplitude; however, they enhance the width of the drift solitary waves. It is observed that the soliton formation for the weakly nonlinear limit is possible for a restricted range of $v_d^*/u < 0$ values. Figure 7 explores the behavior of the ion drift solitary waves for increasing values of the electrons in the regions of high phase space density. This shows a

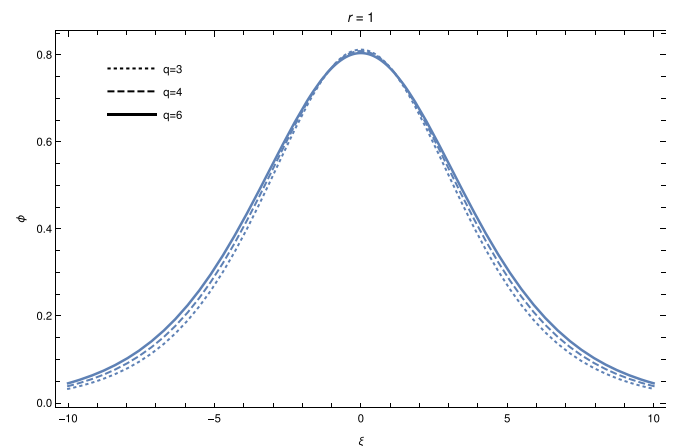


FIG. 4. Solitary drift wave for the flat-topped electron distribution function ($r=1$) for different values of q and $v_d^*/u = 0.3$.

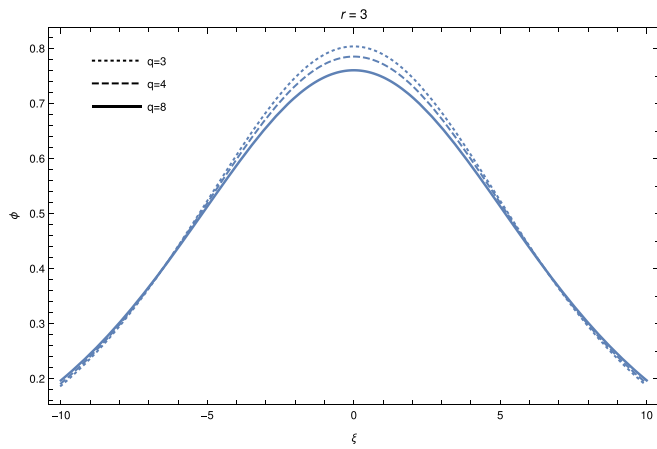


FIG. 5. Solitary drift wave for the flat-topped electron distribution function ($r = 3$) for different values of q and $v_d^*/u = 0.3$.

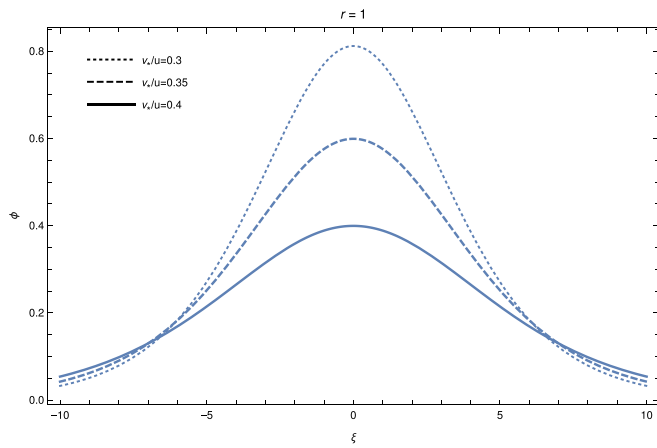


FIG. 6. Solitary drift wave for the flat-topped electron distribution function ($r = 1$) for different values of v_d^*/u with $q = 3$.

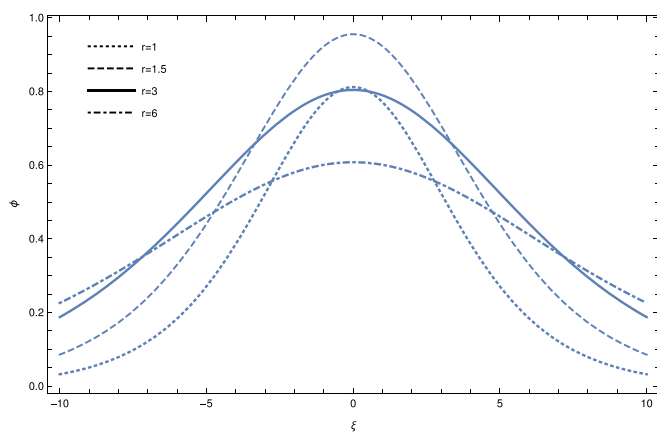


FIG. 7. Solitary drift wave for the flat-topped electron distribution function for different values of r with $q = 3$ and $v_d^*/u = 0.3$.

very interesting and surprising result. For $1 < r < 2$, the amplitude of the drift solitary wave increases but for $r > 2$, the amplitude of the solitary wave begins to decrease. The width of the solitary wave for small r values is much less by comparison with width for large r values. It is observed that

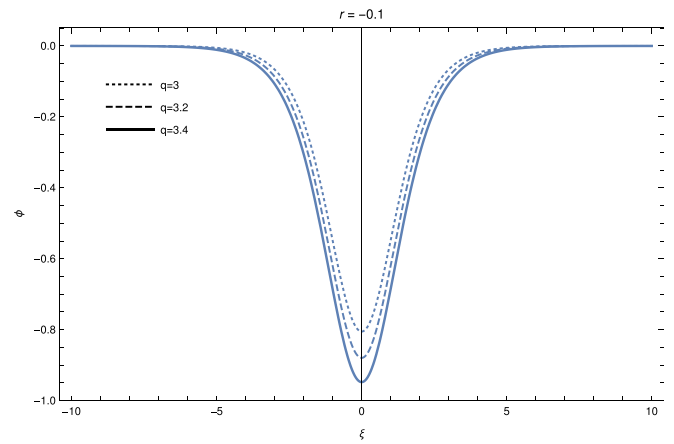


FIG. 8. Solitary drift wave for the spiky electron distribution function ($r < 0$) for different values of q and $v_d^*/u = 0.8$.

the flat topped distribution does not admit any rarefactive structures for any range of r values.

Figure 8 studies the effect of the spiky electron distribution function ($r = -0.1$) on the drift ion solitary waves with increasing number of energetic electrons in the tail of the distribution function. It is found that for this value of spiky distribution, rarefactive drift solitary structures are obtained and that the increasing number of electrons in regions of low phase density enhance (in terms of magnitude) the amplitude and width of the drift solitary waves. Like the kappa-like case, it is again found that there is a narrow range of the variation of the energetic electrons for which the soliton formation in the weakly nonlinear limit is possible. Without showing the figure, it is sufficient to say that the increasing values of ratio v_d^*/u enervate the amplitude, whereas they enhance the width of the drift solitary waves for the spiky electron distribution function. This behavior is akin to the one found for kappa-like distribution but very different from flat-topped electron distribution. Figure 9 shows the effect of change of values of spiky distribution on the behavior of the ion drift solitary waves. Interestingly, as we enhance the values of the spiky distribution, the nature of the drift solitary waves changes from rarefactive to compressive. This is the unique feature of spiky electron distribution function that it allows the formation of both compressive and rarefactive

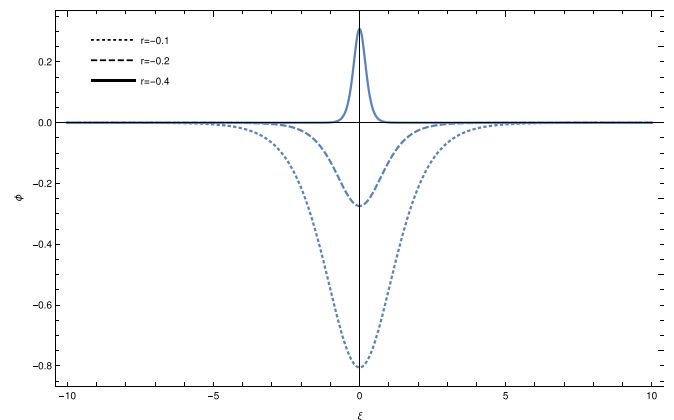


FIG. 9. Solitary drift wave for the spiky electron distribution function for different values of r with $q = 3$ and $v_d^*/u = 0.8$.

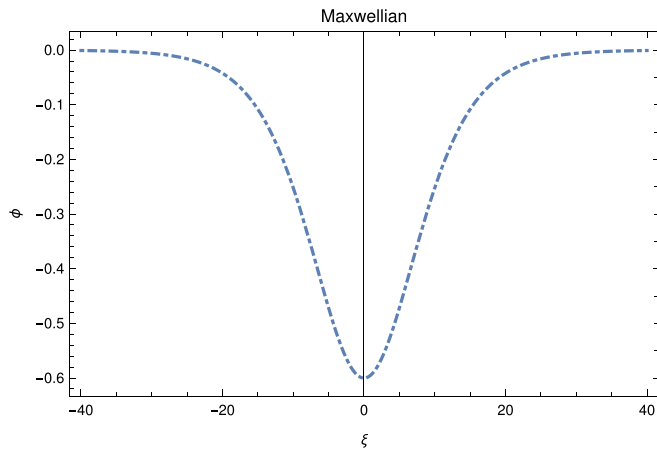


FIG. 10. Solitary drift wave for the Maxwellian electron distribution function.

solitary drift waves. Figure 10 shows the behavior of the ion drift solitary waves for Maxwellian electrons and it is observed that we obtain rarefactive drift solitary structures in this case. It is, however, noteworthy that the spatial scales over which the drift solitary structures form for Maxwellian electrons are much longer by comparison with the product bi (r, q) distribution.

Finally, we would like to highlight some unique features of the product bi (r, q) distribution. It should be noted that contrary to Maxwellian plasmas where the dispersion coefficient does not depend on the shape of the distribution function, the dispersion coefficient of the product bi (r, q) distribution does depend on the shape of the distribution function. Note that $r > 0$ is a particularly unique case. It shows features which are drastically different from the other cases. Unlike the $r \leq 0$ case, it shows that only a certain range of values of ratio v_d^*/u are permissible for the existence of drift solitary structures. For the rest, the dispersive coefficient becomes negative and hence gives rise to imaginary values of the wave potential. It is found that for Maxwellian and kappa-like distributions, the drift solitary structures do not show any departure from the usual behavior; however, for $r \leq 0$ the behavior of the nonlinear drift waves shows unique properties. This means that changing r or the percentage of low energy electrons in the distribution function brings the major change. This can be understood by keeping in mind the fact that the maximum number of electrons lie in the low energy range. Another important feature is that unlike the $r > 0$ case, the $r \leq 0$ cases show that the weakly nonlinear structures are very sensitive to the change of percentage of electrons in the regions of low phase density or in the tail of the distribution function and it is observed that the permissible drift solitary structures form for a very narrow range of q values.

IV. CONCLUSION

To recapitulate, linear and nonlinear structures of drift waves have been investigated in an inhomogeneous electron-ion magnetoplasma with the inclusion of velocity spread of electrons in phase space that is represented by product bi (r, q) distribution. It has been found that unlike the

Maxwellian case, the dispersion relation for the product bi (r, q) distribution gets affected by the shape of the distribution function. It has been observed that the nonlinear drift waves exhibit unique features for this distribution function which are distinctly different from the Maxwellian distribution function. It has been found that for $r = 0$, meaning the kappa-like case, the behavior of drift solitary structures is akin to their Maxwellian counterpart and in both the cases, we obtain rarefactive solitary structures. However, the spatial scales over which the nonlinear structures form in Maxwellian plasmas have been found to be much longer than the ones for the kappa-like electron distribution function. It has been shown that for $r \leq 0$ cases, the drift solitary waves exhibit a fascinating deviation from kappa-like and Maxwellian distribution functions and admit both compressive and rarefactive structures. The $r > 0$ case is particularly unique in that it allows only the formation of compressive solitary structures for a certain range of ratio of diamagnetic drift velocity to nonlinear structure velocity ratio v_d^*/u . It has also been found that the major reason for the extraordinary behavior of nonlinear drift waves in this case is the change in the percentage of electrons in regions of high phase density where the majority of electrons reside. The kappa-like and spiky electron distributions have been found to be more sensitive to the change in the tail of the distribution function and allow the formation of weakly nonlinear structures only for a narrow range of q values. The present study can be extended to study dissipative and dispersive structures like shocks and vortices. The linear and nonlinear coupling between acoustic and drift waves in higher dimensions can also be investigated.

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