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Modulational and three wave decay instabilities in degenerate electron-ion dense plasmas

Ch. Rozina,[1,a)] S. Ali,[2] H. A. Shah,[3] and M. Jamil[4]
[1]*Department of Physics, Lahore College for Women University, Lahore 54000, Pakistan*
[2]*National Centre for Physics (NCP) at QAU Campus, Shahdra Valley Road, Islamabad 44000, Pakistan*
[3]*Department of Physics, Government College University, Lahore 54000, Pakistan*
[4]*Department of Physics, COMSATS University Islamabad, Lahore Campus 54000, Pakistan*

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A formalism for nonlinear interaction of degenerate upper-hybrid waves (DUHWs) with degenerate ion-cyclotron waves (DICWs) and degenerate Alfven waves (DAWs) is revisited to account for quantum corrections owing to quantum Bohm potential, quantum exchange-correlations, and quantum statistical pressure. For this purpose, different nonlinear dispersion relations are derived with quantum settings. By using the phasor matching techniques, the growth rates of three wave decay and modulational instabilities are analyzed by identifying the nonlinear coupling of high-frequency DUHWs with low-frequency DICWs and DAWs. Numerically, it is revealed that parametric three wave decay and modulational instabilities are significantly influenced by the impact of Fermi pressure and exchange-correlation in a degenerate magnetoplasma. The present results are important to understand the dispersive properties of nonlinear waves and their mutual couplings at quantum scales in degenerate environments like white dwarfs, neutron stars, and magnetars. *Published by AIP Publishing.* https://doi.org/10.1063/1.5031931

I. INTRODUCTION

Quantum plasmas and quantum effects have been successfully, recognized for investigating collective dynamics in different research areas, for example, semiconductor devices[1] quantum computers, quantum dots, quantum wires[2] quantum wells, carbon nanotubes and diodes,[3] ultracold,[4] microplasmas,[5] biophotonic,[6] superdense giant planets,[7] electron-hole plasmas,[8] laser-produced plasma experiments,[9] etc. To study a quantum mechanical system, we assume that free particles are present in a finite volume that can occupy a discrete set of energies called quantum energy levels or states. These energy states can be filled by the Pauli exclusion principle which simply illustrates that no more than two identical (i.e., indistinguishable) fermions can occupy the same quantum state as long as the thermal energy of particles is negligibly small (e.g., absolute temperature $T = 0$). However, it is well established that two fermions will only exist in the same quantum state provided they have opposite spins. A gas with all the filled lowest energy quantum states is known as degenerate and the corresponding pressure is called degeneracy pressure or Fermi pressure.[10] One of the important features of the degenerate gas is that Fermi pressure is not a function of temperature but it depends on the fermion number density. It is the Fermi pressure that keeps the dense stars in equilibrium against the gravitational pull. At room temperature, the electron gas in the ordinary metal is a good example of quantum plasma exhibiting high density and low temperature.

Several collective modes[11,12] have been investigated in quantum plasmas with quantum corrections both analytically and numerically. Relying on the Thomas-Fermi theory,[13] the quantum hydrodynamic (QHD) model has been utilized to study low-frequency electrostatic streaming instability in a viscoelastic quantum electron-ion-dust plasma. The parametric domain of streaming instability has also been discussed in quantum plasmas by using the modified QHD model.[14] Quantum exchange and correlations of electrons may be arisen from the electron half spin effects,[15] while degenerate pressure identifies the statistical behavior of electrons in degenerate gases. Collective interactions[16,17] with different approaches[16] and nonlinear quantum waves using the QHD model[18–20] have specifically been studied to account for quantum mechanical effects. The interaction of high-frequency nonlinear Langmuir waves with low-frequency nonlinear ion-acoustic waves has also successfully been described in degenerate plasmas.[21] Marklund[22] has presented theoretical and numerical results on quantum plasmas to investigate modulational instability. Nonlinear dispersion relations[23] that have been derived to demonstrate three wave decay and modulation instabilities due to nonlinear coupling of mode-converted electron Bernstein with low-frequency waves, such as ion-acoustic waves, electron-acoustic waves, ion-cyclotron waves, quasimodes, magnetosonic waves, and Alfven waves. In particular, Rozina *et al.*[24] have recently employed the QHD equations to analyze parametric instability existing due to nonlinear interaction between quantum upper hybrid waves and the quantum lower hybrid, ion-cyclotrons, and Alfven waves to studying three wave decay and modulation instabilities. The authors[25] have examined the impact of exchange-correlation effects on the profiles of nonlinear quantum ion-acoustic waves and an analysis of exchange correlations with boundary effects on semi bounded dense plasma has been carried out in a quantum dense plasma.[26]

[a)]E-mail: plasmaphysics07@gmail.com

Following the drift kinetic and fluid descriptions, the effects of electron density and temperature were studied on the parametric instability during the propagation of lower hybrid waves in tokamak.[27] The parametric decay of Alfven waves in solar wind was explored both in turbulent and non-turbulent plasmas.[28] Simulation studies were also carried out by using the ideal MHD module of PLUTO code to identify the existence of instability in those plasmas. Laboratory observations were performed to show parametric shear Alfven wave instability.[29] It has been experimentally observed that intense high-frequency electric fields indeed excite the parametric instabilities of electron and ion waves resulting in the heating of plasma.[30,31] More specifically, the dissipation of electromagnetic energy into a plasma by damping of electrostatic waves and/or anomalous absorption[32] leads to heating the plasma with significant high temperatures. Physically, parametric instabilities contribute to coronal heating,[33] the observed spectrum and cross helicity of solar wind turbulence[34] as well as the damping of fast magnetosonic waves in fusion plasmas.[35,36] Thus, recognizing the importance of parametric instabilities in different plasmas, we extend the work[24] to include statistical Fermi pressure that is consistent with degenerate dense environments, like white dwarfs, neutron stars, and magnetars.

In this model, we investigate[24] nonlinear behavior of parametrically coupled high-frequency degenerate upper hybrid waves (DUHWs) with ion cyclotron and Alfven waves to showing the impact of Fermi pressure on three wave decay and modulational instabilities. We therefore derive various nonlinear dispersion relations and calculate the corresponding parametric growth rates with quantum settings. The plasma under consideration is comprised of degenerate electrons obeying the quantum-mechanical degenerate Fermi distribution and classical ions. Since the electrons are lighter than ions such that their thermal De Broglie wavelength is larger than that of ions as well. Hence, the quantum behavior of electrons is reached faster than the ions. The equation of state describing the degenerate electrons in terms of Fermi pressure law is given by

$$P_{Fe} = \frac{m_e v_{Fe}^2 n_e^3}{3n_{e0}}, \tag{1}$$

where the electron mass is denoted by m_e and n_e represents the total density of Fermi electrons with equilibrium state n_{e0}.$v_{Fe}(= 2K_BT_{Fe}/m_e)^{1/2}$ being the electron Fermi velocity with Fermi temperature[37] as

$$T_{Fe} = \frac{\hbar^2 n_e^{\frac{2}{3}}(3\pi^2)^{\frac{2}{3}}}{2m_e K_B}. \tag{2}$$

This shows that Fermi temperature is dependent on the equilibrium electron density and an electron energy distribution becomes a step-like function in the degenerate limit (i.e., absolute temperature $T = 0$).[38]

II. DEGENERATE UPPER HYBRID WAVES (DUHWs)

To investigate the properties of nonlinear propagation of electrostatic degenerate upper hybrid waves (DUHWs), we assume that the external magnetic is applied along the z–axis, i.e., $B_0\hat{z}$, where B_0 is the strength of magnetic field and $\hat{z}$ is the unit vector in a Cartesian coordinates system. The electric field associated with DUHWs is $\mathbf{E} \approx \hat{x}E_{x0}\exp(ik_0x - i\omega_0 t) + c.c$ with $\mathbf{k}_0$ the wave number and ω_0 the wave frequency. For our purpose, we consider the basic QHD equations including the electron momentum equation in the presence of quantum corrections through the Fermi pressure, quantum Bohm potential, and exchange-correlation forces as

$$\frac{\partial \mathbf{V}_e^{(1)}}{\partial t} = -\frac{e}{m_e}\left\{\mathbf{E}^{(1)} + \frac{1}{c}\mathbf{V}_e^{(1)} \times \mathbf{B}^{(0)}\left(1 + \frac{\mathbf{B}^{(1)}}{\mathbf{B}^{(0)}}\right)\right\} - \frac{\nabla P_{Fe}}{m_e n_e^{(0)}} + \frac{\hbar^2}{4m_e^2 n_e^{(0)}}\nabla\nabla^2 n_e^{(1)} - v_{e,xc}\frac{\nabla n_e^{(1)}}{m_e n_e^{(0)}} \tag{3}$$

and the continuity equation for electrons

$$\frac{\partial n_e^{(1)}}{\partial t} + n_e^{(0)}\frac{\partial}{\partial x}(1 + N_S)V_{ex}^{(1)} = 0, \tag{4}$$

where $\mathbf{V}_e^{(1)}$ is the perturbed electron fluid velocity associated with DUHWs, $\hbar(= h/2\pi)$ being the scaled Planck constant, e is the magnitude of electronic charge, and c is the speed of light in vacuum, P_{Fe} represents the Fermi statistical pressure defined in Eq. (1). The third term on R.H.S is the Bohm potential showing the quantum diffraction effects and the last term of Eq. (3) pinpoints the electron exchange-correlation potential in terms of high density, which is one of the important quantum effects in dense quantum plasmas.[39] A general model that shows such exchange-correlation potentials is usually described by the density functional theory.[40] In the present study, we are looking at the interactions between the electrons governed by the electrostatic (Hartree) and exchange-correlation potentials, the normalized electron exchange-correlation potential in terms of complicated function of electron density is given as $v_{e,xc} = \frac{0.985e^2}{\epsilon}\left\{1 + \frac{0.034}{a_{Be}^* n_e^{1/3}}\ln\left(1 + 18.37a_{Be}^* n_{0e}^{1/3}\right)\right\}n_{0e}^{1/3}$, where $a_{Be}^*(= \epsilon\hbar^2/m_e e^2)$ represents the effective Bohr atomic radius and ϵ is the relative dielectric constant of the material, and $N_S(= \frac{n_e^{(1)}}{n_e^{(0)}})$ is the ratio of electron density oscillations-to-its equilibrium density at a slow timescale. Rewriting Eq. (3) into scalar components and solving together Eqs. (1), (2), and (4) into the x–component of the Poisson equation

$$\frac{\partial}{\partial x}E_x^{(1)} + 4\pi e n_e^{(1)} = 0, \tag{5}$$

we finally obtain a dispersion relation for DUHWs as

$$\left[\frac{\partial^2}{\partial t^2} + \omega_{UH}^2 + 2\Omega_{ce}^2\frac{B^{(1)}}{B^{(0)}} + \omega_{pe}^2 N_S + \left(\frac{\hbar^2}{4m_e^2}\frac{\partial^2}{\partial x^2}\nabla^2 - V_{e,xc}^2\nabla^2 - v_{Fe}^2\nabla^2\right)(1 + N_S)\right]E_x^{(1)} = 0, \tag{6}$$

where $\omega_{UH}^2 = \Omega_{ce}^2 + \omega_{pe}^2$ and $\nabla^2 = \nabla_x^2 + \nabla_z^2.\Omega_{ce}(\omega_{pe})$ is the electron gyrofrequency (electron plasma frequency). In deriving Eq. (6), we have utilized $\frac{v_{e,xc}\nabla n_e^{(1)}}{m_e n_e^{(0)}} = V_{e,xc}^2 \nabla \frac{n_e^{(1)}}{n_e^{(0)}}$ with $V_{e,xc}$ being the electron exchange-correlation speed, which can be defined as

$$V_{e,xc} = \frac{1}{m_e^{1/2}}\left\{\frac{0.985e^2}{\epsilon}\frac{1}{3}\left(n_e^{(0)}\right)^{1/3} + \frac{0.985e^2}{\epsilon}\frac{0.034}{a_{Be}^*} \times \frac{18.37a_{Be}^*\frac{1}{3}\left(n_e^{(0)}\right)^{1/3}}{1+18.37a_{Be}^*\left(n_e^{(0)}\right)^{1/3}}\right\}^{1/2}.$$

It is noticeable here that in the absence of density and magnetic field oscillations, i.e., $N_s = 0 = B^{(1)}$, Eq. (6) is reduced to $\left(\partial_t^2 + \omega_{UH}^2 + \frac{\hbar^2}{4m_e^2}\partial_x^2\nabla^2 - V_{e,xc}^2\nabla^2 - v_{Fe}^2\nabla^2\right)E_x^{(1)} = 0$, showing that the upper hybrid pump wave frequency is significantly modified by the electron Fermi pressure in addition to electron exchange-correlation effects. By seeking a plane wave solution, we may arrive at the dispersion relation in a quantum dense plasma as $\omega_0 = \left(\omega_{UH}^2 + \frac{\hbar^2}{4m_e^2}\ k_{x0}^2 k_0^2 + V_{e,xc}^2\ k_0^2\ \ + v_{Fe}^2 k_0^2\right)^{1/2}$, where $k_0 = (k_{x0}^2 + k_{z0}^2)^{1/2}$ represents the pump wave number. In the limit $k_{z0}^2 \ll k_{x0}^2$, the quantum pump frequency becomes as $\omega_0 = \left\{\omega_{UH}^2 + \frac{\hbar^2}{4m_e^2}k_0^4 + V_{e,xc}^2 k_0^2 + v_{Fe}^2 k_0^2\right\}^{1/2}$. By neglecting the electron Fermi speed $v_{Fe}^2 = 0$, we can easily retrieve the previous result[24] from Eq. (6).

III. DEGENERATE ION-CYCLOTRON WAVES (DICWs)

These are the electrostatic perturbation $(\Omega \simeq \Omega_{ci})$ that propagate almost perpendicular to the magnetic field in a degenerate dense magnetoplasma. On the ion time scales, the electrons are assumed to be massless (inertialess) particles $(\Omega \simeq k_z V_{Te})$ along the z–direction. The dynamics of electrons in the perpendicular direction is neglected here. To calculate the electrostatic potential associated with DICWs in the presence of upper-hybrid Ponderomotive force and electron Fermi pressure, we simplify the momentum equation for inertialess electrons using the charge-neutrality $(i.e., n_e^{(1)} = n_i^{(1)})$, as

$$-\frac{e^2}{4m_e\omega_{pe}^2}\nabla_z\langle|E_x^2|\rangle + e\nabla_z\phi + \frac{\hbar^2}{4m_e}\nabla_z\nabla^2 N_S - m_e V_{e,xc}^2\nabla_z N_S - \frac{2}{3}K_B T_{Fe}\nabla_z N_S = 0$$

or

$$\phi = \frac{e}{4m_e\omega_{pe}^2}\langle|E_x^2|\rangle - \frac{\hbar^2}{4m_e e}\nabla^2 N_S + \frac{m_e}{e}V_{e,xc}^2 N_S + \frac{2}{3}\frac{K_B T_{Fe}}{e}N_S. \quad (7)$$

Similarly, the dynamics of magnetized classical ions under the influence of electric and magnetic fields can be described by the ion equation of motion in component form as

$$\frac{\partial V_{ix}^{(1)}}{\partial t} = -\frac{e}{m_i}\nabla_x\phi - \Omega_{ci}V_{iy}^{(1)}, \quad (8)$$

since in our consideration, the electric field is along the x-axis, so

$$\frac{\partial V_{iy}^{(1)}}{\partial t} = -\Omega_{ci}V_{ix}^{(1)}, \quad (9)$$

and the x–component of the continuity equation is

$$\frac{\partial N_S}{\partial t} + \nabla_x V_{ix}^{(1)} = 0. \quad (10)$$

Now, solving together (8)–(10), we eventually arrive at

$$\left(\frac{\partial^2}{\partial t^2} + \Omega_{ci}^2\right)\hat{N}_s = \frac{e}{m_i}\frac{\partial^2}{\partial x^2}\phi. \quad (11)$$

Further elimination of ϕ from Eqs. (7) and (11) leads to a nonlinear dispersion relation of DICWs as

$$\left[\frac{\partial^2}{\partial t^2} + \Omega_{IC}^2\right]N_S = \frac{e^2}{4m_i m_e\omega_{pe}^2}\frac{\partial^2}{\partial x^2}\langle|E_x|^2\rangle, \quad (12)$$

where the degenerate ion-cyclotron frequency containing the ion-acoustic Fermi speed $C_F\left(=\frac{2}{3}\frac{K_B T_{Fe}}{m_i}\right)^{1/2}$ and given by

$$\Omega_{IC}^2 = \Omega_{ci}^2 + \frac{\hbar^2}{4m_e m_i}\frac{\partial^2}{\partial x^2}\nabla^2 - C_F^2\nabla^2. \quad (13)$$

The nonlinear term appears on the R.H.S of Eq. (12) due to the presence of upper-hybrid Ponderomotive force. If this force is neglected, then a modified dispersion relation of the ion-cyclotron waves is obtained by seeking the plane wave solution. Note that quantum electron density correlations and Fermi statistical pressure have strong dependence on the dispersion relation of DICWs.

IV. NONLINEAR INTERACTION OF DUHWs WITH DICWs

In this section, we study the amplitude modulation of the wave packet due to nonlinear interaction of electrostatic degenerate upper hybrid waves (DUHWs) and degenerate ion-cyclotron waves (DICWs). In our consideration, the electrons behave quantum mechanically and therefore the quantum effects owing to Fermi pressure, exchange-correlations, and quantum diffraction are entirely associated with electrons, while ions act as classical species. In order to derive a nonlinear dispersion relation for parametric instabilities in a degenerate electron-ion plasma, we consider nonlinear interaction of high-frequency pump $(\omega_{0,}k_0)$ DUHWs having the electric field oscillations $E_x = E_{x0}\exp(i\mathbf{k}_0.\mathbf{r} - i\omega_0 t) + c.c$ with low-frequency electrostatic DICWs (Ω, k) having the density fluctuations $N_S = \hat{N}_S\exp(i\mathbf{k}.\mathbf{r} - i\Omega t)$. This would produce two upper hybrid sidebands, i.e., $E_{x\pm}\exp(i\mathbf{k}_\pm.\mathbf{r} - i\omega_\pm t)$ involving the frequencies $\omega_\pm = \Omega \pm \omega_0$ and wave numbers $k_\pm = k \pm k_0$. Assuming the magnetic field oscillations to be absent $B^{(1)} = 0$ and following the standard method,[23] we then apply the Fourier transform and collect the same phasor terms on both sides of Eq. (6) to obtain

$$C_{\pm}E_{x\pm} = \hat{N}_S \omega_{pe}^2 E_{x0\pm}, \tag{14}$$

where

$$C_{\pm} = \left(\omega_{\pm}^2 - \omega_{UH}^2 - \frac{\hbar^2}{4m_e^2}k_{x\pm}^2 k_{\pm}^2 - V_{e,xc}^2 k_{\pm}^2 - v_{Fe}^2 k_{\pm}^2\right). \tag{15}$$

Similarly, the Fourier transforms and matching of phasor terms on both sides of Eq. (12) lead to

$$\left[\Omega^2 - \Omega_{IC}^2\right]\hat{N}_S = \frac{k_x^2}{16 m_i \pi n_e^{(0)}}\langle |E_{x0}E_{x-} + E_{x0}^{\star}E_{x0+}|\rangle. \tag{16}$$

Taking into account the inequality $\Omega \ll \omega_0$ for high-frequency pump wave, Eq. (15) will become as

$$C_{\pm} = \pm 2\omega_0[\Omega \mp \alpha - \beta], \tag{17}$$

where

$$\omega_0^2 = \omega_{UH}^2 + \frac{\hbar^2}{4m_e^2}\left(k_{x0}^2 k_0^2\right) + (V_{e,xc}^2 + v_{Fe}^2)k_0^2,$$

$$\alpha = \left\{\frac{\hbar^2}{8m_e^2\omega_0}\left(k_x^2 k_0^2 + k_{x0}^2 k^2 + k_x^2 k_0^2 + 4k_x k_{x0}\mathbf{k}.\mathbf{k}_0\right) + \frac{1}{2\omega_0}(V_{e,xc}^2 + v_{Fe}^2)(k^2 + k_0^2)\right\},$$

$$\beta = \left(\frac{\hbar^2}{4m_e^2\omega_0}k_x k_{x0}\left(k^2 + k_0^2\right) + \left(k_x^2 + k_{x0}^2\right)\mathbf{k}.\mathbf{k}_0 + \frac{1}{\omega_0}(V_{e,xc}^2 + v_{Fe}^2)\mathbf{k}.\mathbf{k}_0\right). \tag{18}$$

It is noticeable here that the upper-hybrid frequency ω_0 is significantly affected by the Fermi speed, whereas the modified frequency shifts are appearing due to nonlinear coupling of degenerate upper hybrid waves with degenerate ion cyclotron waves. Furthermore, we may obtain from Eqs. (14) and (16), the nonlinear dispersion relation for parametrically coupled DUHWs and DICWs as

$$\Omega^2 - \Omega_{IC}^2 = \frac{\omega_{pe}^2 k_x^2 |E_{x0}|^2}{16 m_i \pi n_e^{(0)}}\sum_{+,-}\frac{1}{C_{\pm}}. \tag{19}$$

Equation (19) appears as a consequence of nonlinear interaction of high frequency DUHWs with low-frequency DICWs and is the required dispersion equation to investigate both three wave decay and modulation instabilities. To calculate the nonlinear damping of three wave decay interaction, we consider the lower sideband C_- to be resonant, while the upper sideband C_+ is assumed to be off-resonant, then in this case Eq. (19) may be reduced to

$$(\Omega^2 - \Omega_{IC}^2)(\Omega + \alpha - \beta) = -\frac{\omega_{pe}^2 k_x^2 |E_{x0}|^2}{32\omega_0 m_i \pi n_e^{(0)}}. \tag{20}$$

Then by using the coinciding roots $\Omega = \Omega_{IC} + i\gamma_{IC}$ and $\Omega = \beta - \alpha + i\gamma_{IC}$ where $\Omega_{IC} \gg i\gamma_{IC}$, we may obtain from Eq. (20) as

$$\gamma_{IC} = \frac{\omega_{pe}k_x|E_{x0}|}{8\sqrt{\Omega_{IC}\omega_0 m_i \pi n_e^{(0)}}}. \tag{21}$$

This is the growth rate of three wave decay instability which has strong dependence on Fermi statistical speed through the parameter Ω_{IC}, i.e., the increase in Fermi speed leads to the reduction of the growth rate of three wave decay instability. However, the growth rate is directly proportional to the strength of electric field involving the DUHWs that is quite evident from the above expression. Next, for modulational instability, both the upper and lower sidebands $C_{\pm}$ are assumed to be resonant, then one may simply obtain from Eq. (19)

$$\left(\Omega^2 - \Omega_{IC}^2\right)\left(\Omega^2 + \beta^2 - 2\Omega\beta - \alpha^2\right) = \frac{\omega_{pe}^2 k_x^2 |E_{x0}|^2}{16\omega_0 m_i \pi n_e^{(0)}}\alpha. \tag{22}$$

Further if we assume $\Omega \gg \beta$, the growth rate of modulational instability γ_{mIC} may be reduced to

$$\gamma_{mIC} = \left(\frac{\omega_{pe}^2 k_x^2 \alpha}{16\omega_0 m_i \pi n_e^{(0)}}\right)^{\frac{1}{4}}|E_{x0}|^{\frac{1}{2}}. \tag{23}$$

Note that modulational growth rate instability γ_{mIC} is a strong function of electric field of DUHWs and Fermi speed through the parameter α.

V. DEGENERATE ALFVEN WAVES (DAWs)

Here to study the nonlinear dispersion relation of degenerate Alfven waves in a magnetized degenerate electron-ion plasma accounting for Fermi statistical pressure, we consider the momentum equations of inertialess degenerate electrons and classical mobile ions, respectively, as

$$-e\left\{E^{(1)} + \frac{1}{c}\left(V_e^{(1)} \times B^{(0)}\right)\right\} - \frac{e^2}{4m_e}\nabla\frac{\omega_H^2}{\omega_{pe}^4}\langle|E_x|^2\rangle + \frac{\hbar^2}{4n_e^{(0)}m_e}\nabla\nabla^2 n_e^{(1)} - v_{e,xc}\frac{\nabla n_e^{(1)}}{n_e^{(0)}} - \frac{2}{3}K_B T_{Fe}\nabla\frac{n_e^{(1)}}{n_e^{(0)}} = 0 \tag{24}$$

and

$$m_i\frac{\partial V_i^{(1)}}{\partial t} = e\left[E^{(1)} + \frac{1}{c}\left(V_i^{(1)} \times B^{(0)}\right)\right]. \tag{25}$$

Adding Eqs. (24) and (25) and expressing the total current density as $J^{(1)} = e(n_i^{(0)}V_i^{(1)} - n_e^{(0)}V_e^{(1)})$, also by making use of the Maxwell equation $\nabla \times B^{(1)} = \frac{4\pi J^{(1)}}{c}$ and quasineutrality condition, $n_i^{(0)} \approx n_e^{(0)}$ to finally arrive at

$$\frac{\partial V_i^{(1)}}{\partial t} = -\frac{1}{4\pi m_i n_i^{(0)}}B^{(0)} \times \left(\nabla \times B^{(1)}\right) - \frac{e^2}{4m_e m_i}\nabla\frac{\omega_H^2}{\omega_{pe}^4}\langle|E_x|^2\rangle + \frac{\hbar^2}{4m_e m_i n_e^{(0)}}\nabla\nabla^2 n_e^{(1)} - \frac{m_e}{m_i}V_{e,xc}^2\nabla\frac{n_e^{(1)}}{n_e^{(0)}} - C_F^2\nabla\frac{n_e^{(1)}}{n_e^{(0)}}, \tag{26}$$

where $V_A = \frac{B^{(0)}}{\sqrt{4\pi m_i n_i^{(0)}}}$ shows the Alfven speed and $\frac{v_{e,xc}\nabla n_e^{(1)}}{m_e n_e^{(0)}} = V_{e,xc}^2 \frac{\nabla n_e^{(1)}}{n_e^{(0)}}$. Further using the x–component of Eq. (26) into Eq. (10), which gives us

$$\left(\frac{\partial^2}{\partial t^2} - V_{eff}^2 \frac{\partial^2}{\partial x^2}\right) N_S = \frac{e^2}{4m_e m_i}\frac{\omega_H^2}{\omega_{pe}^4}\frac{\partial^2}{\partial x^2}\langle |E_x|^2\rangle, \tag{27}$$

where

$$V_{eff} = \left(V_A^2 - \frac{\hbar^2}{4m_e m_i}\nabla^2 + C_F^2\right)^{1/2} \tag{28}$$

is the effective Alfven speed influenced by the quantum diffraction and electron degenerate pressure effects. It is important to note here that in deriving Eq. (28), we have made the use of the frozen-in field condition as $\left(\frac{n_e^{(1)}}{n_e^{(0)}}\right) = \left(\frac{B^{(1)}}{B^{(0)}}\right) \equiv N_S$. It is also clear from Eq. (28) that in the absence of nonlinear interaction of the degenerate upper hybrid pump wave, one may obtain the dispersion relation of degenerate Alfven waves, as

$$\Omega^2 = V_{eff}^2 k_x^2 \equiv \left(V_A^2 - \frac{\hbar^2\nabla^2}{4m_e m_i} + C_F^2\right) k_x^2, \tag{29}$$

whereas for classical regime ($\hbar^2 \to 0$) and $C_F = 0$, one can easily retrieve from Eq. (29) the dispersion equation of the usual Alfven waves in the case of electron-ion plasma. Equations (6) and (27) are the governing equations for investigating nonlinear interaction between DUHWs and DAWs in dense quantum magnetoplasma.

VI. NONLINEAR INTERACTION OF DUHWs WITH DAWs

The nonlinear interaction between the degenerate upper-hybrid pump wave and degenerate Alfven waves can be studied by solving Eqs. (6) and (27), whereas in this case the pump wave contains the oscillating magnetic field as well through the frozen field condition. Accordingly, by using the Fourier transformation and after matching the phasors, we can express Eqs. (6) and (27), respectively, as

$$C_\pm E_{x\pm} = \left(\omega_{pe}^2 + 2\Omega_{ce}^2\right)\hat{N}_S E_{xo\pm} \tag{30}$$

and

$$\left[\Omega^2 - V_{eff}^2 k_x^2\right]\hat{N}_S = \frac{e^2 k_x^2 \omega_H^2}{4m_i m_e \omega_{pe}^4}\left(E_{x0}E_{x-} + E_{x0}^\star E_{x+}\right), \tag{31}$$

where $E_{x0+} = E_{x0}$ and $E_{x0-} = E_{x0}^*$, and the upper and lower sidebands $C_\pm$ are defined in Eq. (15). Similarly, we solve Eqs. (30) and (31) along with (15) to eventually obtain the nonlinear dispersion equation, as

$$\left[\Omega^2 - V_{eff}^2 k_x^2\right] = \frac{e^2 k_x^2 \omega_{UH}^2\left(\omega_{pe}^2 + 2\Omega_{ce}^2\right)}{4m_i m_e \omega_{pe}^4}|E_{x0}|^2 \sum_{+,-}\frac{1}{C_\pm}. \tag{32}$$

This is the dispersion relation for parametrically coupled DUHWs and Alfven waves in a degenerate magnetoplasma. For the three wave decay interaction, the lower sideband C_- is assumed to be resonant and the upper sideband C_+ is assumed to be off-resonant, and thus, one may obtain from Eq. (32)

$$\left[\Omega^2 - V_\alpha^2 k_x^2\right]([\Omega + \alpha - \beta]) = -\frac{e^2 k_x^2}{8m_i m_e}\frac{\omega_H^2\left(\omega_{pe}^2 + 2\Omega_{ce}^2\right)}{\omega_0\omega_{pe}^4}|E_{x0}|^2. \tag{33}$$

Next, for the growth rate, we suppose $\Omega = V_\alpha k_x + i\gamma_{AL}$ and $\Omega = \beta - \alpha + i\gamma_{AL}$ to obtain

$$\gamma_A = \frac{e\left(\omega_{pe}^2 + 2\Omega_{ce}^2\right)^{1/2}\omega_{UH}}{4\omega_{pe}^2}|E_{x0}|\sqrt{\frac{k_x}{V_{eff}\,\omega_0 m_i m_e}}, \tag{34}$$

which clearly reflects that the growth rate of three wave decay instability is a function of magnetic field, while it is an inverse square function of effective Alfven speed V_{eff}. For modulational instability, the upper and lower sidebands $C_\pm$ are both to be assumed resonant; thus Eq. (32) gives

$$\left(\Omega^2 - V_\alpha^2 k_x^2\right)\left((\Omega - \beta)^2 - \alpha^2\right) = \frac{e^2 k_x^2 \omega_H^2\left(\omega_{pe}^2 + 2\Omega_{ce}^2\right)}{4m_i m_e \omega_{pe}^4\omega_0}|E_{x0}|^2\alpha. \tag{35}$$

Assuming that $\Omega \gg \beta$, we can obtain the growth rate of modulational instability, as

$$\gamma_{mA} = \left(\frac{e^2 k_x^2}{4m_i m_e}\frac{\omega_{UH}^2\left(\omega_{pe}^2 + 2\Omega_{ce}^2\right)}{\omega_{pe}^4\omega_0}\alpha\right)^{1/4}|E_{x0}|^{1/2}. \tag{36}$$

This shows that the growth rate associated with modulational instability in a magnetized dense plasma is strongly influenced by the perpendicular electric field owing to DUHWs and the electron Fermi speed through the parameter α. Thus, Eqs. (21), (23), (34), and (36) are the main results that can be solved numerically to ascertain that the growth rates of three wave decay and modulation instabilities have strong electric field dependence $E_{x0}(= 10^{10}\ \mathrm{V/m})$ involving the DUHWs[41] that are scattered by the nonresonant electron density perturbations.

VII. RESULTS AND DISCUSSION

To show the impact of quantum corrections owing to Fermi pressure and electron exchange-correlations on the growth rates of three wave decay and modulational instabilities, we solve numerically Eqs. (21) and (23) arising from the nonlinear interaction of DUHWs with DICWs as well as Eqs. (34) and (36) resulting from the nonlinear interaction DUHWs with DAWs. We also choose some typical parameters in cgs units such as $n_0 \simeq 10^{25}$–$10^{27}, B_0 \simeq 10^6$, and $T_F \simeq 1.94 \times 10^6$ consistent with white dwarfs, magnetars, and interiors of neutron stars.[42] For graphical representation,

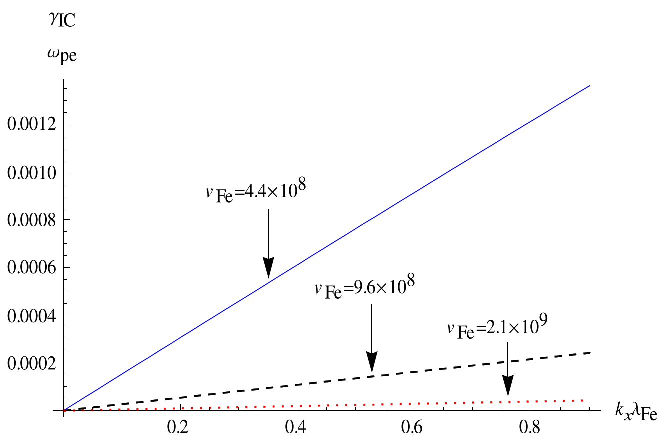

FIG. 1. The normalized growth rate ($\tilde{\gamma}_{IC}$) of the three wave decay instability [by scaling Eq. (21)] is plotted against the normalized wave number $\tilde{k}_x$ for different values of electron density $n_{e0} = 10^{25}\,\mathrm{cm}^{-3}$ (blue solid line), $10^{26}\,\mathrm{cm}^{-3}$ (black dashed line), and $10^{27}\,\mathrm{cm}^{-3}$ (red dotted line). These values correspond to electron Fermi speeds $v_{Fe} \sim 4.4\times10^{8}$ cm/s, 9.6×10^{8} cm/s, and 2.1×10^{9} cm/s, respectively. The electric field is fixed at value $E_{x0} = 0.3\times10^{6}$statvolt/cm.

Eq. (21) can be normalized by the scaled parameters, as $\tilde{k} = k\lambda_{Fe}, \tilde{k}_0 = k_0\lambda_{Fe}, \tilde{k}_x = k_x\lambda_{Fe}, \tilde{c}_F = \frac{c_F}{v_{Fe}}, \tilde{v}_{XC} = \frac{v_{XC}}{v_{Fe}}, \tilde{\omega}_0 = \frac{\omega_0}{\omega_{pe}}, \tilde{\Omega}_{IC} = \frac{\Omega_{IC}}{\omega_{pe}}$, and $\tilde{\gamma}_{IC} = \frac{\gamma_{IC}}{\omega_{pe}}$. It may be noticed from Fig. 1 that when the electron density increases, the associated electron Fermi temperature, electron Fermi speed, and electron exchange-correlation speeds would also increase and consequently, the magnitude of the growth rate due to three wave decay instability reduces. Figure 2 displays that how the variation of electron Fermi pressure via the electron density modifies the growth rate of modulational instability ($\tilde{\gamma}_{mIC}$) as a function of normalized wave number ($\tilde{k}_x$). However, Figs. 3 and 4 represent the growth rates of the three wave decay and modulational instabilities produced due to the parametric coupling of DUHWs with DAWs. Note that growth rate instabilities decrease by increasing the electron density concentration. Additionally, the growth rates of these instabilities have strong dependence on the strength of electric field.

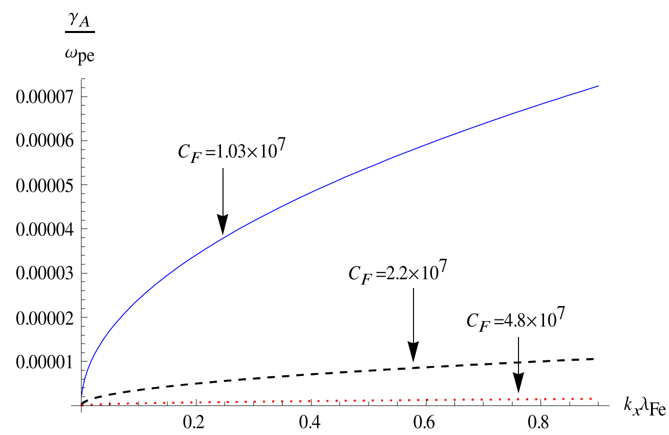

Fig. 3

FIG. 3. The normalized growth rate $\tilde{\gamma}_A (= \gamma_A/\omega_{pe})$ of the three wave decay instability [by normalizing Eq. (34)] is plotted against the normalized wave number $\tilde{k}_x$ for different values of ion-acoustic Fermi speed $C_F \sim 1.03\times10^{7}$ cm/s (blue solid curve), 2.2×10^{7} cm/s (black dashed curve), and 4.8×10^{7} cm/s (red dotted curve). These values correspond to the electron density values $n_{e0} = 10^{25}\,\mathrm{cm}^{-3}$, $10^{26}\,\mathrm{cm}^{-3}$, and $10^{27}\,\mathrm{cm}^{-3}$, respectively, with a fixed electric field strength $E_{x0} = 0.3\times10^{6}$ statvolt/cm.

VIII. CONCLUSION

To conclude, we have examined the effects of Fermi pressure and exchange-correlations on the profiles of three wave decay and modulational instabilities in an electron-ion degenerate magnetoplasma. Following the standard techniques, we have obtained the nonlinear dispersion equations for DUHWs, DICWs, and DAWs in terms of density and magnetic field fluctuations. High-frequency DUHWs are nonlinearly coupled with DICWs and DAWs which lead to three wave decay and modulational instabilities. It is worthwhile to mention that quantum Bohm potential represents the density fluctuations at quantum scales and introduces a dispersion term to modify the frequencies of DUHWs, DICWs, and DAWs. On the other hand, Fermi pressure can be coupled with the exchange-correlation terms to significantly alter the growth rates of three wave decay and

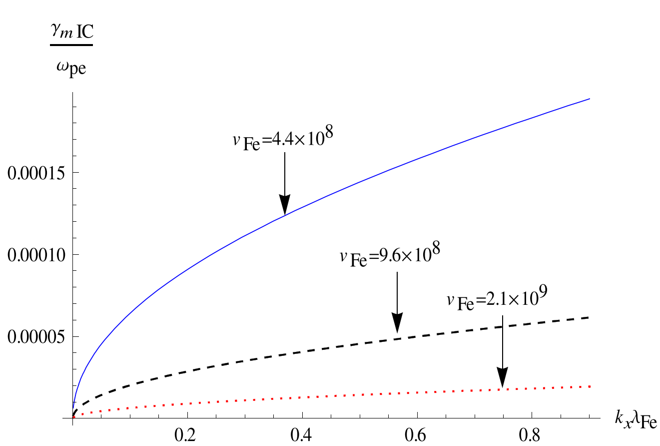

Fig. 2

FIG. 2. The normalized growth rate ($\tilde{\gamma}_{mIC}$) of the modulational instability [by normalizing Eq. (23)] is plotted against the normalized wave number $\tilde{k}_x$ for different values of electron density $n_{e0} = 10^{25}\,\mathrm{cm}^{-3}$ (blue solid curve), $10^{26}\,\mathrm{cm}^{-3}$ (black dashed curve), and $10^{27}\,\mathrm{cm}^{-3}$ (red dotted curve). Other parameters are the same as in Fig. 1.

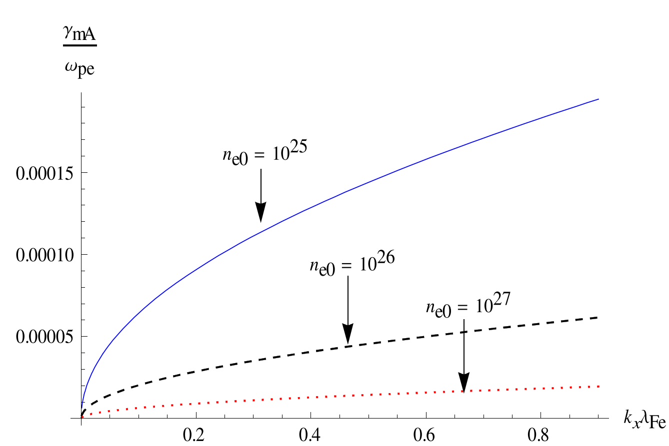

Fig. 4

FIG. 4. The normalized growth rate $\tilde{\gamma}_{mA} (= \gamma_{mA}/\omega_{pe})$ of the modulation instability [by normalizing Eq. (36)] is plotted against the normalized wave number $\tilde{k}_x$ for different values of electron density $n_{e0} = 10^{25}\mathrm{cm}^{-3}$ (blue solid curve), $10^{26}\,\mathrm{cm}^{-3}$ (black dashed curve), and $10^{27}\,\mathrm{cm}^{-3}$ (red dotted curve). Other parameters are the same as Fig. 3.

modulational instabilities. Nonlinear coupling between DUHWs and DICWs gives rise to three wave decay instability, which is an inverse square function of Fermi pressure through parameter Ω_{IC}, whereas modulation instability is the direct function of Fermi pressure through parameter α. Similar results are obtained for nonlinear coupling between DUHWs and DAWs. The present results provide effective information on density fluctuations that may be relevant in microplasmas and micro mechanical systems, like semiconductors, quantum diodes, etc., as well as in astrophysical plasmas like neutron stars, white dwarf, magnetars, etc. Since the electron density in semiconductors is much smaller as compared to metals, therefore, in many electronic devices, the de Broglie wavelength involving the electrons is assumed to be of the order of spatial variation of the doping. Consequently, tunneling effects at quantum scales become important in ultrasmall electronic devices. However, in astrophysical and cosmological environments, the density of electrons is extremely high which strongly modifies the Fermi temperature and pressures, playing a vital role in the dynamical study of waves.[7]

[1]P. A. Markowich, C. A. Ringhofer, and C. Schmeiser, *Semiconductor Equations* (Springer-Verlag, New York, 1990).
[2]G. V. Shpatakovskaya, J. Exp. Theor. Phys. **102**, 466 (2006).
[3]L. K. Ang, T. J. T. Kwan, and Y. Y. Lau, Phys. Rev. Lett. **91**, 208303 (2003); L. K. Ang, IEEE Trans. Plasma Sci. **32**, 410 (2004); L. K. Ang, W. S. Koh, Y. Y. Lau, and T. J. T. Kwan, Phys. Plasmas **13**, 056701 (2006); L. K. Ang and P. Zhang, Phys. Rev. Lett. **98**, 164802 (2007).
[4]T. C. Killian, Nature (London) **441**, 297 (2006).
[5]K. H. Backer, K. H. Schoenback, and J. G. Eden, J. Phys. D: Appl. Phys. **39**, R55 (2006).
[6]W. L. Barnes, A. Dereux, and T. W. Ebbesen, Nature (London) **424**, 824 (2003); D. E. Chang, A. S. Sørensen, P. R. Hemmer, and M. D. Lukin, Phys. Rev. Lett. **97**, 053002 (2006).
[7]G. Chabier, F. Douchin, and A. Y. Potekhin, J. Phys.: Condens. Matter **14**, 9133 (2002).
[8]Y. D. Jung, Phys. Plasmas **8**, 3842 (2001); M. Opher, L. O. Silva, D. E. Dauger, V. K. Decyk, and J. M. Dawson, Phys. Plasmas **8**, 2454 (2001); S. H. Glenzer, O. L. Landen, and P. Neumayer, Phys. Rev. Lett. **98**, 065002 (2007).
[9]G. Barak, H. Steinberg, L. N. Pfeiffer, K. W. West, L. Glazman, F. von Oppen, and A. Yacoby, Nat. Phys. **6**, 489 (2010).
[10]G. Andrew Truscott, E. Kevin Strecker, W. I. McAlexander, G. Partridge, and R. G. Hulet, "Observation of Fermi pressure in a gas of trapped atoms," Science **291**, 2570–2572 (2001).
[11]A. Luque, H. Schamel, and R. Fedele, Phys. Lett. A **324**, 185 (2004); P. K. Shukla, *ibid.* **352**, 242 (2006).
[12]B. Shokri and S. M. Khorashady, Pramana **61**, 1 (2003).
[13]P. K. Karmakar and H. P. Goutam, Phys. Plasmas **23**, 112121 (2016).
[14]H. P. Goutam and P. K. Karmakar, IOP Conf. Ser.: J. Phys.: Conf. Ser. **836**, 012004 (2017).
[15]P. K. Shukla, B. Eliasson, and L. Stenflo, Phys. Plasmas **19**, 072302 (2012).
[16]G. Manfredi, Fields Inst. Commun. **46**, 263 (2005).
[17]P. K. Shukla and B. Eliasson, Phys. Rev. Lett. **96**, 245001 (2006).
[18]C. L. Gardner and C. Ringhofer, Phys. Rev. E **53**, 157 (1996).
[19]P. K. Shukla and B. Eliasson, New J. Phys. **9**, 98 (2007).
[20]O. P. Sah and J. Manta, Phys. Plasmas **16**, 032304 (2009).
[21]L. G. Garcia, F. Haas, L. P. L. de Oliveira, and J. Goedert, Phys. Plasmas **12**, 012302 (2005).
[22]M. Marklund, Phys. Plasmas **12**, 082110 (2005).
[23]R. P. Sharma and P. K. Shukla, Phys. Fluids **26**, 87 (1983).
[24]Ch. Rozina, S. Ali, N. Maryam, and N. Amina, Phys. Plasmas **24**, 102312 (2017).
[25]K. Ourabah and M. Tribeche, Phys. Rev. E **88**, 045101 (2013).
[26]H. Cai-Xia and X. Ju-Kui, Chin. Phys. B **22**, 025202 (2013); Y.-T. Ma, S.-H. Mao, and J.-K. Xue, Phys. Plasmas **18**, 102108 (2011).
[27]Z. Aihui and G. Zhe, Nucl. Fusion **53**, 083015 (2013).
[28]S. Mijie, L. Hui, X. Chijie Xiao, and W. Xiaogang, Astrophys. J. **842**, 63 (2017).
[29]S. Dorfman and T. A. Carter, Phys. Rev. Lett. **116**, 195002 (2016).
[30]H. Eubank, Phys. Fluids **14**, 2551 (1972).
[31]R. P. H. Chang, M. Porkola, and B. Grek, Phys. Rev. Lett. **28**, 206 (1972).
[32]J. M. Dawson and C. Oberman, Phys. Fluids **6**, 394 (1963).
[33]F. Pruneti and M. Velli, in *Proceedings of the Fifth SOHO Workshop: The Corona and Solar Wind near Minimum Activity*, edited by A. Wilson, ESA Special Publication Vol. 404 (European Space Agency, Noordwijk, 1997), p. 623.
[34]B. Inhester, J. Geophys. Res. **95**, 10525, https://doi.org/10.1029/JA095iA07p10525 (1990).
[35]J. H. Lee, W. A. Peebles, E. F. Jaeger, E. J. Doyle, N. C. Luhmann, C. C. Petty, R. I. Pinsker, R. Prater, and T. L. Rhodes, Phys. Rev. Lett. **80**, 2330 (1998).
[36]T. Oosako, Y. Takase, A. Ejiri, Y. Nagashima, Y. Adachi, H. Kasahara, T. Yamada, O. Watanabe, H. Tojo, S. Kainaga, J. Sugiyama *et al.*, Nucl. Fusion **49**, 065020 (2009).
[37]L. D. Landau and E. M. Lifshitz, *Statistical Physics, Part 1* (Butterworth-Heinemann, Oxford, 1998); N. L. Tsintsadze and L. N. Tsintsadze, Europhys. Lett. **88**, 35001 (2009).
[38]P. Misra and A. R. Chowdhury, Phys. Plasmas **13**, 072305 (2006); W. F. El-Taibany and M. Wadati, Phys. Plasmas **14**, 042302 (2007).
[39]F. Brosens and J. Devreese, Phys. Rev. B **29**, 543 (1984); S.-H. Mao and J.-K. Xue, Phys. Scr. **84**, 055501 (2011).
[40]L. Brey, J. Dempsey, N. F. Johnsen, and B. I. Halperin, Phys. Rev. B **42**, 1240 (1990).
[41]K. Morawetz and A. P. Jauho, Phys. Rev. E **50**(1), 474 (1994).
[42]Ch. Rozina, N. L. Tsintsadze, and A. Khadija, Phys. Plasmas **24**, 083704 (2017).