Physica Scripta

PAPER

Cusp solitons in piezoelectric semiconductor plasmas

IOP Publishing Physica Scripta

Phys. Scr. **94** (2019) 045601 (6pp) https://doi.org/10.1088/1402-4896/aaf952

Cusp solitons in piezoelectric semiconductor plasmas

Amna Fayyaz[1], **H A Shah**[1], **M N S Qureshi**[1] **and Z Ehsan**[2]

[1] Department of Physics, Government College University, Lahore 54000, Pakistan

[2] Space & Plasma Astrophysics Research Group (SPAR), Department of Physics, COMSATS University Islamabad, Lahore Campus, 54000, Pakistan

E-mail: hashah.gcl@gmail.com

Received 11 July 2018, revised 23 November 2018
Accepted for publication 18 December 2018
Published 29 January 2019

Abstract

Using the semi-classical model, the piezoelectric coupling of Langmuir waves with lattice ion vibrations has been studied in linear as well as nonlinear regime in n-type piezoelectric semiconductors. It is shown that there is not any significant coupling in the linear regime. In the nonlinear regime, we have developed a set of coupled nonlinear evolution equations whose solution leads to cusp solitons. These equations are analyzed numerically to investigate the conditions for the significant coupling using the standard parameters for n-type piezoelectric semiconductor plasmas.

Keywords: piezoelectric semiconductors, cusp solitons, semiconductor plasmas

(Some figures may appear in colour only in the online journal)

1. Introduction

The piezoelectric effect was first observed in a variety of solid materials [1]. The reverse piezoelectric effect, that is the change in the dimensions of a crystals by applying the electric field, was later proposed on the basis of the thermodynamic principles and then experimentally confirmed by Curie brothers [2–4]. Voigt [5], developed the theory of elasticity and made two major contributions by pointing out the relation between piezoelectricity and crystal structure by giving the fundamental equations for crystal vibrations. He later found out the effect of internal electric field on the elastic stiffness of the medium and also expressed the electrical and mechanical states of a crystal as eighteen piezoelectric constants, as there are three possible components of electric polarization and six components of the stress. Kyame [6] studied the propagation of plane waves in piezoelectric crystals and illustrated that a simultaneous solution to mechanical-piezoelectric and Maxwell's field equations provide the propagation conditions in a medium. He worked out five-by-five secular determinant for any given direction corresponding to the coupling of three acoustic waves and two shear electromagnetic waves by virtue of the piezoelectricity. For a certain propagation direction and piezoelectric tensor, the acoustic waves accompanied by the longitudinal electric field can increase the elastic stiffness significantly. The solutions of this determinant gave three acoustic waves and two shear electromagnetic waves propagating with the speed of sound and speed of light, respectively. These velocities were found to be frequency independent. Depending on the considered mode and piezoelectric tensor, the acoustic waves generate an electric field. The longitudinal component of this electric field has a significant effect which has been observed in most of the piezoelectric materials. For the electromagnetic wave, its electric component may produce stress in the crystal. Since this forced acoustic wave traversing at speed of light has a very small amplitude so its effect is negligible on electromagnetic wave propagation. It was later found that the piezoelectric stiffening could be relaxed if electrical conductivity was taken into account [7].

Plasmons were first discovered in metals by Ruthemann [8] which introduced the study of solid state plasmas. After the seminal papers on collective effects or plasma like behavior in solids [9, 10], the detailed characteristics of the normal modes were investigated theoretically. It is extensively reported in literature [11–13] that the interplay between the acoustic waves and carriers lead to three novel effects due to the momentum and energy exchange between the carriers and lattice ions; these effects are (i) the amplification or attenuation of acoustic waves, (ii) the electrons will tend to shield out any electric field produced due to elastic deformation which will modify the elastic constant and thus the speed of sound and (iii) the electrons experience an additional force due to the transfer of momentum from the sound wave which gives rise to the so called electroacoustic field that causes the electroacoustic current in semiconductors.

Hutson [14] took a leap forward by discovering the piezoelectric effect in semiconductors and developed the linear theory for both the intrinsic and extrinsic semiconductors with the effects of diffusion, trapping and drift of carriers taken into account. It was argued that these electric fields generate the space charge and currents which leads to acoustic loss and dispersion. The effect of these results and directional properties were demonstrated for GaAs and CdS. A year later, the converse piezoelectric effect in piezoelectric semiconductor was studied to probe the amplification and attenuation properties of ultrasonics [15]. It was seen that an ultrasonic wave in a particular direction can be attenuated or amplified if a dc electric field is applied to the medium unlike the non-piezoelectric medium. Amplification was observed when the drift velocity of the carriers exceeded the characteristic sound velocity for CdS. However, gain was reduced at high frequencies due to the diffusion of carriers. This lead to the new arena of the ultrasonic studies as well as its applications [16].

Piezoelectric semiconductors since then have found widespread applications in industry and certain of their aspects have been experimentally very well investigated. From the point of view of MEMS and using piezoelectrics for harvesting energy have been very thoroughly investigated [17–19] and many of these findings have been reviewed in literature [20]. More recently, with advent of nanotechnology piezoelectric materials have found new applications which are being actively pursued [21, 22].

Durkan *et al* [23] (and references therein) have provided a comprehensive review of nonlinear piezoelectricity and pointed out its need keeping in view new theoretical understandings related to the nature of piezoelectric polarization and electric fields. We are, however, concerned here with the interplay of normal modes due to electromechanical coupling factor in the crystal. For plasma to interact with lattice waves (employing hydrodynamic or Vlasov model) it was asserted that the plasmons will resonate with lattice waves provided the frequencies are close to the characteristic plasma frequency if temperature is kept low enough [12]. The theoretical work was extended to incorporate nonlinear effects by Ridley and Wilkinson [24–26] to investigate the sound waves in the nonlinear regime by modifying the Krylov–Bogoluibov–Mitropolskii technique for nonlinear oscillations to illustrate the propagation nonlinear sound waves in piezoelectric semiconductor media [27]. This method is based on asymptotic expansions of two space and time scales segregating the slow change in amplitude which happens due to the nonlinear interactions with local density perturbations. The expressions for the nonlinear frequency shifts, wave vectors and growth rates were also obtained by incorporating the self-interaction and coupling with other linear modes existed in the crystals. The amplitudes and growth constants were said to be slowly varying functions of time and space. Nonlinearities sought to modify phase, amplitude as well as group structure owing to the energy exchange between the modes. The nonlinearity of acoustic waves was further probed by using the modified perturbative method [28] which yielded the evolution equation of wave envelope whose amplitude was found to satisfy the well-known nonlinear Schrodinger (NLS) equation having complex coefficients and cubic nonlinearity [29]. Furthermore, the modulational instability was studied using NLS equation having a solitary wave solution [30, 31]. For the plasmas, the NLS equation with the derivative of cubic nonlinearity was first derived for the circular polarized Alfven wave [32]. The stationary solutions of this derivative NLS give peculiar spiky soliton which exhibits a cusp at the crest in contrast to ordinary soliton [33] and explains nonlinear modulation of its field. The exact solutions of derivative NLS were found by Kaup and Newell [34]. This opened a new research avenue of studying the nonlinear mode coupling to obtain the ordinary, periodic wave trains and singular spiky soliton solutions [35].

As can be seen that investigations in the area of coupled mode nonlinear interactions have been few and far between, thus in the present work we consider the coupling between the lattice ions and electrons and then attempt to find the singular spiky soliton solutions in a piezoelectric semiconductor media for the first time by employing the semi-classical hydrodynamic model. We will consider the conditions and comparisons for their coupling in the linear as well as nonlinear regime. The layout of the manuscript is given as follows. In section 2, the basic theoretical framework is introduced followed by the linear dispersion relation for the coupled lattice ion waves with electron waves. In section 3, we carry out the nonlinear analysis. Section 4 deals with the solitary solutions of resulting nonlinear equations. In section 5, results and conclusions are discussed.

2. Theoretical formulation

In the present section we start by giving the mathematical formulation necessary for studying the coupled lattice-electron waves in an n-type piezoelectric semiconductors. We consider the following piezoelectric equations of state along with the one-dimensional hydrodynamic fluid equations for electrons [16, 35]

$$T = cS - \beta E, \tag{1}$$

$$D = \varepsilon E + \beta S, \tag{2}$$

$$\frac{\partial n_e}{\partial t} + \frac{\partial (n_e v_e)}{\partial x} = 0, \tag{3}$$

$$\frac{\partial v_e}{\partial t} + v_e \frac{\partial v_e}{\partial x} = -\frac{e}{m_e}E - \frac{1}{m_e n_e}\frac{\partial P_e}{\partial x}. \tag{4}$$

Here equations (1), (2) are the piezoelectric equations of state and equations (3), (4) are the electron's continuity and momentum equations, respectively; where T is the stress, c is the elastic constant, S is the strain, β is the piezoelectric constant, E is the electric field, D is the electric displacement,

ε is the relative permittivity, n is the number density, v_e is the velocity of electrons, e is the electronic charge, m_e is the effective electron mass which incorporates the quantum mechanical effects and the electron pressure is given as $P_e = n\,\kappa_b\,T_e$ where κ_b is the Boltzmann's constant and T_e is the electron's temperature. It should be noted that all the variables in equations (1) and (2) are scalar quantities as we have considered one-dimensional propagation (x-direction) only. The strain S may be written as $S = \partial u_i/\partial x$ where u_i is the physical displacements of lattice ions. This gives us the elastic wave equation for the medium as follows

$$\frac{\partial^2 u_i}{\partial t^2} = c_s^2\frac{\partial^2 u_i}{\partial x^2} - \frac{\beta}{\rho_i}\frac{\partial E}{\partial x}. \tag{5}$$

Here $c_s = \sqrt{c/\rho_i}$ is the speed of sound and ρ_i is the lattice ion density. Equation (2) yields the following modified Poisson equation which accounts for the effect of charge separation.

$$-\frac{e}{\varepsilon}n - \frac{\beta}{\varepsilon}\frac{\partial^2 u_i}{\partial x^2} = \frac{\partial E}{\partial x}. \tag{6}$$

From equations (3)–(6), using plane wave solution for all the perturbed quantities of the form $\sim e^{ikx - i\omega t}$ we derive the following linear dispersion relation for the coupled ion-electron mode due to piezoelectricity.

$$\left(\frac{\omega^2}{k^2} - \left(c_s^2 + \frac{\beta^2}{\varepsilon\,\rho_i}\right)\right)(\omega^2 - k^2 v_{Te}^2 - \omega_{pe}^2) = \frac{\omega_{pe}^2\beta^2}{\varepsilon\,\rho_i}, \tag{7}$$

where $v_{Te} = \sqrt{\kappa_B T_e/m_e}$ is the electron thermal velocity and $\omega_{pe} = \sqrt{e^2 n_0/m_e\varepsilon}$ is the electron plasma frequency. The first term on the left-hand side of equation (7) is the linear dispersion relation of lattice acoustic waves in a piezoelectric semiconductor and the second term is the linear dispersion of electron plasma waves. The term on the right-hand side of equation (7) is the coupling term. It is interesting to note that under the limiting case $\beta \to 0$ this dispersion relation decouples into the lattice acoustic mode and Langmuir mode.

Generally, the piezoelectric coupling constant β for such materials ranges from 0.045 to 0.35 C m^{-2} [36, 37]. We have used the parameters of InSb $n = 10^{21}/m^3$, $\varepsilon = 15.8$, $T = 77$ K, $v_{Te} = 2.9\times10^5$m s^{-1}, $\omega_{pe} = 3.7\times10^{12}$ rad s^{-1}, $\rho_i = 5.8\times10^3$ kg m^{-3}, $\beta = 0.054$ C m^{-2}, $c_s = 2500$ ms^{-1} and $m_e = 0.014\,m_0$ (m_0 is the rest mass of the electron). The right-hand side is a very small in comparison to the right-hand side of the equation (7), thereby signifying the in the linear regime the coupling between the lattice acoustic and electron plasma modes is not effective.

3. Nonlinear analysis

From the preceding section, it is clear that there is no effective piezoelectric coupling in the linear regime between the lattice and electron modes. Thus in this section we advance our problem to nonlinear regime where we expect that the coupling will be more effective. In order to investigate the coupling of two modes (lattice ions wave with electron wave) which excite at two different time scales owing to the ions and electron mass difference, we split all the variables into low 'l' and high 'h' frequency components. The inclusion of these two time scales makes it possible to derive the nonlinear evolution equation [35, 38]

$$n_e(x, t) = n_0 + n_{le}(x, t) + n_{fe}(x, t), \tag{8}$$

$$v_e(x, t) = v_{le}(x, t) + v_{fe}(x, t), \tag{9}$$

$$u_i(x, t) = u_{li}(x, t), \tag{10}$$

$$E(x, t) = E_l(x, t) + E_f(x, t). \tag{11}$$

It may be noted that the motion of the lattice ions is only slowly varying due to their large mass. However the electron motion has the freedom to respond at both time scales. Solving the equations (3), (4) and (6) for the high frequency part only, we get

$$\frac{\partial^2 E_f}{\partial t^2} + \omega_{pe}^2\left(1 + \frac{n_{le}}{n_0}\right)E_f - v_{Te}^2\frac{\partial^2 E_f}{\partial x^2} = 0. \tag{12}$$

We note here that the nonlinear effect is included in the second term via the slow variation of the electron number density n_{le}. Proceeding further in the standard manner, we write the fast-time scale electric field as [35, 38]

$$E_h(x, t) = \frac{1}{2}[\tilde{E}(x, t)e^{-i\,\omega_0 t} + \text{c.c.}], \tag{13}$$

where c.c. refers to the complex conjugate, $\tilde{E}(x, t)$ is slowly varying in time and assume that ($\omega_0 \approx \omega_{pe}$) for the long wavelength, small finite amplitude electron wave and thus approximate $\omega_0^2 - \omega_{pe}^2 = 2\Delta\omega$ and $\Delta\omega = \omega_0 - \omega_{pe}$. Since we have defined $\tilde{E}$ to be the slowly varying amplitude so we can neglect its second derivative in equation (12) to obtain the complex nonlinear evolution equation

$$i\frac{\partial\tilde{E}}{\partial t} + \frac{1}{2}\frac{v_{Te}^2}{\omega_0}\frac{\partial^2\tilde{E}}{\partial x^2} + \left(\Delta\omega - \frac{\omega_{pe}}{2n_0}n_{le}\right)\tilde{E} = 0. \tag{14}$$

This equation depicts the slow variation of local electron number density with the complex amplitude $\tilde{E}(x, t)$ of the rapidly oscillating electric field. Next, we consider the slow frequency part of electrons by averaging over the fast oscillations which gives us the following set of equations

$$\frac{\partial v_{le}}{\partial t} + \frac{e}{m_e}E_l + \frac{\kappa_B T_e}{m_e n_0}\frac{\partial n_{le}}{\partial x} + \frac{e^2}{4m_e^2\omega_{pe}^2}\frac{\partial\,|\tilde{E}|^2}{\partial x} = 0, \tag{15}$$

$$-\frac{e}{\varepsilon}n_{le} - \frac{\beta}{\varepsilon}\frac{\partial^2 u_i}{\partial x^2} = \frac{\partial E_l}{\partial x}, \tag{16}$$

$$\frac{\partial n_{le}}{\partial t} + n_0\frac{\partial v_{le}}{\partial x} = 0. \tag{17}$$

The last term on the rhs of equation (15) is the nonlinear ponderomotive force which acts to direct the particles away from the regions of high electric field and predominantly acts on the lighter particles at lower frequencies. This term basically arises from the convective derivative term as described by Thornhill and ter Haar [39]. Eliminating E_l and v_{le} from equations (15)–(17), we get

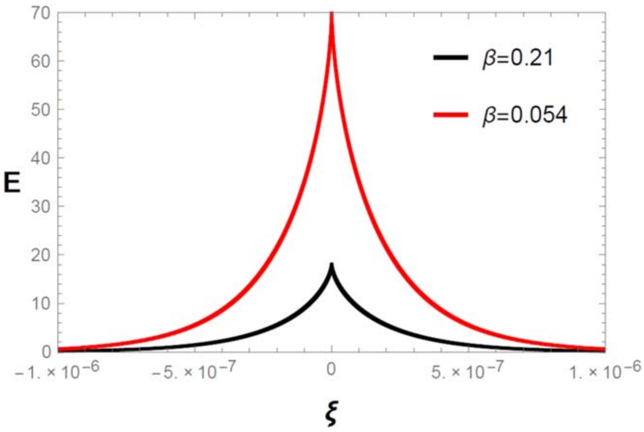

Figure 1. Cusp soliton with the variation of piezoelectric coupling constant.

$$\frac{\partial^2 n_{le}}{\partial t^2} - v_{Te}^2 \frac{\partial^2 n_{le}}{\partial x^2} + \omega_{pe}^2 n_{le} + \frac{\beta\omega_{pe}^2}{e}\frac{\partial^2 u_i}{\partial x^2} - \frac{\varepsilon}{4m_e}\frac{\partial^2 |\tilde{E}|^2}{\partial x^2} = 0. \quad (18)$$

The contribution of lattice ions on the slow scale dynamics is given by equations (5), (6) as

$$\frac{\partial^2 u_i}{\partial t^2} - \left(c_s^2 + \frac{\beta^2}{\varepsilon\ \rho_i}\right)\frac{\partial^2 u_i}{\partial x^2} = \frac{\beta e}{\varepsilon\ \rho_i} n_{le}. \quad (19)$$

The equations (14), (18) and (19) are the set nonlinear evolution equations coupling the electron wave with lattice ion wave in a piezoelectric medium.

4. Soliton solutions

In an attempt to find the solitary solution of the coupled nonlinear evolution equations equations (14), (18) and (19), we shall limit ourselves to traveling wave solutions which have been extensively studied in gaseous plasmas in comparison to other plasma like media. So, we define a co-moving frame given by $\xi = x - v_g t$ moving with velocity $v_g = \mathrm{d}\omega/\mathrm{d}k = v_{Te}^2 k/\omega_0$. Transforming the equation (14) by introducing $\tilde{E} \sim \mathrm{e}^{\mathrm{i}\Delta\omega t}$ where $\Delta\omega$ is defined above and further by eliminating u_i from equations (18), (19) to get

$$2\Delta\omega\tilde{E} + \frac{1}{2}\frac{\mathrm{d}v_g}{\mathrm{d}k}\frac{\partial^2\tilde{E}}{\partial\xi^2} - \frac{\omega_{pe}}{2n_0}n_{le}\tilde{E} = 0, \quad (20)$$

$$\left[(v_g^2 - v_{Te}^2)\frac{\partial^2}{\partial\xi^2} + \omega_{pe}^2 + \frac{\omega_{pe}^2\beta^2}{\rho_i\varepsilon(v_g^2 - c_s^2) - \beta^2}\right] n_{le} = \frac{\varepsilon}{4m_e}\frac{\partial^2 |\tilde{E}|^2}{\partial\xi^2}. \quad (21)$$

The equations (20) and (21) are the modified Zakharov equations for the electron waves coupled to the lattice ion vibrations due to piezoelectricity. The classical Zakharov equations reduces to nonlinear Schrodinger equation under the static limit which admits soliton solutions. Under this limit for our case, we consider $\partial^2/\partial t^2 - v_{te}^2\partial^2/\partial x^2 \ll \omega_{pe}^2 + \omega_{pe}^2\beta^2/\rho_i\varepsilon(v_g^2 - c_s^2) - \beta^2$ to obtain the expression for perturbed number density from equation (21) which comes out to be proportional to the second derivative of electric field strength

$$n_{le} = \frac{\varepsilon}{4m_e\omega_{pe}^2}\left[1 - \frac{\beta^2}{\rho_i\varepsilon\ (v_g^2 - c_s^2)}\right]\frac{\partial^2 |\tilde{E}|^2}{\partial\xi^2}. \quad (22)$$

It shows that the regions of higher amplitude of electric field correspond to highly depleted regions of local number density. Substituting this equation (22) in equation (14), we have

$$\frac{\partial^2\tilde{E}}{\partial\xi^2} - \frac{2\Delta\ \omega\ \omega_0}{v_{Te}^2}\tilde{E} - \frac{\varepsilon}{4m_e n_0 v_{Te}^2}\left[1 - \frac{\beta^2}{(v_g^2 - c_s^2)\rho_i\varepsilon}\right] \times \tilde{E}\frac{\partial^2 |\tilde{E}|^2}{\partial\xi^2} = 0. \quad (23)$$

Integrating it once and using the boundary condition that all the perturbations vanish at infinity i.e. $\tilde{E} \to 0$ as $\xi \to \infty$

$$\frac{\mathrm{d}\tilde{E}}{\mathrm{d}\xi} = \frac{\sqrt{2a}\ \tilde{E}}{(1 - 8b\tilde{E}^2)^{1/2}}, \quad (24)$$

where $a = \frac{2\Delta\omega\ \omega_{pe}}{v_{Te}^2}$ and $b = \frac{\varepsilon}{m_e n_0 v_{Te}^2}\left[1 - \frac{\beta^2}{(v_g^2 - c_s^2)\ \rho_i\varepsilon}\right]$. We observe from the above equation (24) that the conditions $1 \sim \beta^2/(v_g^2 - c_s^2)\rho_i\varepsilon$ and $v_g < c_s$ must hold for the piezoelectricity to have a substantial effect and the last term retains the negative sign to obtain the standard form of cusp equation. This shows that this equation only holds for subsonic case. The above expression shows that $\mathrm{d}\tilde{E}/\mathrm{d}\xi \to \infty$ at the maxima ($\xi = 0$) and thus here $\widetilde{E} = 1/\sqrt{8b}$. Integrating equation (24), we obtain

$$\xi = \frac{1}{\sqrt{2a}}[\sqrt{1 - 8b\tilde{E}^2} + \mathrm{Log}|\sqrt{8\ b}\ \tilde{E}| - \mathrm{Log}|1 + \sqrt{1 - 8b\tilde{E}^2}|] \quad (25)$$

which is a singular spiky soliton solution (cusp soliton) [32, 40].

5. Results and conclusion

We have studied the coupling of the electron wave with the lattice ions via nonlinear ponderomotive force using the two time scale theory in a piezoelectric semiconductor plasma. The piezoelectric effects, nonlinearities and the plasma effects in semiconductors have been extensively studied owing to its widespread technological and industrial applications but to the best of the author's knowledge, their stationary cusp soliton solution and the condition under which piezoelectric effect become crucial have been studied for the first time. In this section, we analyze the cusp soliton solution numerically and for that we have used the same physical parameters of n-type piezoelectric semiconductors that were used to evaluate the equation (7).

In the figure 1, we have plotted the equation (25) to trace the variation in the profile of cusp soliton by altering the piezoelectric constant. As it has been mentioned that the piezoelectric coupling constant ranges from 0.045 C m^{-2} to 0.35 C m^{-2} [36]

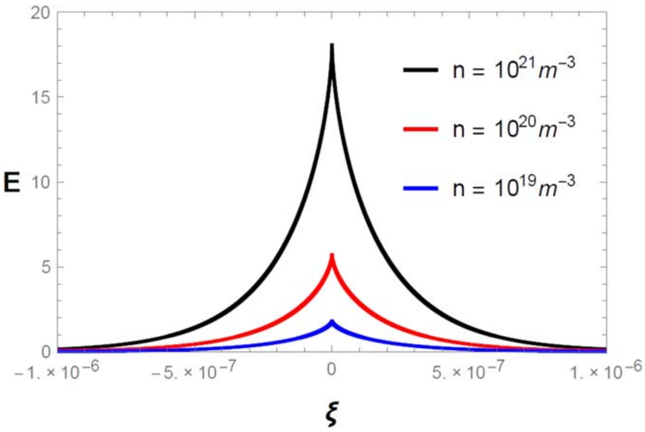

Figure 2. Cusp soliton with the variation of number density.

therefore we have used $\beta = 0.21\ \mathrm{C\ m^{-2}}$for GaSb and $\beta = 0.054\ \mathrm{C\ m^{-2}}$for InSb to keep our work applicable for piezoelectric semiconductors in general. We observe from the graphical analysis here that the amplitude of the electric field is significantly affected by the change in strength of the piezoelectric coupling constant.

In the figure 2, we have examined the trend of variation in the cusp with number density at constant coupling factor $0.21\ \mathrm{C\ m^{-2}}$. The pattern shows that the amplitude of the electric field varies substantially as the number density changes. It is shown that electric field amplitude decreases with decreasing number density. This result is in accordance with the equation (21) which shows that the perturbed density is proportional to the second derivative of the electric field.

Our work shows that for coupled lattice-electron plasma waves, piezoelectricity plays a significant role in the nonlinear regime. The nonlinear evolution equation that we have derived gives the subsonic cusp soliton solution.

Although we have presented a simplified one-dimensional analysis, our investigations fill a gap in theoretical work as theoretical work in this area has not been actively studied, as stated in section 1. This may also have a bearing in experimental and device fabrication work.

Acknowledgments

Authors acknowledge the Higher Education Commission (HEC) grant no. 20-2595/NRPU/R&D/HEC/13.

ORCID iDs

H A Shah https://orcid.org/0000-0003-1236-8825
Z Ehsan https://orcid.org/0000-0003-1614-5944

References

[1] Curie P and Curie J 1880 *C. R. Acad. Sci.* **91** 294
[2] Lipmann G 1875 *Ann. Chim. Phys.* **5** 494
[3] Cady W G 1946 *Piezoelectricity—An Introduction to the Theory and Applications of Electromechanical Phenomena in Crystals* (New York: Dover)
[4] Mason W P 1981 Piezoelectricity, its history and applications *J. Acoust. Soc. Am.* **70** 1581
[5] Voigt W 1894 On an apparently necessary extension of the theory of elasticity *Ann. Phys.* **52** 536
[6] Kyame J J 1949 Wave propagation in piezoelectric crystals *J. Acoust. Soc. Am.* **21** 159
[7] Kyame J J 1954 Conductivity and viscosity effects on wave propagation in piezoelectric crystals *J. Acoust. Soc. Am.* **26** 990
[8] Ruthemann G 1948 *Ann. Phys.* **2** 113
[9] Bohm D and Pines D 1951 A collective description of electron interactions: I. Magnetic interactions *Phys. Rev.* **82** 625
[10] Pines D and Bohm D 1952 A collective description of electron interactions: II. Collective vs Individual particle aspects of the interactions *Phys. Rev.* **85** 338
[11] Kuzmany H 1974 Acoustoelectric interaction in semiconductors *Phys. Status Solidi* a **25** 9
[12] Robson R E and Paranjape B V 1973 Interaction of plasma and lattice waves *Phys. Status Solidi* b **59** 641
[13] Blotekaer K and Quate C F 1964 The coupled modes of acoustic waves and drifting carriers in piezoelectric crystals *Proc. IEEE* **52** 360
[14] Hutson A R 1960 Piezoelectricity and conductivity in ZnO and CdS *Phys. Rev.* **4** 505
[15] Hutson A R and White D L 1962 Elastic wave propagation in piezoelectric semiconductors *J. Appl. Phys.* **33** 40
[16] White D L 1962 Amplification of ultrasonic waves in piezoelectric semiconductors *J. Appl. Phys.* **33** 2547
[17] Roundy S and Wright P K 2004 A piezoelectric vibration based generator for wireless electronics *Smart. Mater. Struct.* **13** 1131
[18] Lefeuvre E, Badel A, Richard C and Guyomer D 2005 Piezoelectric energy harvesting device optimization by synchronous electric charge extraction *J. Intell. Mater. Syst. Struct.* **16** 865
[19] Sodano H A, Inman D G and Park G 2005 Comparison of piezoelectric energy harvesting devices for recharging batteries *J. Intell. Mater. Syst. Struct.* **16** 799
[20] Cook-Chennault K A, Thambi N and Sastry A M 2008 Powering MEMS portable devices—a review of non-regenerative and regenerative power supply systems with special emphasis on piezoelectric energy harvesting systems *Smart. Mater. Struct.* **17** 043001
[21] Wang Z L, Wang X, Song J, Liu J and Gao Y 2008 Piezoelectric nanogenerators for self-powered nanodevices *IEEE Comput. Soc.* **7** 49
[22] Li P, Jin F and Jianxun M A 2018 One-dimensional dynamic equations of a piezoelectric semiconductor beam with a rectangular cross section and their application in static and dynamic characteristic analysis *Appl. Math. Mech.-Engl. Ed.* **39** 685
[23] Durkan C, Chu D P and Migliorato P 2000 Investigations into local piezoelectric properties by atomic force microscopy *Appl. Phys. Lett.* **76** 366
[24] Ridley B K and Wilkinson J 1969 A theory of acoustoelecric domains in piezoelectric semiconductors: I. Linear regime *J. Phys. C: Solid State Phys.* **2** 1299
[25] Ridley B K and Wilkinson J 1969 A theory of acoustoelecric domains in piezoelectric semiconductors III. Domain solution *J. Phys. C: Solid State Phys.* **2** 1321
[26] Ridley B K 1970 Nonlinear amplification of sound in a piezoelectric semiconductor *J. Phys. C: Solid State Phys.* **3** 935
[27] Ridley B K 1971 Acoustoelectric effects in semiconductors *Phys. Bull.* **22** 461
[28] Taniuti T and Yajima N 1969 Perturbation method for a nonlinear wave modulation: I *J. Math. Phys.* **10** 1369

[29] Pawlik M and Rowlands G 1975 The propagation of solitary waves in piezoelectric semiconductors *J. Phys. C: Solid State Phys.* **8** 1189

[30] Abdullah T, Durrani I U R and Shah H A 1988 Modulational instability of acoustic waves in a piezoelectric semiconductor *J. Phys. C: Solid State Phys.* **21** L123

[31] Shah H A, Hamed M and Murtaza G 2002 Elliptic solitary electron drift vortices in semiconductor piezoelectrics *Phys. Scr.* **65** 181

[32] Ichikawa Y H and Konno K 1980 Spiky soliton in circular polarized Alfven wave *J. Phys. Soc. Japan* **48** 279

[33] Ichikawa W M, Ichikawa Y H and Shimizu T 1980 Cusp soliton of a new integrable nonlinear evolution equation *Prog. Theor. Phys.* **64** 1959

[34] Kaup D J and Newell A C 1978 A new integrable nonlinear evolution equation *J. Math. Phys.* **19** 798

[35] Nisihikawa K, Hojo H, Mima K and Ikezi H 1974 Coupled nonlinear electron-plasma and ion-acoustic waves *Phys. Rev. Lett.* **33** 148

[36] Arlt G and Quadflieg P 1968 Piezoelectricity properties in III–V compounds with a Phenomenological analysis of piezoelectric effect *Phys. Status Solidi* b **25** 323

[37] Ghazala A, Shah H A and Rashid A 1996 Modulational instability of coupled non-linear helicon-acoustic waves in a piezoelectric semiconductor plasma *J. Phys.: Condens. Matter* **8** 1207

[38] Dendy R O 1990 *Plasma Dynamics* (Oxford: Clarendon)

[39] Thornhill S G and ter Haar D 1978 Langmuir turbulence and modulational instability *Phys. Lett.* C **43** 45

[40] Ehsan Z, Tsintsadze N L, Murtaza G and Shah H A 2009 Two types of lower-hybrid waves in dusty plasmas and cusp solitons *Phys. Plasmas* **16** 23702