

# Distinct features of Alfvén wave in non-extensive plasmas

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## ABSTRACT

Since the time of its discovery, Alfvén wave has been the subject of attention in both theoretical and experimental plasma physics. To date, many different aspects of this wave have been studied using various models. In this study, we adopt linearized Vlasov–Maxwell system and investigate very important characteristics of the wave: electromagnetic field perturbation and Poynting flux, which gives a measure of the electromagnetic energy stored in the field. The plasma which supports the wave is considered to be non-extensive. It is found that the electromagnetic field behaves totally different from the Maxwellian case, and so does the resultant transport of energy. Due to the wide spectrum of the non-extensive parameter  $q$ , the results can incorporate a broad range of possibilities which suggest that the energy transport of the wave over short and long distances may somehow be controlled by the non-extensivity of the plasmas.

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## 1. Introduction

Due to their phenomenal applications in laboratory and space plasmas, Alfvén waves have mesmerized scientists for decades. The reason for the great interest in these waves is the transport of energy which is a burgeoning aspect from both experimental and theoretical points of view [1]. From the laboratory plasmas in fusion devices to the naturally occurring plasmas in space, Alfvén waves have been proven to be the best candidate to carry energy from one place to another via electromagnetic fields [2].

The electromagnetic field, apart from playing a key role in the energy transport mechanism, is an important tool for the identification of waves in a plasma. For example, if, in some region of plasma, ratio of the electric field to the magnetic field is equal to Alfvén speed, then in that particular environment, Alfvén wave is most likely at work. But, as ample observations have confirmed, sometimes the ratio slightly deviates from the Alfvén speed [3–9].

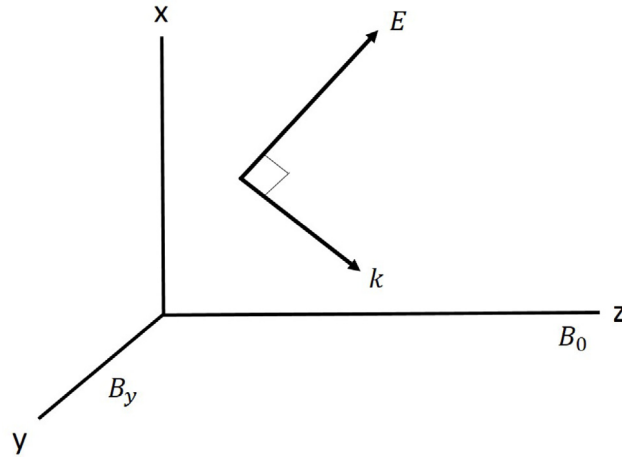
Such deviation may be due to the kinetic effects of plasma [10]. The kinetic effects also give rise to additional component of the electric field in the direction of ambient magnetic field. The wave in this case no longer remains pure Alfvén wave; it is called kinetic Alfvén wave [11].

The kinetic Alfvén wave gives its energy to the plasma in such a way that the transfer may vary from point to point in space as the wave moves forward. The magnitude of the energy per unit time per unit area is given by the Poynting flux vector which evolves according to the Poynting theorem [10]. From the practical point of view, the Poynting flux vector can give the information of whether or not the wave can carry or deliver its energy far away from its source.

In the case of kinetic Alfvén wave, the Poynting flux vector has been mostly studied in the Maxwellian distributed plasmas (Ref. [10] and references therein). However, mountains of data have shown that Maxwellian distribution of particles is not always observed in the plasmas (Refs. [12–14] and references therein). Indeed, in the realistic plasma

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**Fig. 1.** Geometry of the system. The wave vector  $\mathbf{k}$  and perturbed electric field  $\mathbf{E}$  are perpendicular to each other in  $x$ - $z$  plane. The perturbed magnetic field  $B_y$ , perpendicular to both  $\mathbf{k}$  and  $\mathbf{E}$ , is in the  $y$ -direction. The ambient magnetic field  $B_0$  points in the  $z$ -direction.

environments, the distribution of particles frequently exhibits non-Maxwellian character—suprathermal tail or cut off on the maximum value allowed for the velocity of the particles [15].

The departure of the systems from the Maxwellian distribution is not just a coincidence; rather, it is an inescapable consequence of the non-extensive statistical mechanics, first advanced by Tsallis [16,17]. The non-extensive statistical mechanics offers a broad range of possible states of the system, characterized by the index  $q$ . The velocity distribution of particles in a state  $q$  is termed as non-extensive distribution which has found wide applications in both laboratory and space plasmas [18–24].

In the present study, we examine the influence of the non-extensivity on the electromagnetic field and the associated Poynting flux of the kinetic Alfvén wave. To the best of our knowledge, this issue has not been reported so far.

## 2. Method

We assume low frequency waves emitted from a source at position  $z = 0$  in a homogeneous, collisionless, and low  $\beta$  plasma where the geometry of the system is such that the wave vector  $\mathbf{k}$  and the perturbed electric field lie in  $x$ - $z$  plane; the perturbed magnetic field is along  $y$ -axis; and the ambient magnetic field is along  $z$ -axis (Fig. 1). Employing linearized Vlasov–Maxwell system of equations, the dispersion relation in such a plasma can be written as [10,25,26]

$$\begin{pmatrix} \epsilon_{xx} - N_{\parallel}^2 & N_{\parallel}N_{\perp} \\ N_{\parallel}N_{\perp} & \epsilon_{zz} - N_{\perp}^2 \end{pmatrix} \begin{pmatrix} E_x \\ E_z \end{pmatrix} = 0, \quad (1)$$

where  $N_{\perp,\parallel} = k_{\perp,\parallel}^2 c^2 / \omega^2$  is the perpendicular/parallel component of refractive index. In the preceding equation,  $\omega$  is assumed to be complex, i.e.,  $\omega = \omega_r + i\omega_i$  with  $\omega_r \gg \omega_i$ . The two epsilons,  $\epsilon_{xx}$  and  $\epsilon_{zz}$ , are the components of the permittivity tensor. These are distribution dependent [27]:

$$\epsilon_{xx} = 1 + \sum_{\alpha} \frac{\omega_{p\alpha}^2}{\omega} \int d^3v \sum_{n=-\infty}^{\infty} \frac{n^2}{\zeta^2} \frac{v_{\perp} J_n^2(\zeta)}{\omega - k_{\parallel}v_{\parallel} - n\Omega_{\alpha}} \frac{\partial f_0}{\partial v_{\perp}}, \quad (2)$$

and

$$\epsilon_{zz} = 1 + \sum_{\alpha} \frac{\omega_{p\alpha}^2}{\omega} \int d^3v \sum_{n=-\infty}^{\infty} \frac{J_n^2(\zeta)}{\omega - k_{\parallel}v_{\parallel} - n\Omega_{\alpha}} \frac{v_{\parallel}^2}{v_{\perp}} \frac{\partial f_0}{\partial v_{\perp}}, \quad (3)$$

where  $f_{0\alpha}$  represents unperturbed distribution function;  $\omega_{p\alpha} (= \sqrt{4\pi n_{\alpha} e^2 / m_{\alpha}})$  stands for the plasma frequency; and  $J_n(\zeta)$  is Bessel function of the argument  $\zeta = k_{\perp} v_{\perp} / \Omega_{\alpha}$  with  $\Omega_{\alpha} = q_{\alpha} B_0 / m_{\alpha} c$ .

In the above Eqs. (2) and (3), we choose the following non-extensive velocity distribution [15,28–32]:

$$f_{0\alpha}(v) = \frac{A_q}{\pi^{\frac{3}{2}} v_{T\alpha}^3} \left[ 1 - (q-1) \frac{v^2}{v_{T\alpha}^2} \right]^{\frac{1}{q-1}}, \quad (4)$$

where  $v_{T\alpha}$  ( $= \sqrt{2T_\alpha/m_\alpha}$ ) is the thermal velocity of species  $\alpha$  having mass  $m_\alpha$  and temperature  $T_\alpha$ . In this study, the species will be electrons and ions. The normalization constant  $A_q$  in Eq. (4) reads:

$$A_q = \frac{3q - 1}{2} \frac{\sqrt{1 - q} \Gamma\left(\frac{1}{1 - q}\right)}{\Gamma\left(\frac{1}{1 - q} - \frac{1}{2}\right)},$$

if  $-1 < q < 1$ ; and

$$A_q = \frac{q + 1}{2} \frac{3q - 1}{2} \frac{\sqrt{q - 1} \Gamma\left(\frac{1}{q - 1} + \frac{1}{2}\right)}{\Gamma\left(\frac{1}{q - 1}\right)},$$

if  $q > 1$ . One may easily check that when  $q \rightarrow 1$  in Eq. (4), Maxwellian distribution is retrieved. Clearly, Eq. (4) exhibits thermal cutoff, i.e.,  $v_{\max} = \sqrt{v_{T\alpha}^2/(q - 1)}$  when  $q > 1$ . Hence, the allowed range of velocities is  $v \in (-\infty, \infty)$  when  $-1 < q < 1$ , and  $v \in [-v_{\max}, v_{\max}]$  when  $q > 1$ .

Substituting Eq. (4) in Eqs. (2) and (3), the relevant components of the permittivity tensor for non-extensive isotropic distribution in the low frequency domain ( $\omega \ll \Omega_i$ ) and small gyroradii limits ( $k_\perp^2 \rho_e^2 \ll k_\perp^2 \rho_i^2 \ll 1$ ) turn out to be

$$\epsilon_{xx} = \frac{c^2}{v_A^2} \left( 1 - \frac{3}{2} \frac{k_\perp^2 \rho_i^2}{5q - 3} \right), \tag{5}$$

and

$$\epsilon_{zz} = \frac{2\omega_{pe}^2}{k_\parallel^2 v_{Te}^2} \left( \frac{3q - 1}{2} + i\eta \right), \tag{6}$$

where  $v_A$  ( $= B_0/\sqrt{4\pi m_i n_0}$ ) is the Alfvén speed and  $\eta = A_q \xi_{0e} \sqrt{\pi} [1 - (q - 1) \xi_{0e}^2]^{\frac{1}{q-1}}$  with  $\xi_{0e} = \omega_r/k_\parallel v_{Te}$ .

In order to find the non-trivial solution of  $\omega$ , we solve determinant of the matrix in Eq. (1) which gives the real frequency as

$$\omega_r = k_\parallel v_A \sqrt{1 + \frac{3}{4} \frac{2k_\perp^2 \rho_i^2}{5q - 3} + \frac{2k_\perp^2 \rho_s^2}{3q - 1}}, \tag{7}$$

and the imaginary part of the frequency as

$$\omega_i = -\alpha k_\parallel v_A, \tag{8}$$

with

$$\alpha = \frac{2\sqrt{\pi} A_q k_\perp^2 \rho_s^2}{(3q - 1)^2} \frac{v_A}{v_{Te}} [1 - (q - 1) \xi_{0e}^2]^{\frac{1}{q-1}}, \tag{9}$$

where  $\rho_s = \rho_i T_e/T_i$ , and  $q \neq 3/5$  and  $1/3$ . According to Eq. (8),  $\alpha$  can be interpreted as damping rate, and indeed it is the Landau damping rate of the kinetic Alfvén wave.

The above Eqs. (5) to (9) can further be used as recipe for the most important part of this paper: the electric and magnetic fields and the associated Poynting flux. So moving on to the electric field part, we solve the first row of the matrix Eq. (1) and find that the ratio of the transverse electric field  $E_x$  to parallel electric field  $E_z$  is

$$\frac{E_z}{E_x} = \frac{N_\parallel^2 - \epsilon_{xx}}{N_\parallel N_\perp}. \tag{10}$$

Using Eqs. (5) to (9), the electric field ratio simplifies to

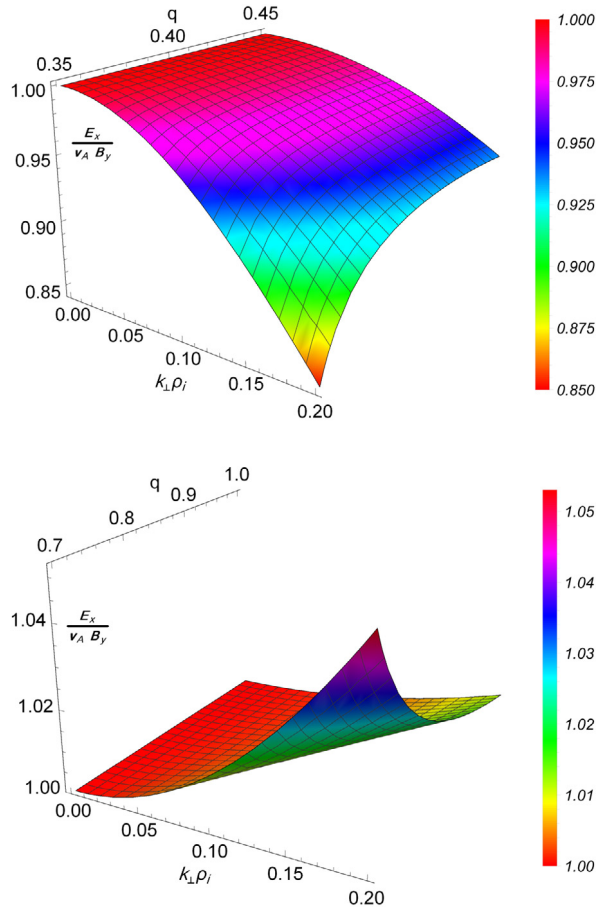
$$\frac{E_z}{E_x} = -\frac{k_\parallel}{k_\perp} \left( \frac{2k_\perp \rho_s^2}{3q - 1} - 2i\alpha \right). \tag{11}$$

The magnetic field perturbation does not explicitly appear anywhere in the equations discussed so far; however, it can be found by invoking Faraday's law,  $N_\parallel E_x - N_\perp E_z = cB_y$ . After doing a little algebra, it is found that

$$\frac{E_x}{B_y} = \frac{cN_\parallel}{\epsilon_{xx}}, \tag{12}$$

which upon simplification reduces to

$$\frac{E_x}{B_y} = v_A \frac{1 + \frac{3}{2} \frac{k_\perp^2 \rho_i^2}{5q - 3}}{\sqrt{1 + \frac{3}{2} \frac{k_\perp^2 \rho_i^2}{5q - 3} + \frac{2k_\perp^2 \rho_s^2}{3q - 1}}}. \tag{13}$$



**Fig. 2.** Normalized ratio  $E_x/v_A B_y$  as a function of normalized wavenumber  $k_{\perp} \rho_i$  and non-extensivity parameter  $q$ . Different colors correspond to different values of  $E_x/v_A B_y$  which is shown by the bar legend positioned on the right of the figure. In the upper and lower panels, the range of non-extensive parameter  $q$  is different.

The aforementioned field ratios, Eqs. (11) and (13), can be used to calculate the Poynting flux using the steady state Poynting theorem:  $\nabla \cdot \mathbf{S} = -P$ , where the real parts of the complex Poynting vector  $\mathbf{S}$  and the power dissipation  $P$  are given by  $\mathbf{S} = \text{Re}(\mathbf{E}^* \times \mathbf{B}) / 2\mu_0$  and  $P = \text{Re}(\mathbf{j}^* \cdot \mathbf{E}) / 2$ , respectively. In the preceding two equations,  $\mu_0$  is the permeability of free space, and  $\mathbf{j}$  is the current density.

Kinetic Alfvén waves carry energy across the field lines, but calculations show that the Poynting flux in the perpendicular direction is not significant [10], so we only take the parallel component:  $S = \text{Re} E_x B_y^* / 2\mu_0$ . In the parallel direction, the current  $j_z$  in the expression  $P = \text{Re}(j_z^* E_z) / 2$  can be found from Ampère's law  $j_z = (\partial B_y / \partial x) / \mu_0 = ik_{\perp} B_y / \mu_0$  with which  $P$  becomes

$$P = -\frac{1}{2\mu_0} \text{Re} (ik_{\perp} B_y^* E_z) = -k_{\perp} \text{Re} \left( i \frac{E_z}{E_x} \right) S = 2\alpha k_{\parallel} S, \quad (14)$$

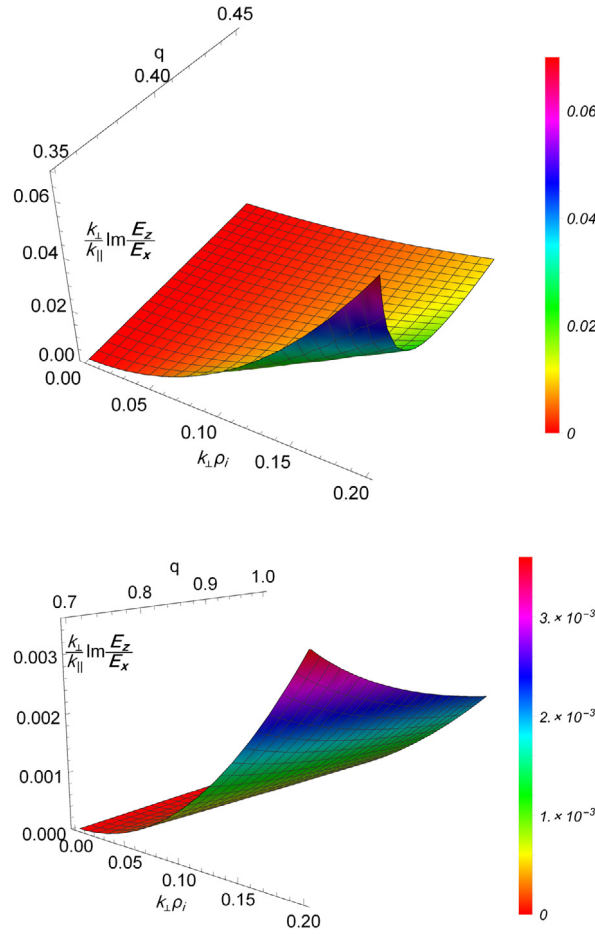
where we have used Eq. (11). Now we consider the Poynting flux theorem  $\partial S / \partial z = -P = -2\alpha k_{\parallel} S$  which gives us the solution

$$S(z) = S(0) e^{-2\alpha k_{\parallel} z}, \quad (15)$$

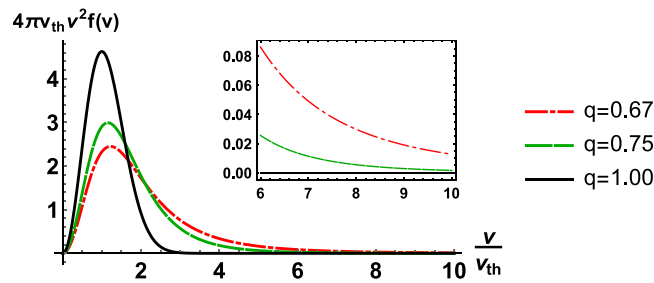
where  $S(0)$  represents energy per unit time per unit area at the location from where the wave starts its journey.

### 3. Results and discussion

To visualize our analytical results of kinetic Alfvén wave, we choose the following appropriate parameters for the magnetopause [33]:  $B_0 = 30$  nT,  $v_A = 400$  km/s,  $T_e/T_i = 0.2$ ,  $\lambda_{\parallel} = 2\pi k_{\parallel}^{-1} = 10$  Re,  $\rho_s = 50$  km,  $v_{Te} = 6157$  km/s. As to the best of our knowledge, there is no evidence for  $q > 1$  case in space plasmas [34], so we restrict ourselves only



**Fig. 3.** The normalized ratio  $\text{Im}k_{\perp}E_z/k_{\parallel}E_x$  as a function of normalized wavenumber  $k_{\perp}\rho_i$  and non-extensivity parameter  $q$ . Different colors correspond to different values of  $\text{Im}k_{\perp}E_z/k_{\parallel}E_x$  which is shown by the bar legend positioned on the right of the figure. In the upper and lower panel the range of non-extensive parameter  $q$  is different.

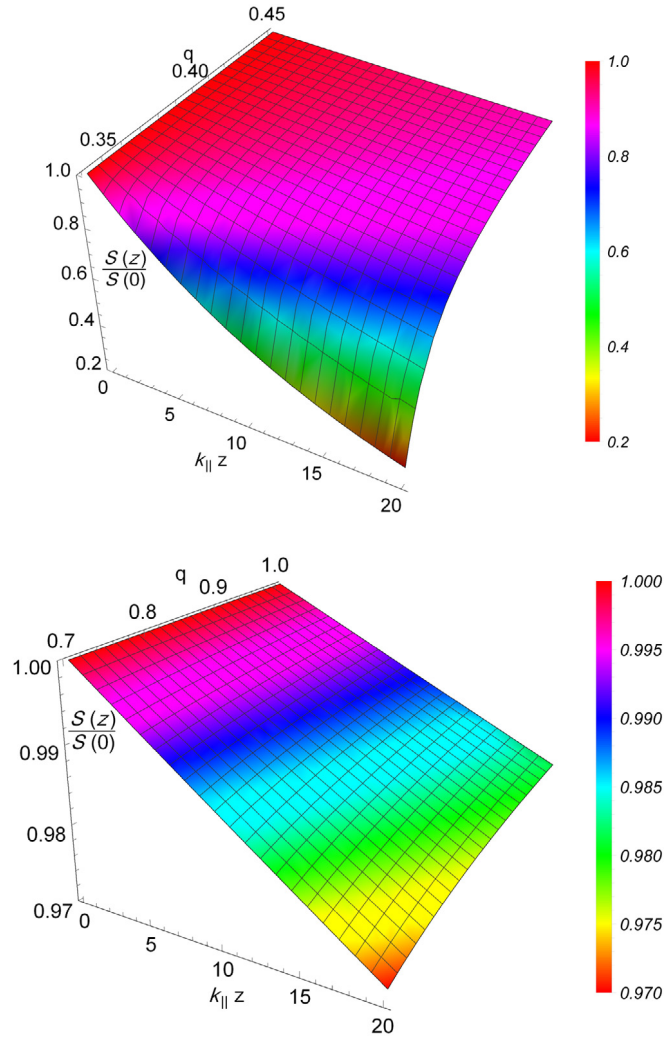


**Fig. 4.** Plot of  $4\pi v_{th} v^2 f(v)$  vs  $v/v_{th}$  for different values of  $q$ . The resonance point lies somewhere in the tail of the distribution function. The inset is the zoom region in the tail which shows that the resonant particles are large in number for small values of  $q$ .

to  $q < 1$ . Moreover, due to the singularities in Eqs. (9) and (11) at point  $q = 1/3$  and in Eq. (13) at point  $q = 3/5$ , we consider, for the sake of clarity, the ranges of  $q$  to be  $0.35 < q < 0.45$  and  $0.7 < q < 1$  throughout the figures.

Under these ranges, the 3D plot of Eq. (13) displays some interesting features. When  $0.35 < q < 0.45$ , the normalized ratio  $E_x/B_y v_A$  decreases with  $k_{\perp}\rho_i$  more rapidly for small values of  $q$  (Fig. 2 top panel). However, if  $0.7 < q < 1$ , the variation of  $E_x/B_y v_A$  with  $k_{\perp}\rho_i$  reverses (Fig. 2 bottom panel). This time, instead of decreasing, the ratio  $E_x/B_y v_A$  increases with  $k_{\perp}\rho_i$  more quickly for values of  $q$  close to 0.7.

The important thing here to be noted is that, in the non-extensive case, there is broad spectrum of the non-extensive parameter  $q$  which can take into account tiny details of the fluctuations. These fluctuations of the electromagnetic



**Fig. 5.** The ratio  $S_z/S_0$  as a function of  $k_{\parallel}z$  and non-extensivity parameter  $q$ . Different colors correspond to different values of  $S_z/S_0$  which is shown by the bar legend positioned on the right of the figure. In the upper and lower panel the range of non-extensive parameter  $q$  is different.

field have indeed been routinely observed in the space plasmas [35]. Thus, despite other possibilities, for example, the gyroradius correction terms [10], the non-extensivity of the system may play a key role in the dynamics of the electromagnetic field of kinetic Alfvén wave.

Like  $E_x/B_y v_A$ , the ratio  $E_z/E_x$  strongly depends on the non-extensive parameter  $q$  (Fig. 3). It is seen that the imaginary part of the normalized  $E_z/E_x$  is large in magnitude for small values of  $q$  compared to Maxwellian case ( $q \rightarrow 1$ ).

The possible explanation for why this is to be the case can be given in terms of resonant particles participating in the wave-particle interaction. Eq. (11) shows that the imaginary part of  $E_z/E_x$  is directly related to Landau damping rate  $\alpha$  which results from the resonance condition  $\omega = k_{\parallel} v_{\parallel}$  (or equivalently stated when phase speed of the wave is equal to the speed of particles moving along the direction of the wave). In the case of kinetic Alfvén waves, the ion dynamics shifts the resonance point towards the tail of the electron distribution Ref. [25]. And, in the tail, it is clearly seen (Fig. 4), the resonant particles are large in number for small values of  $q$  which means that the small values of  $q$  should increase the magnitude of the imaginary part of the electric field ratio  $E_z/E_x$  for a given  $k_{\perp} \rho_i$ .

Since the electric and magnetic fields of kinetic Alfvén waves carry the electromagnetic energy stored in them, the question is how the transfer occurs spatially when the waves move forward? This information is contained in the Poynting flux vector which explicitly depends on spatial coordinate  $z$ .

The variation of the normalized Poynting flux versus normalized distance  $k_{\parallel}z$  is shown in Fig. 5. The Poynting flux decays rapidly for small values of  $q$  but decays gradually for large values. All this is due to the Landau mechanism. For small  $q$ , the electromagnetic energy of the wave is converted to the Landau resonant electrons over short distances. This should be the case, because, in the small  $q$  regime, the energy is distributed over a large number of suprathermal

particles which causes the wave to run out of energy quickly. On the other hand, as one might expect, for small number of suprathermal particles, the wave lingers on and can transport its energy over long distances. If the non-extensivity of the system effects the transport of energy – which seems to be the case – then these results would play an important role in the space plasmas.

For example, observations suggest that Poynting flux associated with kinetic Alfvén waves at the plasma sheet boundary layer at 4–6  $R_E$  varies from 1–2 mW/m<sup>2</sup> to over 100 mW/m<sup>2</sup> at the ionospheric altitudes. Moreover, the Poynting flux decays with distance and about 10% of  $S$  is converted to particle energy over locations in the range 1.5–15  $R_E$  [10]. As non-extensivity is widely considered to be the most important feature in space plasmas, we suggest that our results may find potential applications in these environments. In the observed variations, non-extensivity of the plasmas may play a significant role.

Up to now, the focus of our attention has been on the space plasmas, but the generalized results could usefully be employed in laboratory plasmas. For example, in fusion plasma devices, kinetic Alfvén wave has been proven to be the best candidate to carry energy away from the core regions [36,37].

Moreover, as experimental observations have shown that there is considerable difference between experimentally observed and theoretically predicted electric fields at different locations in Large Plasma Device at UCLA [3]. The claim for large uncertainty present in the measurement of  $k_{\perp}$  is fully justified for such a substantial difference in the measured electric fields. However, we propose that, in addition to the uncertainty in  $k_{\perp}$ , non-extensivity may also play a key role in the resultant fluctuations because the plasma in this experimental study was treated to be Maxwellian.

With regard to the Maxwellian distribution, our results slightly differ from that of Ref. [10] [compare Eqs. (6) and (7) of Ref. [10] with Eqs. (11) and (13) of this paper]. The reason is the following: in that reference, Padé approximation for the Bessel function was used in the mathematical treatment of Eqs. (2) and (3). If instead Taylor approximation was used, as in the earlier work [25], and the same procedure was followed, then the results in Ref. [10] and ours in the limit  $q \rightarrow 1$  would become identical.

To summarize, in the present work, we have seen that the characteristics of kinetic Alfvén waves are significantly influenced by the non-extensivity of the system. The electric and magnetic fields do vary with a lot of possibilities, depending on which  $q$ -state the system resides in. Through the electromagnetic field, the waves interact with particles and transfer their electromagnetic energy to the plasma in such a way that in the small  $q$  limit, the waves transfer energy in their immediate vicinity. However, for large values of  $q$ , the waves can give their energy to the plasma a long way away. Hence the transport of the stored energy over short and long distances seems to be controlled by the non-extensivity of the system. If that is the case, then the results would be of particular importance, because if the driving source of the non-extensivity of the system is controlled by some means, the wave could be localized or sent to remote regions for practical purpose such as heating of the plasma.

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