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Coupled Drift Ion Acoustic Shock waves with trapped electrons in Quantum Magnetoplasma

Amna Fayyaz[1], H. A. Shah[2,*], M. N. S. Qureshi[1] and W. Masood[3]

[1]Department of Physics, Government College University, Lahore 54000, Pakistan.
[2]Department of Physics, Forman Christian College (A Chartered University), Lahore 54000, Pakistan.
[3]Department of Physics, COMSATS University Islamabad (CUI), Islamabad Campus, Islamabad, 45550, Pakistan.

E-mail: hashah.gcl@gmail.com

Abstract

Coupled drift ion acoustic shock waves are studied in inhomogeneous, dissipative quantum magnetoplasmas in the presence of adiabatically trapped electrons. We have derived a nonlinear Burgers like evolution equation having a fractional nonlinearity. The quantum magnetohydrodynamics model (QMHD) has been employed to derive this nonlinear equation (2+1) (two spatial and one temporal) dimensions. Nonlinear analysis of the equation is carried out to show that it admits shock solutions. These shock structures are numerically investigated by using parameters of neutron stars where quantum effects are expected to dominate the dynamics of the system. These results show significant dependence on the physical parameters namely inhomogeneity, number density, ambient magnetic field, collisional frequency and the angle of propagation.

Introduction:

The elementary theory for the existence of ion acoustic waves (IAW) in an ionized gas was formulated by employing fluid model which had been studied extensively as a fundamental normal mode in plasmas [1]. The propagation characteristics of linear IAW [2] in the presence of two-electron population are found to be dominated by the lower temperature electrons. Bychenkov [3] developed a Vlasov-Poisson's model for Maxwellian plasmas with temperature gradient, ion-electron and ion-ion collisions to study the wave damping analytically. It is a well-established idea that large amplitude ion acoustic waves in dispersive media give rise to solitary waves which were first observed using a double-plasma device [4]. These electrostatic nonlinear solitary structures were also observed by Freja [5] and Viking spacecraft [6] in Earth's magnetosphere. The theory for IAW was further extended to include various nonlinear effects such as trapped electron populations [7-8], Landau damping (Collisionless damping), higher order nonlinearities, finite ion temperature [9], kinematic viscosity and inter particle collisions. For Landau damping, Korteweg de Vries (KdV) equation is derived which shows that particle density remains conserved but the amplitude and energy of solitary wave decay as time progresses [10]. Furthermore, the relativistic effects play a crucial role on the dynamics of these solitary waves [11] in the presence of ion streaming.

During the last few decades, quantum plasmas have been investigated significantly due to their ample applications in astrophysical plasmas [12], nonlinear optics and microelectronic devices etc. [13]. Such plasmas are studied by employing Quantum Hydrodynamic Model (QHD) and Wigner-Poisson model which are analogous to classical fluid model and kinetic model respectively [14]. Haas et al. [15] studied one dimensional quantum ion acoustic wave by using QHD and showed that this model reduces to its classical counterpart when the quantum parameter is sufficiently small. However, this system exhibits KdV like solutions with nontrivial dependence on quantum parameter in the weakly nonlinear regime while it gives periodic wave patterns in fully nonlinear regime. This problem was further studied for nonplanar geometries to formulate a deformed Korteweg-de Vries (dKdV) [16] and it was shown that the solitary wave diminishes beyond a critical value of quantum parameter. The damping characteristics of solitons have also been studied for collisional dusty plasmas with trapped nonthermal distributions of electrons in the realm of earth's mesosphere [17-18].

Bernstein et al. [19] investigated the one-dimensional stationary longitudinal wave and illustrated that stationary wave solutions can be obtained for sufficient number of particles trapped in the potential well. For the nonstationary electrostatic wave analysis, kinetic formalism was developed by Gurevich to explain the behavior of trapped particles in a slowly varying field [20] which gives a $3/2$ order nonlinearity instead of quadratic nonlinearity. The number density of these trapped particles in a slowly varying electrostatic potential energy trough has been worked out for partially and fully degenerate plasma [21]. This adiabatic trapping of electrons is studied in a series of papers for dense plasmas and relativistic dense plasmas to obtain the stationary wave solutions by Tangent hyperbolic method *tanh* and pseudopotential approach [22-24].

On the other hand, the inhomogeneous magnetized plasmas exhibit various drift oscillations due to density/temperature gradients which tend to transport energy and momentum in the plane perpendicular to ambient magnetic field [25]. Such waves arise in two component quantum plasmas in the presence of a strong quantizing magnetic field. Solitary solutions of drift waves were also discussed by deriving Korteweg-de Vries Burgers KdVB equation in electron-positron-ion plasmas. [26-27]

The interplay of nonlinearity and dissipation leads to the formation of shock waves in collisional plasmas. Nonlinear ion acoustic shock waves were modelled for cold ions and two populations of Boltzmann electrons at the altitude of one Earth radius in the auroral zone [28]. Such NIAW were also studied in dispersive media in the presence of trapped electrons in nonhomogeneous plasmas by using the reductive perturbation technique in the small amplitude limit. This analysis yielded rarefactive shocks for ionospheric plasmas parameters of ionosphere. Further it was also noted that relativistic plasmas are ubiquitously observed in Van Allen radiation belts, laser plasma interactions and space plasmas. These plasmas also support the existence of trapped electrons populations and were studied to obtain solitary solutions [29].

In the present work, we undertake the study of drift ion acoustic shock waves with trapped degenerate electron density bearing the fractional nonlinearity of the form $(1+\Phi)^{3/2}$. We will seek to establish the dependence of the shock waves steepness on the different parameters i.e. number density, magnetic field strength and collisional frequency etc. The arrangement for this manuscript is as follows. In section II, the governing equations for our physical system are given. In section III, the nonlinear analysis of our evolution equation is presented followed by the results and discussion in the subsequent section IV.

I. Mathematical Model Equations:

We consider a two-component inhomogeneous quantum magnetoplasma where electrons are considered to be fully degenerate and ion dynamics is treated classically due to its large inertia. The plasma is assumed to be collisional with a constant ambient magnetic field taken along the z-axis and a weak density inhomogeneity along the x-axis. Thus, we write the momentum equation for ions as

$$m_i n_i(\partial_t + v_i.\nabla)v_i = en_i\left(E + \frac{1}{c}v_i \times B_o\right) - m_i n_i \nu_{in} v_i \tag{1}$$

For ion-ion collisions ν_{ii}, the momentum remains conserved when scattered and scattering particles are added together. The ion-electron collisions ν_{ie} are considered unimportant because the momentum gained or lost by ions from scattering electrons is negligible due to their large mass. Thus, ion-neutral collisions ν_{in} are taken into consideration only in Eq (1). The continuity equation for ions and Poisson's equation are given as

$$\frac{\partial n_i}{\partial t} + \nabla.(n_i v_i) = 0 \tag{2}$$

$$\nabla^2\varphi = -4\pi e(n_i - n_e) \tag{3}$$

where m_i is the mass, e is the charge and n_i is the number density. For the low frequency electrostatic wave i.e. ($E = -\nabla\varphi$ and $\partial_t \ll \Omega_{ci} \equiv e\, B_o/cm_i$), the perpendicular component of ion's velocity is given as

$$v_{i\perp} = \frac{c}{B_o}(\hat{z} \times \nabla\varphi) - \frac{c}{B_o\Omega_{ci}}\partial_t\nabla_\perp\varphi - \frac{c\,\nu_{in}}{B_o\Omega_{ci}}\,\nabla_\perp\varphi \tag{4}$$

The first term on the RHS of Eq. (4) is $E \times B$ drift, second term is the polarization drift and the third term is the collisional drift term. By using the standard drift wave approximation [30], the parallel component of velocity can be written as

$$(\partial_t + v_E.\nabla_\perp + v_{iz}\partial_z)v_{iz} = -\frac{e}{m_i}\partial_z\varphi \tag{5}$$

On the other hand, the number density of the degenerate electrons trapped in the potential energy is given by [19]

$$n_e = n_o\left\{(1 + e\,\varphi/\varepsilon_F)^{3/2} + \pi^2 T^2/8\,\varepsilon_F^2\,(1 + e\,\varphi/\varepsilon_F)^{-1/2}\right\} \tag{6}$$

where $\varepsilon_F = \hbar^2/2m_e(3\pi^2 n_0)^{2/3}$ is the Fermi energy. The second term in the above expression is a small correction term which accounts for the usual temperature which can be neglected for fully degenerate plasmas. Thus Eq. (6) becomes

$$n_e = n_o(1 + e\,\varphi/\varepsilon_F)^{3/2} \tag{7}$$

We substitute Eqs. (3-6) in Eq. (2) and obtain

$$\partial_t^2(1+\Phi)^{3/2} - \lambda_{Fe}^2\partial_t^2\left(\partial_y^2 + \partial_z^2\right)(1+\Phi) - \rho_i^2\partial_t^2\partial_y^2(1+\Phi) - \nu_{in}\rho_i^2\partial_t\partial_y^2(1+\Phi) + \frac{3}{2}v_*\partial_y\partial_t(1+\Phi) - c_s^2\partial_z^2(1+\Phi) = 0 \tag{8}$$

where $\Phi = e\,\varphi/\varepsilon_F$ is the normalized electrostatic potential, $c_s = \sqrt{\varepsilon_F/m_i}$ is the ion acoustic speed, $\rho_i = c_s/\Omega_{ci}$ is the ion Larmor radius, $\lambda_{Fe} = \sqrt{\varepsilon_F/4\pi e^2 n_0}$ is electron Fermi length and $v_* = (-2c\varepsilon_F/3eB_o)|d_x \ln n_0|$ is the fluid drift velocity. This equation depicts the nonlinear evolution of drift-ion acoustic wave in a dissipative and dispersive medium in (2+1) dimensions with fractional power $3/2$ nonlinearity. The predominant direction of propagation is along y-axis to retain the drift character of the wave. It is imperative to note here that in Eq [5], polarization drift is a small effect in comparison to $E \times B$ drift and it vanishes if the plasma approximation is used instead of Eq [3]. The inclusion of polarization drift leads to dispersive effects on solitary structures which has been studied earlier [30]. Therefore, we write a new nonlinear Burgers like equation in (2+1) dimensions to study the dissipative effects on solitary structures in inhomogeneous plasmas i.e.

$$\partial_t^2(1+\Phi)^{3/2} - \nu_{in}\rho_i^2\partial_t\partial_y^2(1+\Phi) + \frac{3}{2}v_*\partial_y\partial_t(1+\Phi) - c_s^2\partial_z^2(1+\Phi) = 0 \tag{9}$$

II. Nonlinear Analysis:

In order to further analyze Eq. (9), we use a comoving frame of reference defined as $\xi = \eta_y(y + \beta\,\mathrm{z} - v\,t)$ where $\beta = \eta_z/\eta_y = \eta\,sin\theta/\eta\,cos\theta$, θ is the angle of propagation between y and z axis, η_y, η_z are the nonlinear wavenumbers along y and z-axis respectively, $v = \Omega/\eta_y$ is the velocity and Ω is the frequency of the nonlinear structure. We set $\Psi = 1 + \Phi$ and the dimensionless form of Eq. (9) reads as

$$v^2\frac{d^2\Psi^{3/2}}{d\xi^2} + \frac{\nu_{in}\rho_i^2\eta}{c_s}v\frac{d^3\Psi}{d\xi^3} - (\beta^2 + \frac{3}{2}v\,v_*)\frac{d^2\Psi}{d\xi^2} = 0 \tag{10}$$

Normalizing $v = v/c_s$ and integrating twice for the boundary conditions $\xi \to \infty$, $\Psi \to \Psi_R$; $\xi \to -\infty$, $\Psi \to \Psi_L$ and $d_\xi^n\Psi \to 0$ where Ψ_R and Ψ_L are the right hand and left hand boundary conditions respectively [31]. The first constant of integration $c_1 = 0$ and second constant of integration c_2 is

$$v^2\Psi_R^{3/2} - (\beta^2 + \frac{3}{2}vv_*)\Psi_R = c_2 = v^2\Psi_L^{3/2} - (\beta^2 + \frac{3}{2}vv_*)\Psi_L \tag{11}$$

The value of normalized Φ ranges from ± 1 therefore, we get $\Psi_R = 2$ and $\Psi_L = 0$. Solving Eq. (11) for c_2 and v, we get

$$v = \frac{1}{8}\left[3\sqrt{2}\,v_* \pm \sqrt{18v_*^2 + 32\sqrt{2}\beta^2}\right] \tag{12}$$

$$c_2 = \frac{\left[3\sqrt{2}\,v_* + \sqrt{18v_*^2 + 32\sqrt{2}\,\beta^2}\right]^2}{16\sqrt{2}} - 2\left[\beta^2 + \frac{3}{16}\,v_*\left(3\sqrt{2}\,v_* + \sqrt{18v_*^2 + 32\sqrt{2}\,\beta^2}\right)\right] \tag{13}$$

Using Eqs. (12) and (13) in Eq. (10), we obtain

$$d\xi = \frac{\nu_{in} c_s \eta}{\Omega_{ci}^2}\left(\frac{v}{(\beta^2 + \frac{3}{2}vv_*)\Psi - v^2\Psi^{3/2} + c_2}\right)d\Psi \tag{14}$$

Integrating Eq. (14) yields a solution given as follows

$$\xi = -2\sqrt{2}\,\nu_{in} c_s \eta\left\{3\,v_* + \sqrt{9v_*^2 + 16\sqrt{2}\beta^2}\right\}\left\{2\,Log\left[9v_*^2\left(\sqrt{2} - \sqrt{\Psi}\right) + 3\,v_*\left(\sqrt{2} - \sqrt{\Psi}\right)\sqrt{9v_*^2 + 16\sqrt{2}\beta^2} - 8\beta^2\left(-2 + \sqrt{2\,\Psi}\right)\right] - Log(\Psi)\right\} \times \left[\Omega_{ci}^2\left(9\sqrt{2}\;v_*^2 + 16\beta^2 + 3v_*\sqrt{18\,v_*^2 + 32\sqrt{2}\beta^2}\right)\right]^{-1} \tag{15}$$

We note here that had we taken the standard quadratic nonlinearity or expanded our $(\mathbf{1} + \boldsymbol{\Phi})^{3/2}$ term in Eq. (9), then Eq. (15) could have been expressed in the standard *tanh* form. However, in our case, we numerically investigate Eq. (15) to show that our solution leads to shock structures.

III. Results and discussion:

In the present section, we have applied our results to neutron stars parameters where the quantum effects are expected to dominate. It is in the fitness of the situation to mention here that the *in-situ* observations of waves in dense plasmas in extreme environments are very difficult. However, the rapid development of laser technology which includes chirped pulse amplification [32] and radiative blast waves [33] would hopefully make it possible for us to compare the theory with experiments in future. We plot Eq. (15) for the positive root of $\boldsymbol{v}$ which is found to yield compressive shock structures which we have investigated numerically by employing the standard parameters of dense astrophysical plasmas (neutron stars) whose dynamics are governed by quantum effects i.e. $\boldsymbol{n_o \sim 10^{26} - 10^{29}cm^{-3}}$ and $\boldsymbol{B_o \sim 10^9 - 10^{11}G}$ [12]. We probe the impact of the varying number density $\boldsymbol{n_o}$, collisional frequency $\boldsymbol{\nu_{in}}$ and magnetic field strength $\boldsymbol{B_o}$ on the shock structure. For any system to retain its plasma behavior, the collisional frequency $\boldsymbol{\nu_{in}}$ must be less than the characteristic plasma frequency i.e. $\boldsymbol{\Omega_{ci}} \gg \boldsymbol{\nu_{in}}$ as in our case. From Figs. 1 and 2, we see that the rise in the electrostatic potential becomes sharper or thickness of shock decreases as the number density and collisional frequency decrease respectively. It originates from the fact that if we decrease the collisional frequency or number density, the nonlinear effects tend to dominate and makes shock steeper as it propagates [34]. On the contrary, from Fig. 3 we see that the shock wave becomes sharper with the increase in magnetic field strength which is evident from Eq. (15).

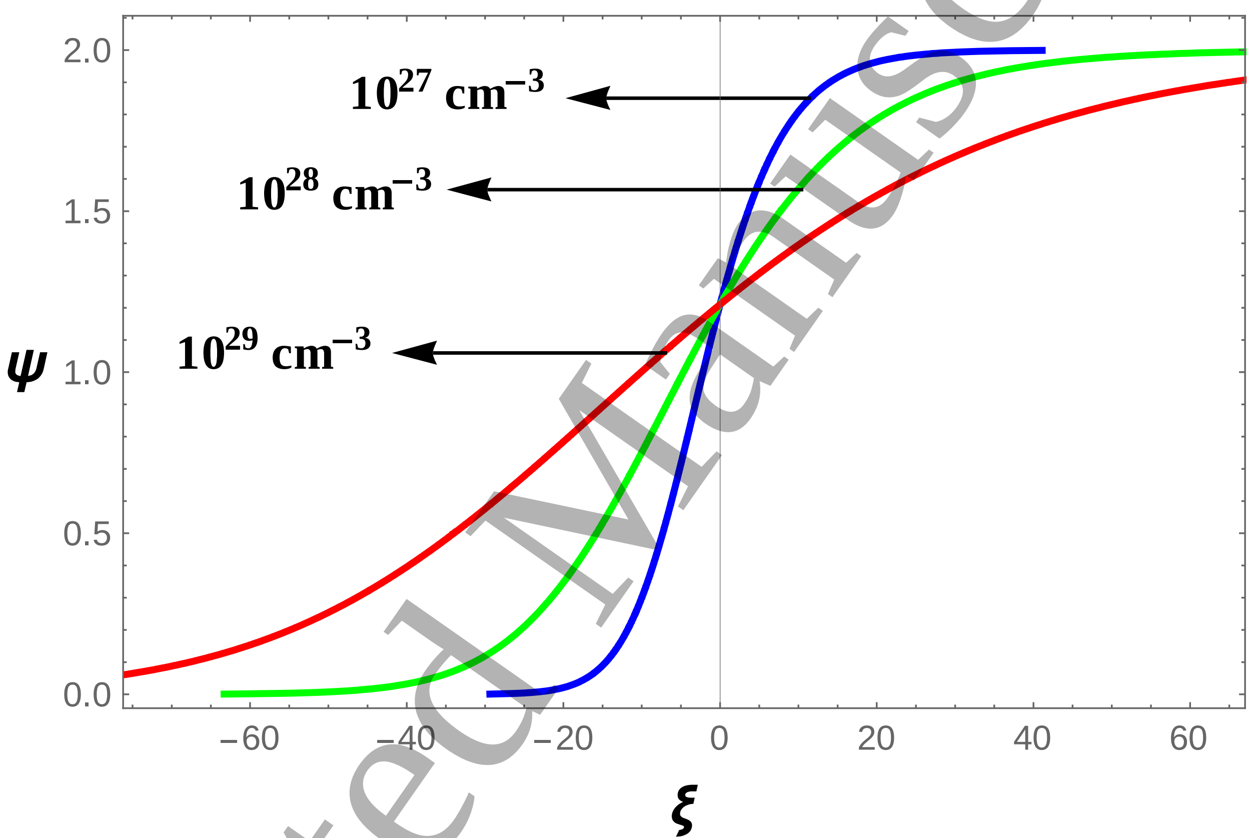

Fig. 1: Variation of Shock profile with the number density ($n_o = 10^{27}, 10^{28}, 10^{29} cm^{-3}$)($B_o = 10^{10}\ G$, $\nu_{in} = 3 \times 10^{13} s^{-1}$, $v_*/c_s = 0.4$ and $\theta = 10^0$)

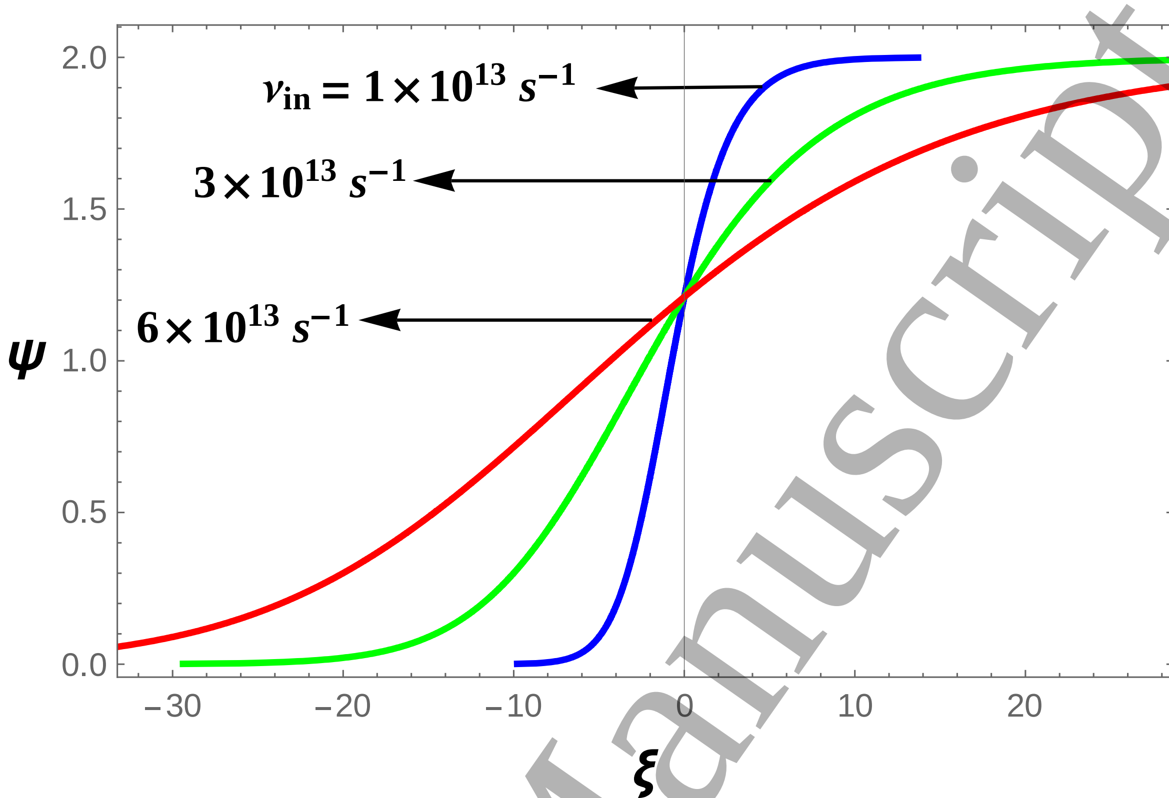

Fig. 2: Variation of Shock profile with collisional frequency ($n_o = 10^{27} cm^{-3}$, $B_o = 10^{10}\ G$, $v_*/c_s = 0.4$ and $\theta = 10^0$)

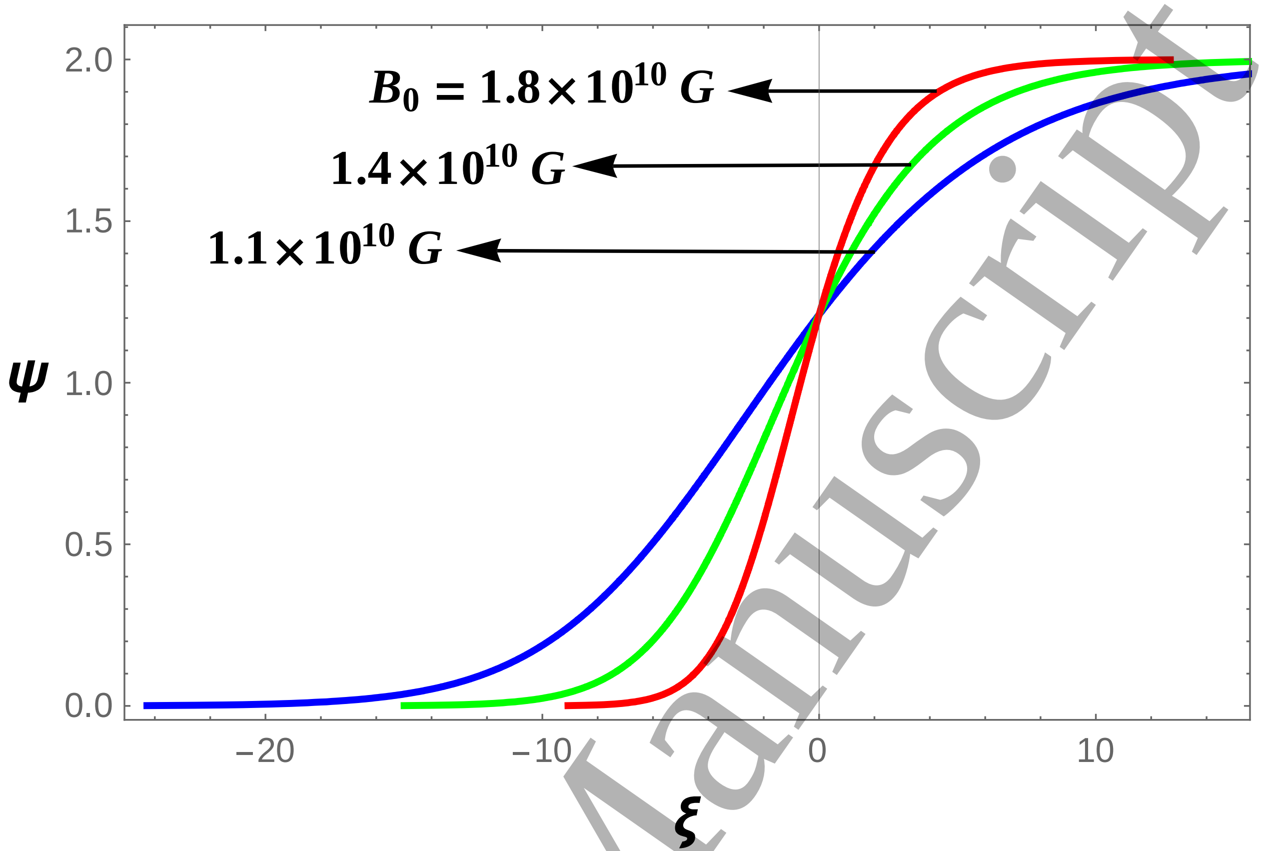

Fig. 3: Variation of Shock profile with the magnetic field ($n_o = 10^{27} cm^{-3}$, $\nu_{in} = 3 \times 10^{13} s^{-1}$, $v_*/c_s = 0.4$ and $\theta = 10^0$)

Furthermore, another parameter which modifies the shock profile is the variation in inhomogeneity which is taken in account from the ratio v_*/c_s. This ratio is varied from $0.3 - 0.7$ which follows from the conditions [35] that (i) $v_* \ll c_s$ (ii) $\theta \sim 10^0$to 30^0or $\eta_z/\eta_y \ll 1$to have coupled drift ion acoustic mode. We observe from Fig. 4 that the thickness of the shock increases significantly with the increase in inhomogeneity. Besides, if we increase the angle of propagation in yz-plane, the shock front becomes sharper and its thickness decreases slightly as exhibited in Fig. 5.

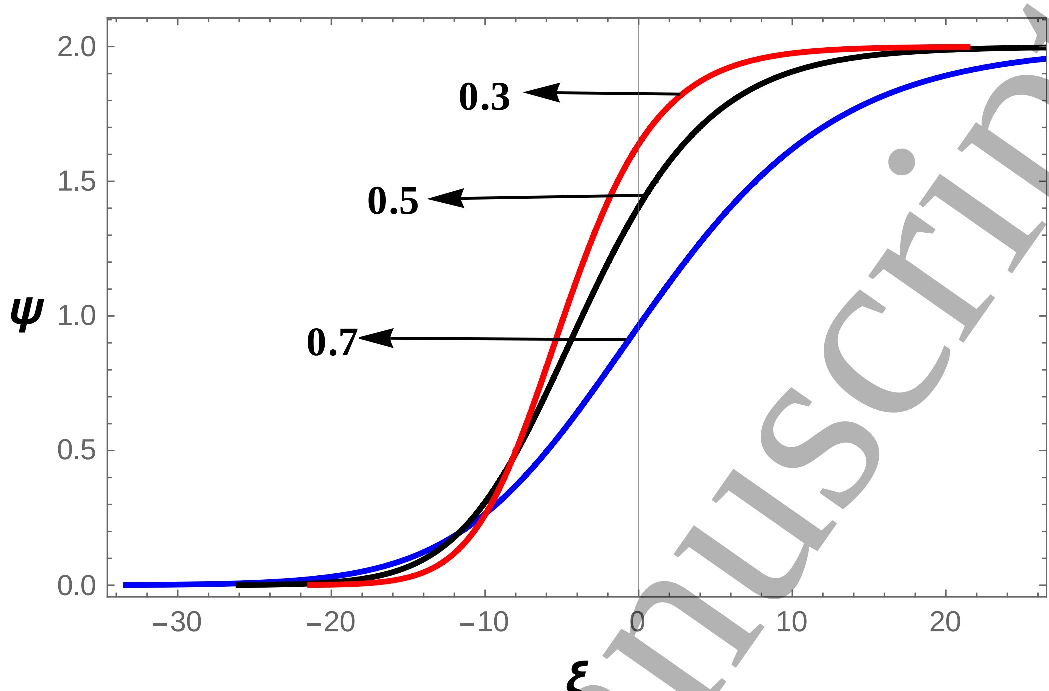

Fig. 4: Variation of Shock profile with the increasing inhomogeneity ($n_o = 10^{27} cm^{-3}$, $B_o = 10^{10}\ G$, $\nu_{in} = 3 \times 10^{13} s^{-1}$ and $\theta = 10^0$)

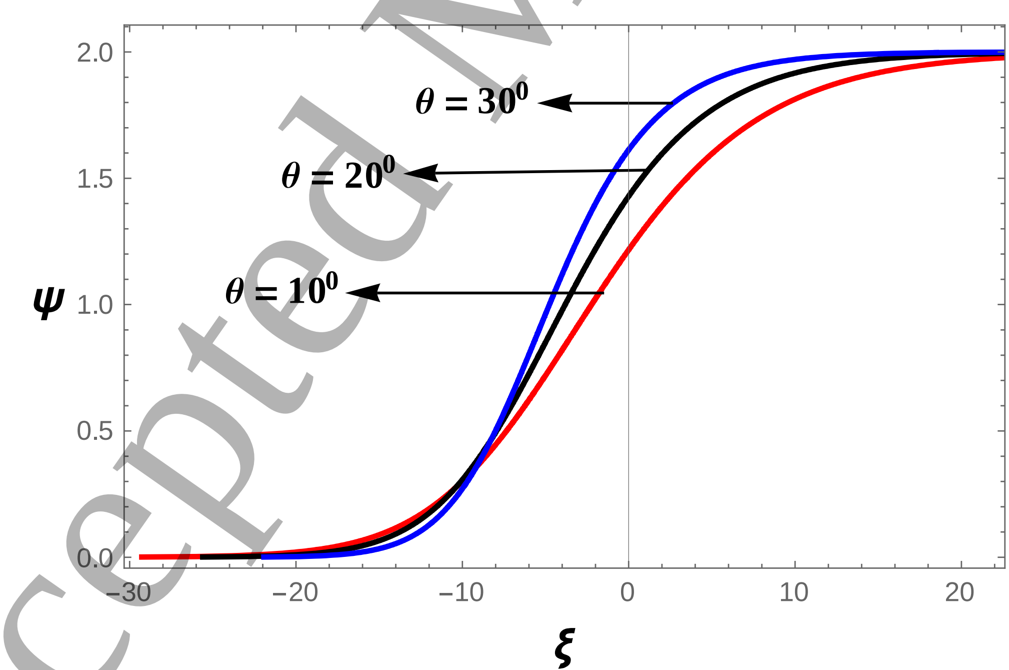

Fig. 5: Variation of Shock profile with angle of propagation ($n_o = 10^{27} cm^{-3}$, $\nu_{in} = 3 \times 10^{13} s^{-1}$, $v_*/c_s = 0.4$ and $B_o = 10^{10}\ G$)

IV. Conclusions and Summary

In this work, we have considered quantum magnetoplasma in the presence of collisions and a weak density gradient along with the effect of Gurevich like trapping to derive a Burgers like nonlinear evolution equation which admits shock solutions. We have further obtained an analytical solution to this equation and investigated the dependence of the shock strength (via its steepness/thickness) for the parameters of neutron stars in regard to the variation of number density, collisional frequency, ambient magnetic field, inhomogeneity and angle of propagation. Our work incorporates the effect of fractional power nonlinearity i.e. $(1+e\,\varphi/\varepsilon_F)^{3/2}$ term instead of quadratic $(e\,\varphi/\varepsilon_F)^2$ nonlinearity for shock structures in degenerate plasmas. Earlier work on the classical effect of trapping [27] on shock waves had a $\varphi^{3/2}$order nonlinearity. Thus, our present work makes a modest contribution to shock waves with the effect of microscopic trapping.

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