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ORIGINAL ARTICLE

Contributions to Plasma Physics

Excitation of kinetic Alfvén waves by streaming ions in a dusty magnetoplasma with generalized (r, q) distribution function

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Abstract

We investigate the parallel streaming effects on the dispersion characteristics of a kinetic Alfvén wave (KAW) in a low β dusty magnetoplasma. To analyse the influence of streaming ions obeying generalized (r, q) distribution function, hot and magnetized electrons, and mobile charged dust, a theoretical approach has been used for the instability analysis by employing two potential theories. A linear kinetic dispersion relation of Alfvén waves is derived, whose solutions are used to interpret the numerical and analytical results. The solutions of dispersion relation indicate that the characteristics of KAWs are transformed when generalized (r, q) distribution function is employed instead of its Maxwellian counterpart. We also found that the unstable modes have a strong dependence on spectral indices r and q, dust parameters, and plasma β. For the excitation of KAWs, the streaming velocity has been observed to be within the sub-Alfvén range, whereas when it extends to the super-Alfvén range, the growth rates are significantly suppressed. The observations further show that an ambient magnetic field and superthermal particles inhibit the growth of an electromagnetic wave to a significant degree and have a stabilizing effect on the wave mode, whereas an increasing concentration of low-energy particles contributes to enhancing growth rates.

KEYWORDS

Alfven waves, dusty plasma, magnetized plasma

1 | INTRODUCTION

Alfvén waves are low-frequency electromagnetic waves propagating along the ambient magnetic field. These waves tend to develop a longitudinal electric field as their wavelength becomes comparable to the Larmor radii of ions. Kinetic properties of the Alfvén wave, originating from the finite perpendicular wavelength effects, lead the Alfvén waves to couple with electrostatic waves, resulting in the generation of kinetic Alfvén waves (KAWs), which have been studied for both linear and nonlinear regimes. The wave has a parallel electric field that helps it to propagate in the direction perpendicular to the ambient magnetic field. The KAWs are linearly compressional as they develop electric field E fluctuations along the magnetic field B_0. Due to these E field fluctuations, the KAWs are likely to generate effects like Landau damping and plasma heating.[1]

The KAWs have been a subject of interest in environments where conditions like strong perpendicular gradients to the external magnetic field B_0 and electric field fluctuations parallel to the B_0 are present.[1] Moreover, the presence of effects associated with these conditions, like plasma energization and Landau damping, can suggest the occurrence of KAW waves. In literature, the existence of KAWs in Earthcs auroras,[2] planetary magnetospheres,[3] solar radio bursts,[4] solar flares,[5] heating of solar coronal loops,[6] comets,[7] and extragalactic jets has been discussed.[8] The presence of solid matter like dust in interplanetary and interstellar space drastically influences the properties of KAW. The mobile/static charged dust particles tend to respond to the external and electromagnetic fields of the plasma, and the interaction between the dust and the electromagnetic fields results in the altering of the dispersion characteristics of various wave modes inside the plasma.

The interaction of various dusty plasma wave modes with the KAWs yields several interesting phenomena. The damping of KAWs in dusty plasma have been observed to be attributed to the dust charge fluctuations along with wave particle interaction.[9] It was further reported that the dispersion and damping of the KAWs in dusty plasmas have been modified due to the imbalance between the electron and ion densities.[10] The correlation of the growth rate of instability with temperature and density of plasma ions/electrons, Alfvén speed, and the velocity of dust beam was studied due to parallel streaming of dust by Salimullah et al.[11] The above-mentioned studies assumed that the energy is uniformly distributed among the plasma particles, that is, the plasma particles' energy is defined by the Maxwellian velocity distribution function.

In a series of recent papers on dust KAWs (DKAWs) and KAWs in dusty plasmas, several aspects of these wave modes have been thoroughly studied for low-beta Lorentzian (non-Maxwellian) plasma for superthermal particle populations.[12–15] The KAW can be excited in a plasma by resonant mode conversion for an Magnetohydrodynamics (MHD) wave by a drift wave and free energy provided by the streaming of plasma particles. The interaction of KAWs with ion beam streaming parallel to B_0 renders the KAW mode unstable and has been analysed in a thorough study of KAW in low-beta plasma.[13] A Lorentzian ion beam has been assumed in the aforementioned study, and a correlation has been reported of beam velocity and superthermal particles with the growth and decay of instability. It is noted that the Lorentzian (κ) distribution does not provide adequate resolution for the low-energy parts (plateau) of the particle energy profile. In comparison to the previous studies, which employ non-Maxwellian distribution function, a more generalized form used is the (r, q) distribution function. In the limiting case, $r = 0$ and $q = \kappa + 1$ reduces to Lorentzian distribution function and approaches Maxwellian when $r = 0$ and $q \to \infty$.[16] On the basis of the theory and observations in many space and astrophysical regions, the generalized (r, q) distribution functions deviate significantly from the classical Maxwellian function in the low-energy portion (shoulders) and in the high-energy portion (tail) of the distribution function. Recently, several investigations have used the (r, q) distribution for the analysis of various electrostatic and electromagnetic wave modes.[17–20] The usefulness of (r, q) distribution is significant due to its dependence upon two indices r and q, which control the low- and high-energy profile of the distribution function.[16] A configuration worth mentioning was previously studied to analyse the power dissipation by obliquely propagating Alfvén waves to heat the solar wind protons, which better fits the observations obtained while employing the generalized (r, q) distribution function.[21]

The present work is the study of the effects of streaming ions with (r, q) distribution function and its effect on the properties of Alfvén–acoustic mode. The (r, q) ion beam is assumed to be moving parallel to B_0, which renders the KAW mode unstable. The energy profile of the ion beam as defined by the (r, q) distribution function is expected to give a deeper insight into the consequences of the interaction between the particle beam and KAWs. This strengthens our case of using the (r, q) distribution in order to define the energy profile of a parallel ion beam and hence its effects on KAW mode.

The structure of the manuscript is as follows. In Section 2, we have discussed the generalized (r, q) distribution function. In Sections 3 and 4, a brief description of the basic assumptions and the governing equations that lead to a dispersion relation of KAW have been presented. We have discussed the results and conclusion in Sections 5 and 6, respectively.

2 | MODEL DISTRIBUTION FUNCTION

Generalized (r, q) distribution function is a generalized form of the kappa distribution function where r is the same spectral index that appears in the Davydov–Druyvestien distribution function, which includes flat top and spikes at low energies, and q represents the high-energy tails of the observed velocity distribution function that are found to closely fit with the observed data in different space plasma regimes, that is, terrestrial magnetosphere and solar wind. In the present investigation, we introduce $f_{i0}(v)$ as an equilibrium distribution of ions obeying a generalized (r, q) distribution function,

$$f_{i0}(v) = \Psi \left[1 + \frac{1}{q-1}\left(\frac{v_\perp^2 + (v_\parallel - V_0)^2}{\alpha_{r,q}^2}\right)^{r+1}\right]^{-q},$$

$$\Psi = \frac{3n_0}{4\pi\alpha_{r,q}^3}\left(\frac{(q-1)^{-\frac{3}{2(1+r)}}\Gamma(q)}{\Gamma\left(q - \frac{3}{2(1+r)}\right)\Gamma\left(1 + \frac{3}{2(1+r)}\right)}\right), \tag{1}$$

where $\alpha_{r,q} = \sqrt{\frac{3(q-1)^{-\frac{1}{1+r}}\Gamma\left(q - \frac{3}{2(1+r)}\right)\Gamma\left(\frac{3}{2(1+r)}\right)}{2\Gamma\left(\frac{5}{2(1+r)}\right)\Gamma\left(q - \frac{5}{2(1+r)}\right)}}v_{ti}$ is the thermal velocity that is related to a particle's temperature; Γ is the standard Gamma function; and r and q are the spectral indices representing the shoulders and tail of the distribution profile, respectively. The coefficient Ψ is obtained from the normalization of the distribution function, which obeys the conditions $q > 1$ and $q(r+1) > 5/2$. The (r, q) distribution function approaches the Maxwellian distribution function in the limit $r \to 0$ and $q \to \infty$, while it reduces to the kappa distribution at $r \to 0$ and $q \to \kappa + 1$.

To describe the electron distribution function and obtain the perturbed distribution function to discuss any electromagnetic wave mode, we have used guiding centre coordinates. The perturbed distribution for electrons is given as:

$$f_{e1}(v) = -\left(\frac{n_{e0}e}{T_e}\right)\Sigma_l\Sigma_n\frac{k_\parallel v_\parallel \psi + n\Omega_{ce}\phi}{\omega - n\Omega_{\text{ce}} - k_\parallel v_\parallel}$$
$$\exp\left[i(n-l)\theta\right]J_n\left(\frac{k_\perp v_\perp}{\Omega_{ce}}\right)J_l\left(\frac{k_\perp v_\perp}{\Omega_{ce}}\right)f_{e0}(v), \tag{2}$$

Ω_{ce} is the electron cyclotron frequency.

The charged dust particles are considered cold and unmagnetized and are thus treated using the hydrodynamic approach.

3 | BASIC ASSUMPTIONS AND GOVERNING EQUATIONS

We investigate the electromagnetic KAW instability in an electron ion dusty magnetoplasma by introducing the finite Larmor radius effect. For oblique propagation, the finite perpendicular wavelength gives rise to the wave dispersion. In this analysis, the electrons are assumed to be strongly magnetized, and their energies are defined by the Maxwellian distribution function, while the dust particles are assumed to be cold and unmagnetized with constant charge. The ambient magnetic field $B_0(0, 0, B_0)$ lies along the z-axis, and a beam of ions with streaming velocity V_0 is defined by the (r, q) distribution function that flows parallel to the magnetic field, that is, $(0, 0, V_0)$ and the propagation vector $\mathbf{k}(k_\perp, 0, k_\parallel)$ lie in the xz-plane such that $k_\parallel = k\cos\theta$ and $k_\perp = k\sin\theta$. The plasma beta β_i is considered to be very small because a strong magnetization of electrons exists due to a strong magnetic field B_0, so $\beta_i < 1$, where $\beta_i = 4\pi n_i T_i / B_0$. The low value of plasma beta and the choice of coordinates (oblique propagation) allow us to use two potential theories and to express the components of electric field in terms of two independent electrostatic potentials, $E_{1\perp} = -\nabla_\perp \phi$, and $E_{1\parallel} = -\frac{\partial\psi}{\partial z}$, where $\phi \neq \psi$.

The above expressions of an electric field can be used to write the Poisson Equation (3) and Ampere's and Faraday's coupled Equation (4) in terms of parallel and perpendicular potentials. The Poisson equation is given as,

$$\nabla^2\phi + \frac{\partial^2\psi}{\partial z} = 4\pi e\left[n_{e1} - n_{i1} - \frac{Q_{d0}}{e}n_{d1} - \frac{n_{d0}}{e}Q_{d1}\right], \tag{3}$$

where n_{e1}, n_{i1}, and n_{d1} are the electron, ion, and dust number densities, respectively. Moreover, Q_{d0} is the equilibrium charge on an average dust grain as dust charge fluctuations are ignored; therefore, $Q_{d1} = 0$.

Furthermore, by combining Ampere's and Faraday's Laws, we obtain the following equation:

$$\frac{\partial}{\partial z}\nabla_\perp^2(\phi - \psi) = \frac{4\pi}{c^2}\frac{\partial}{\partial t}\left[j_{e1z} + j_{i1z} + j_{d1z}\right], \tag{4}$$

where j_{e1z}, j_{i1z}, and j_{d1z} are the field aligned electron, ion, and dust current density perturbations, respectively.

With the aid of the Vlasov equation, the perturbed distribution function is given by

$$f_{j1}(v) = -\frac{ek_{\parallel}\psi}{m_j}\frac{1}{\left(\omega - k_{\parallel}v_{\parallel}\right)}\frac{\partial f_{j0}(v)}{\partial v_{\parallel}}. \tag{5}$$

In order to find the number density of a parallel streaming ion beam, we use the value of $f_{i1}(v)$ from Equation (1) in the definition of number density and obtain

$$n_{i1} = -\frac{2n_{i0}e\psi}{m_i\alpha_{r,q}^2}\left[\Lambda_{r,q} + \mu Z_{r,q}(\mu)\right], \tag{6}$$

where $\Lambda_{r,q} = \frac{(q-1)^{-\frac{1}{(1+r)}}\Gamma\left(\frac{1}{2(1+r)}\right)\Gamma\left(q-\frac{1}{2(1+r)}\right)}{2\Gamma\left(\frac{3}{2(1+r)}\right)\Gamma\left(q-\frac{3}{2(1+r)}\right)}$, and $Z_{r,q}(\mu)$ is the modified dispersion function for (r,q) distribution and is defined as

$$Z_{r,q}(\mu) = \frac{3(q-1)^{-\frac{3}{2(1+r)}}\Gamma(q)}{4\Gamma\left(q-\frac{3}{2(1+r)}\right)\Gamma\left(1+\frac{3}{2(1+r)}\right)}\int_{-\infty}^{\infty}\frac{1}{(x-\mu)}\left(1+\frac{x^{2(1+r)}}{(q-1)}\right)^{-q}dx,$$

with $x = (v_{\parallel} - V_0)/\alpha_{r,q}$ and $\mu = (\omega - k_{\parallel}V_0)/k_{\parallel}\alpha_{r,q}$. In the limit $r \to 0$ and $q \to \infty$, the coefficient $\Lambda_{r,q} \to 1$ and Equation (6) approaches a classical form as obtained with Maxwellian distribution function.

By adopting the same procedure for electrons and using the value of $f_{e1}(v)$ from Equation (5), we obtain

$$n_{e1} = \left(\frac{2en_{e0}}{k_{\parallel}m_e v_{te}^3}\Sigma_n\left\{k_{\parallel}v_{te}\psi\left[1+\xi_{en}Z(\xi_{en})\right] + n\Omega_{ce}\phi Z(\xi_{en})\right\}I_n(b_e)e^{-b_e}\right), \tag{7}$$

$Z(\xi_{en})$ is the plasma dispersion function for the Maxwellian distribution function, which is defined as

$$Z(\xi_{en}) = \int_{-\infty}^{\infty}\frac{\exp\left[-s^2\right]}{(s-\xi_{en})}ds,$$

where $\xi_{en} = \frac{\omega - n\Omega_{ce}}{k_{\parallel}v_{te}}$, and $I_n(b_e)$ is the modified Bessel function with argument $b_e = \frac{k_{\perp}^2 v_{te}^2}{2\omega_{ce}^2}$.

The dust particles are cold and unmagnetized, and hence, their number density is calculated using the momentum balance and continuity equations, that is,

$$n_{d1} = \frac{n_{d0}Q_{d0}}{m_d\omega^2}\left(k_{\perp}^2\phi + k_{\parallel}^2\psi\right). \tag{8}$$

The field-aligned current density perturbations of ions, electrons, and dust are respectively given as

$$j_{i1z} = -\frac{2n_{i0}e^2\psi\mu}{m_i\alpha_{r,q}}\left[\frac{(q-1)^{-\frac{1}{(1+r)}}\Gamma\left(\frac{1}{2(1+r)}\right)\Gamma\left(q-\frac{1}{2(1+r)}\right)}{2\Gamma\left(\frac{3}{2(1+r)}\right)\Gamma\left(q-\frac{3}{2(1+r)}\right)} + \mu Z_{r,q}(\mu)\right], \tag{9}$$

$$j_{e1z} = -\frac{n_{e0}e^2}{T_e k_{\parallel}}\Sigma\left\{\left[1+\xi_{en}Z(\xi_{en})\right]\left(k_{\parallel}v_{te}\xi_{en}\psi + n\Omega_{ce}\phi\right)\right\}I_n(b_e)e^{-b_e}, \tag{10}$$

and

$$j_{d1z} = \frac{n_{d0}Q_{d0}^2}{m\omega}k_{\parallel}\psi. \tag{11}$$

4 | DISPERSION RELATION

To determine the dispersion relation of KAW, we first substitute the electron, ion, and dust number densities in Equation (3), whereas the current density perturbations are substituted in Equation (4) to obtain the following set of equations

$$\begin{aligned}\chi_1\phi + \chi_2\psi &= 0,\\ \chi_3\phi + \chi_4\psi &= 0,\end{aligned} \tag{12}$$

where

$$\begin{aligned}
\chi_1 &= k_\perp^2 + \frac{2\omega_{pe}^2}{v_{te}^2}\left[\frac{n\Omega_{ce}}{v_{te}k_\parallel} Z(\xi_{en}) I_n(b_e) e^{-b_e}\right] - \frac{\omega_{pd}^2}{\omega^2}k_\perp^2,\\
\chi_2 &= k_\parallel^2 + \frac{2\omega_{pe}^2}{v_{te}^2}\Sigma_n\left[1+\xi_{en}Z(\xi_{en})\right] I_n(b_e) e^{-b_e} + \frac{2\omega_{pi}^2}{\alpha_{r,q}^2}\left[\Lambda_{r,q} + \mu Z_{r,q}(\mu)\right] - \frac{\omega_{pd}^2}{\omega^2}k_\parallel^2,\\
\chi_3 &= \frac{\omega\omega_{pe}^2}{v_{te}^2 k_\parallel} n\Omega_{ce}\left[1+\xi_{en}Z(\xi_{en})\right] I_n(b_e) e^{-be} + c^2 k_\parallel k_\perp^2,\\
\chi_4 &= \frac{\omega\omega_{pe}^2}{v_{te}}\xi_{en}\left[1+\xi_{en}Z(\xi_{en})\right] I_n(b_e) e^{-be} + \frac{2\omega\omega_{pi}^2}{\alpha_{r,q}}\mu\left[\Lambda_{r,q} + \mu Z_{r,q}(\mu)\right] - k_\parallel\left(k_\perp^2 c^2 + \omega_{pd}^2\right).
\end{aligned} \tag{13}$$

We follow the standard technique to find the dispersion relation of KAW, which is obtained by solving the homogeneous equation, that is,

$$\chi_1\chi_4 - \chi_2\chi_3 = 0. \tag{14}$$

After substituting the values of coefficients χ_1, χ_2, χ_3, and χ_4 from Equation (13) into Equation (14) and performing rigorous simplification, we finally obtain

$$\begin{aligned}
1 &+ \frac{2\omega_{pe}^2}{k_\parallel^2 v_{te}^2}\left\{\left[\frac{n\Omega_{ce}}{v_{te}k_\parallel}Z(\xi_{en}) I_n(b_e) e^{-b_e}\right]\left(1+\frac{\omega_{pd}^2}{c^2k_\perp^2}\right) + \left(1 - \frac{\omega v_{te}\xi_{en}}{2c^2k_\parallel} + \frac{n\omega\Omega_{ce}}{2c^2k_\perp^2}\right)\left[1+\xi_{en}Z(\xi_{en})\right] I_n(b_e) e^{-be}\right\}\\
&+ \frac{2\omega_{pi}^2}{k_\parallel^2\alpha_{r,q}^2}\left\{\left(1 - \frac{\omega\alpha_{r,q}\mu}{c^2k_\parallel}\right)\left(1 - \frac{\omega_{pd}^2}{\omega^2}\right)\left[\Lambda_{r,q} + \mu Z_{r,q}(\mu)\right]\right\} - \left(1 + \frac{k_\perp^2}{k_\parallel^2}\right)\frac{\omega_{pd}^2}{\omega^2} + \frac{\omega_{pd}^2}{k_\parallel^2c^2} + \frac{k_\perp^2}{k_\parallel^2} = 0.
\end{aligned} \tag{15}$$

Equation (15) is the dispersion relation of KAW streaming instability due to a parallel beam of ions obeying the (r, q) distribution function in a dusty magnetoplasma. To analyse the effect of streaming velocity on the wave dispersion, Equation (15) is further solved by the expansions of plasma dispersion functions $Z(\xi_{en})$ and $Z_{r,q}(\mu)$.

For strongly magnetized electrons, we apply the approximations $\omega < < \omega_{ce}$ and $b_e < < 1$ and expand the plasma dispersion functions for $\xi > > 1$ and $\mu < < 1$. The coefficients χ_1, χ_2, χ_3, and χ_4 after applying the approximations become

$$\begin{aligned}
\chi_1 &= \frac{k_\perp^2}{\omega^2} f_e\left(\omega^2 - \omega_{\mathrm{DLH}}^2\right),\\
\chi_2 &= \frac{1}{\lambda_{Di(r,q)}^2} - k_\parallel^2\frac{\omega_{pd}^2}{\omega^2},\\
\chi_3 &= c^2 k_\parallel k_\perp^2,\\
\chi_4 &= \Lambda_{r,q}\omega\frac{\omega_{pi}^2}{k_\parallel\alpha_{r,q}^2}\left(\omega - k_\parallel V_0\right) + k_\parallel\left(k_\perp^2c^2 + \omega_{pd}^2\right),
\end{aligned} \tag{16}$$

where $f_e = \frac{\omega_{pe}^2}{\omega_{ce}^2}$, $\omega_{\mathrm{DLH}}^2 = \frac{\omega_{pd}^2\omega_{ce}^2}{\omega_{pe}^2}$ is the dust lower hybrid frequency, and $\lambda_{Di(r,q)} = \sqrt{\frac{(q-1)^{1/(1+r)}\Gamma(5/2(1+r))\Gamma(q-5/2(1+r))}{3\Gamma(3/2(1+r))\Gamma(q-3/2(1+r))}}\left(\frac{\alpha_{r,q}}{\omega_{pi}}\right)$ is the Debye length of ions due to (r, q) distribution.[22]

Now, by using Equation (14) again and substituting χ_1, χ_2, χ_3, and χ_4 from Equation (16), we obtain

$$\left[\omega^2-\omega_{\mathrm{DLH}}^2\right]\left[\frac{1}{\lambda_{Di(r,q)}^2}\frac{\omega}{k_\parallel}\left(\omega-k_\parallel V_0\right)-k_\parallel\left(k_\perp^2c^2+\omega_{pd}^2\right)\right]-\frac{V_{Ae}^2k_\parallel}{\lambda_{Di(r,q)}^2}\left[\omega^2-k_\parallel^2\lambda_{Di(r,q)}^2\omega_{pd}^2\right]=0, \tag{17}$$

where $V_{Ae}^2=\frac{B_0^2}{4\pi n_{e0}m_e}$ is the electron Alfvén speed,[23] and $n_{e0}m_e$ is the electron mass density. Equation (17) can be further simplified to obtain the quartic equation, that is,

$$a\omega^4+b\omega^3+c\omega^2+d\omega+e=0, \tag{18}$$

where

$$\begin{aligned}
a&=1,\\
b&=-k_\parallel V_0,\\
c&=-\omega_{\mathrm{DLH}}^2-k_\parallel^2V_{Ae}^2-k_\parallel^2\lambda_{Dir,q}^2\left(k_\perp^2c^2+\omega_{pd}^2\right),\\
d&=k_\parallel V_0\omega_{\mathrm{DLH}}^2,\\
e&=k_\parallel^2\omega_{\mathrm{DLH}}^2\lambda_{Dir,q}^2\left(k_\perp^2c^2+\omega_{pd}^2\right)+k_\parallel^4V_{Ae}^2\lambda_{Dir,q}^2\omega_{pd}^2.
\end{aligned} \tag{19}$$

Equation (18) is the dispersion relation of unstable KAW, which shows the coupled Alfvén–acoustic mode in the presence of (r, q) streaming ions. From Equation (17), we can observe that the dispersion relation is a combination of electromagnetic and electrostatic mode, which shows that the shear Alfvén wave develops a longitudinal component due to strong magnetic field. Moreover, this equation reveals that the dusty plasma harbours dust lower-hybrid waves for perpendicular propagation and dust acoustic wave for the parallel propagation of waves. This can be shown by eliminating the ion streaming velocity V_0 and applying the suitable approximations as follows:

For $V_0=0$, the dispersion relation of Equation (17) takes the form

$$\omega^2=\omega_{\mathrm{DLH}}^2+k_\parallel^2V_{Ae}^2\left[1+\sigma_{r,q}\delta_e\left(1+\frac{\omega_{pd}^2}{k_\perp^2c^2}\right)k_\perp^2\rho_e^2\right], \tag{20}$$

where $\sigma_{r,q}=\left(\frac{\Gamma\left(q-\frac{3}{2(1+r)}\right)\Gamma\left(1+\frac{3}{2(1+r)}\right)\Gamma\left(q-\frac{3}{2(1+r)}\right)\Gamma\left(\frac{3}{2(1+r)}\right)}{\Gamma\left(q-\frac{1}{2(1+r)}\right)\Gamma\left(1+\frac{1}{2(1+r)}\right)\Gamma\left(\frac{5}{2(1+r)}\right)\Gamma\left(q-\frac{5}{2(1+r)}\right)}\right)$, $\delta_e=\frac{n_{e0}}{n_{i0}}$, and $\rho_e^2=\frac{T_i}{m_e\Omega_{ce}^2}$.

Equation (20) can also be rewritten in terms of plasma beta β_i as:

$$\omega^2=\omega_{\mathrm{DLH}}^2+k_\parallel^2V_{Ae}^2\left[1+\sigma_{r,q}\delta_e\frac{c^2}{\omega_{pi}^2}\frac{m_e}{m_i}\left(1+\frac{\omega_{pd}^2}{k_\perp^2c^2}\right)k_\perp^2\beta_i\right], \tag{21}$$

where $\beta_i=\frac{4\pi n_{i0}T_i}{B_0^2}$.

Equation (21) presents a KAW as a dispersion relation and gives a clear picture of the coupling of KAW with dust acoustic wave, where $k_\perp^2\,\rho_e^2$ is the coupling parameter and the dust lower-hybrid wave. The coupling between the kinetic Alfvén and acoustic mode occurs due to the Doppler-shifted (r, q) streaming ions along the direction of the magnetic field. It can also be observed that, for small values of β_i, the coupling becomes weak, and the two modes decouple. Moreover, by ignoring the streaming of ions, we obtain the hydrodynamic relation for the KAW in a dusty magnetoplasma, that is,

$$\omega^2=\omega_{\mathrm{DLH}}^2+k_\parallel^2V_{Ae}^2$$

Now, if we consider Equation (19) and apply the limit $k_\perp^2\rho_e^2 << 1$, the equation becomes:

$$\omega^2=\omega_{\mathrm{DLH}}^2\left(1+\frac{k_\parallel^2c^2}{\omega_{pd}^2}\right),$$

where $\omega_{\mathrm{DLH}}^2=\frac{\omega_{pd}^2\omega_{ce}^2}{\omega_{pe}^2}$ is the dust lower-hybrid frequency.

4.1 | Longitudinal stable/unstable modes

In the limit $\beta_i << 1$ and $\frac{\omega^2}{k_\parallel^2 V_{Ae}^2} \to 0$, Equation (20) reduces to:

$$\omega^2 = \sigma_{r,q} k_\parallel^2 \frac{Z_{d0} T_i}{m_d},$$

which is the dispersion relation for dust acoustic mode with generalized (r, q) distribution function as discussed by Zaheer et al.[17] This relation reduces to Maxwellian for $r = 0$ and $q \to \infty$ and to Lorentzian form in the limit $r \to 0$ and $q \to \kappa + 1$, which further reduces to Maxwellian in the limit $\kappa \to \infty$.

In the limit, $n = 0$, $b_e < 1$ and $k \| B_0$, that is, the wave propagation is along the magnetic field and streaming direction (B_{0z}); then, the particles can flow easily along the magnetic field directions, while their motion in the perpendicular direction is suppressed by the gyromotion of plasma particles; therefore, this is the most plausible case.

$$1 + \frac{2\omega_{pe}^2}{k_\parallel^2 v_{te}^2} \left[1 + \xi_{en} Z\left(\xi_{en}\right)\right] + \frac{2\omega_{pi}^2}{k_\parallel^2 \alpha_{r,q}^2} \left[\Lambda_{r,q} + \mu Z_{r,q}\left(\mu\right)\right] - \frac{\omega_{pd}^2}{\omega^2} = 0. \tag{22}$$

Under the conditions discussed above, Equation (22) can be described in the form of electrostatic two-stream instability in a dusty plasma. It is interesting to note that, in the limit $r \to 0$ and $q \to \kappa + 1$, $\Lambda_{r,q} \to (\kappa - 1/2)/\kappa$ and $Z_{r,q}(\mu) \to Z_{0,\kappa+1}(\mu)$, $\mu \to (\omega - k_\parallel v_0)/k_\parallel \alpha_{0,\kappa+1}$, where $\alpha_{0,\kappa+1} = \{(2\kappa - 3)/\kappa\}^{1/2} v_{th}$.[17] When the flow direction is across the field direction, we obtain a dispersion equation of modified two-stream instability.

5 | NUMERICAL RESULTS

We have computed Equation (18) and carried out instability analysis of KAWs for different values of spectral indices r and q. We have discussed the influence of parameters like plasma beta β_i, streaming velocity V_0, and the dust charge Z_{d0} on the characteristics of KAWs. For the numerical analysis, the dispersion relation given in Equation (18) has been plotted, and the solutions of this equation are discussed for several parameters that are close to the dusty plasma environments native to the interstellar clouds. To study the waves and instabilities in interstellar clouds, heavy dust particles must be considered a component of the neutral fluid. A general analysis has been carried out to indicate that the presence of dust particles affects the electromagnetic instability. The range of numerical values of the fixed parameters is $n_{e0} = 1 - 10^2 \mathrm{cm}^{-3}$, $n_{i0} = 10 - 10^4 \mathrm{cm}^{-3}$, $n_{d0} = 10^{-2} - 10\ \mathrm{cm}^{-3}$, $Z_d = 10 - 10^4$, and $m_d = (10^5 - 10^8) m_i$. For computational convenience, we introduce the following dimensionless variables:

$$\widetilde{\omega} = \frac{\omega}{\Omega_{ce}}, \widetilde{k} = \frac{kV_A}{\Omega_{ce}}, \widetilde{V}_0 = \frac{V_0}{V_A}$$

Figure 1a exhibits the general behaviour of the instability characteristics of KAWs by using a generalized (r, q) distribution function. In Figure 1a, we have fixed $r = 0$ and changed the values of q (=3,4,5,15) in the computation and observed the unstable region, which significantly departs from the Maxwellian distribution function. It can be seen that, as the value of q decreases, the growth rates seem to be suppressed, which is because, for the low values of q, the high-energy component in the tail of the distribution function tends to increase, which increases the number of high-energy particles.

The growth rates tend to enhance as we increase the value of spectral index q and hence approaches the Maxwellian when the contribution of low-energy particles dominates. The real and imaginary parts of the quartic equation are depicted in Figure 1b, while all solutions of Equation (18) are presented in Figure 1c where the unstable modes originate due to the coupling of Alfvén–acoustic mode when ions are streaming along the field direction.

Similarly, Figure 2 shows the effect of the variation of the spectral index r on the growth rates, whereas the spectral index q(=5) remains fixed. We have noticed that the growth rates become higher with increasing r, which is attributed to the dominance of low-energy particles, and hence, the shoulders of the distribution function become more prominent. Furthermore, it may be noticed that, for various values of r, the growth rates are constant for small wavenumber $\widetilde{k}_\parallel$. At large values of $\widetilde{k}_\parallel$, the growth rate tends to increase, which implies that the effect of changing the index r is somehow considerable for small wavelengths, whereas for large wavelengths, it is negligible. It is worth mentioning here that the growth rates for KAW instability are more sensitive to spectral index q than r as presented above.

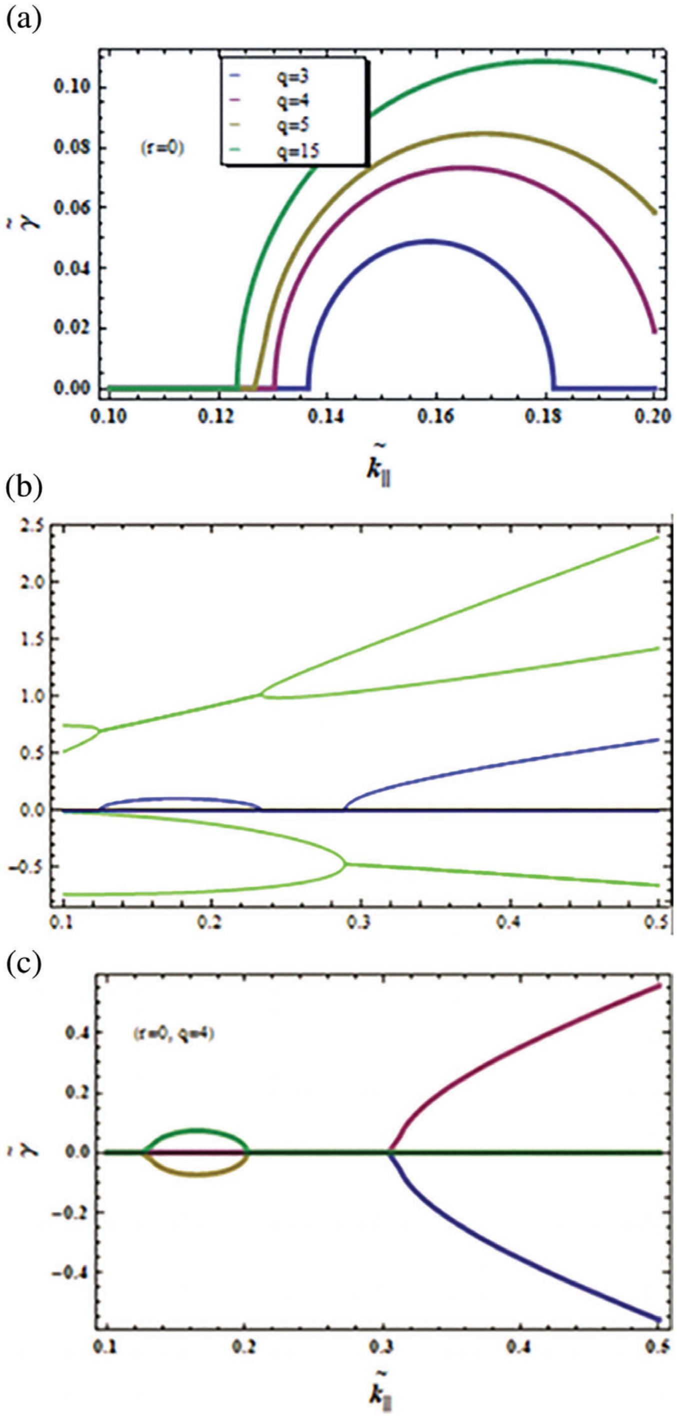


FIGURE 1 (a) Imaginary ($\overline{\gamma}$) part as a function of $\overline{k}_{\parallel}$ plotted for $r = 0$ and different values of q. (b) Graph showing numerical solution of the dispersion relation in situation of one plasma beam penetrating into the plasma background. Imaginary parts are depicted in blue colour and are responsible for instability growth. (c): Imaginary ($\overline{\gamma}$) part of dispersion Equation (18) as a function of ~kk plotted for $r = 0$ and $q = 5$

Figure 3 depicts the effect of various values of β_i on the growth rate of KAW. For $\beta_i < 0.2$, the Alfvén–acoustic modes decouple, and there is no instability, while for $\beta_i \simeq 0.2$, the lower-panel (see Figure 1b) mode is unstable. As we continue increasing $\beta_i(\sim 0.5)$, both upper- and lower-panel unstable regions are observed. Moreover, as β_i increases, the growth rates are enhanced, and the onset of instability is shifted towards a smaller parallel wave number $\widetilde{k}_{\parallel}$. This implies that, for small β_i values, the unstable region is significantly reduced, which indicates that the presence of a stronger magnetic field B_0 ($\beta_i < 1$) acts as a stabilizing factor. However, when the magnetic pressure approaches particle pressure ($\beta_i \simeq 1$), the KAW becomes unstable abruptly.

Figure 4 shows the impact of varying charge numbers on dust grain Z_d, when $\widetilde{\gamma}$ is plotted as a function of $\widetilde{k}_{\parallel}$. It is apparent that the instability becomes stronger as the charge on the dust grain Z_d increases, the unstable regions become wider, and the onset of instability shifts towards a large wavenumber $\widetilde{k}_{\parallel}$. As the number of electrons on a mobile dust

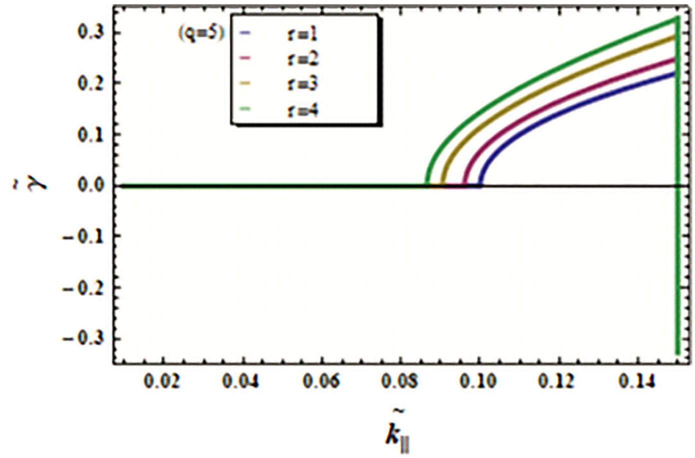


FIGURE 2 Imaginary ($\overline{\gamma}$) part as a function of $\overline{k}_{\parallel}$ plotted for $q = 5$ and various values of r

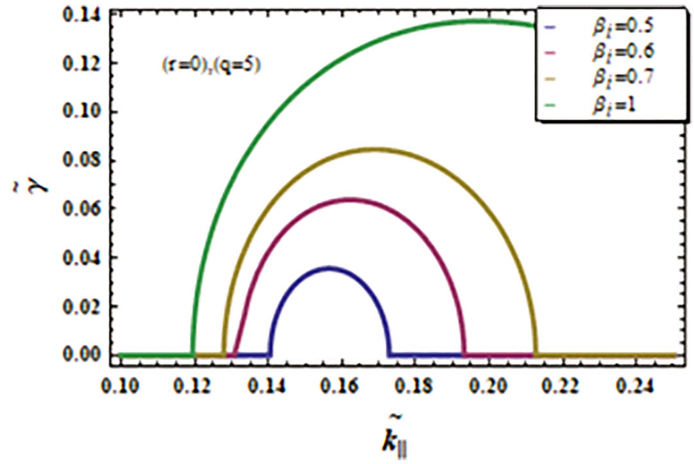


FIGURE 3 Imaginary ($\overline{\gamma}$) part as a function of $\overline{k}_{\parallel}$ plotted for $r = 0$; $q = 5$ and various values of β_i

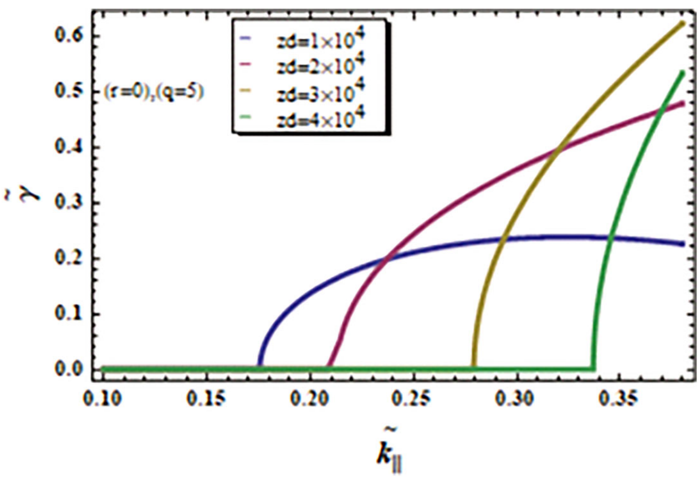


FIGURE 4 Imaginary ($\overline{\gamma}$) part as a function of $\overline{k}_{\parallel}$ plotted for $r = 0$; $q = 5$ and various values of Z_d

grain surface becomes non-zero, it gives rise to the dust lower-hybrid frequency ω_{DLH} due to the hybrid motion of electrons and dust, which significantly modifies the wave characteristics and provides a cut-off for the propagation of an Electromagnetic (EM) wave.

Figure 5a illustrates the consequences of the variation of streaming velocity $\widetilde{V}_0$ on the growth rates when $\widetilde{V}_0 > \widetilde{V}_A$. It is presented in order to excite the KAWs; the streaming velocity should be in the sub-Alfvén range, but, when it extends to super-Alfvén range, the growth rates are significantly suppressed. As we start increasing $\widetilde{V}_0$ from smaller values, that is, ($\widetilde{V}_0 = 2$), the growth rates amplify, whereas a further increase tends to stabilize the growth rates, and its onset shifts

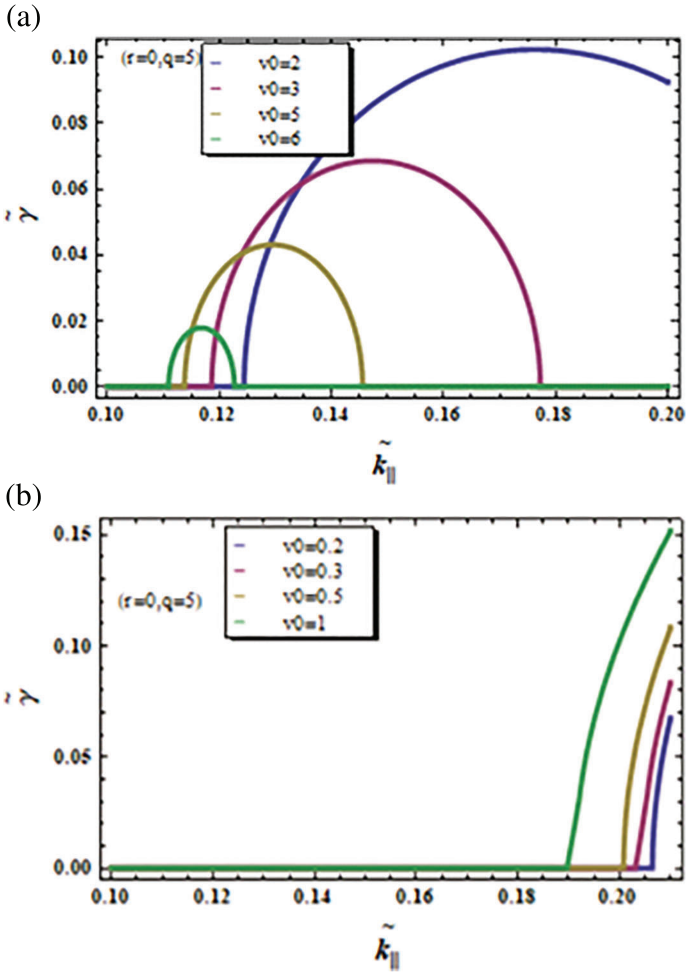


FIGURE 5 (a) Imaginary $(\overline{\gamma})$ part as a function of $\overline{k}_{\|}$ plotted for $r = 0$, $q = 5$, and various values of V_0 when $V_0 > V_A$. (b) Imaginary $(\overline{\gamma})$ part as a function of $\overline{k}_{\|}$ plotted for $r = 0$; $q = 5$ and various values of V_0 when $V_0 \le V_A$

towards smaller $\widetilde{k}_{\|}$, which is depicted in Figure 5a. In the limit $V_0 < V_A$, the growth rates tend to enhance in magnitude, whereas in the limit $V_0 \to V_A$, the instability becomes large and reaches its maximum value when $V_0 = V_A$ as shown in Figure 5b. As the streaming velocity V_0 is a possible source of free energy, when the velocity of parallel streaming ions approaches Alfvén velocity, the interaction between waves and the ion beam increases and contributes towards the growth of instability. When streaming velocity becomes faster than the Alfvén velocity, the interaction reduces, and thus, the instability leads to decay.

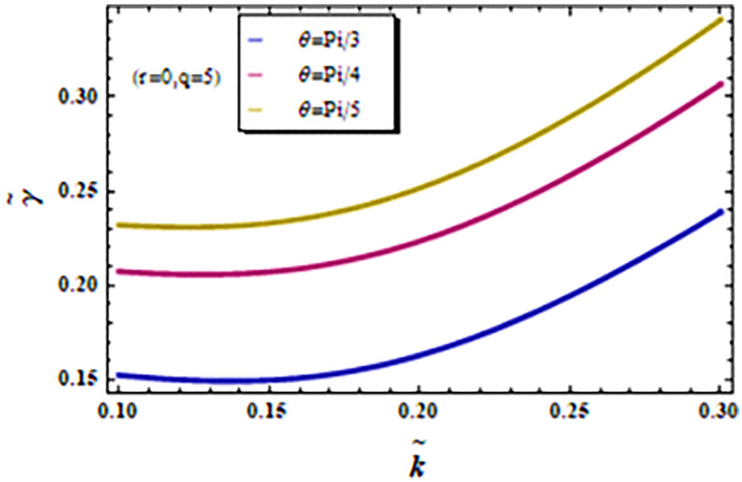
Finally, we have also shown the effect of the direction of propagation on the growth rates (Figure 6), which demonstrates that large propagation angles suppress the growth rates.

6 | CONCLUSION

A theoretical model for KAWs consisting of hot and magnetized electrons, non-Maxwellian ion beam obeying (r, q) distribution function, and mobile charged dust grains is presented. The dispersion analysis of the Alfvén–acoustic coupling in the vicinity of a streaming ion source obeying (r, q) distribution function is analysed when the characteristic scales (wavelength) are comparable to ion gyroradii. This finite Larmor radius (FLR) effect leads the Alfvén waves to couple with electrostatic waves, resulting in the generation of the KAW. When a relative drift between electrons/dust and ions is admissible, the streaming instabilities develop. The KAW is also excited by the electrostatic coupling, which requires density perturbation.

The current study is motivated by a number of investigations on electromagnetic waves, which have been observed in the terrestrial magneotail with accompanying flat-topped distribution of electrons. The (r, q) distribution is favourable as it emulates model plasma waves observed in space plasmas, which show a significant departure and a better fit as

FIGURE 6 Imaginary ($\overline{\gamma}$) part as a function of $\overline{k}$ plotted for $r = 0$, $q = 5$, and various values of θ



obtained for its Maxwellian counterpart.[16,24] The insight obtained from the study of streaming ions with (r, q) distribution function on the instability characteristics of KAW can be applied to understand the significance of low-energy particles in the excitation and stability characteristics of the waves.

The superthermal particles in the interplanetary medium range in energy from solar wind (~keV) to galactic cosmic rays (~GeV), and they appear to originate from various sources, like the galaxy, planetary magnetospheres, and the nearby interstellar medium. The temperature of interstellar molecular clouds is deduced from the balance between the various cooling processes and the heating processes through the ionization caused by the superthermal particles. These high-energy particles are believed to play an important role in the formation of gas and dust clouds of the astrophysical entities. In a quest to study the kinetics and transport of superthermal particles, various mathematical models are formulated that use the probabilistic interpretation of collisions of particles where the distribution function was found to deviate significantly from equilibrium. It was observed using the Monte Carlo method that the velocity distribution approaches stationary states at low and high temperatures, which differ significantly from the Maxwellian distribution function.[25,26]

Much of intercloud and interstellar turbulence is super-Alfvénic waves, which play a vital role in the star formation. In these regions, supertherrmal particles may produce partially ionized dust molecular clouds, and the presence of dust particles modify the propagation characteristics of the wave not only by introducing a great variety of novel modes but also by modifying the previously known classical modes. The low-frequency KAWs (below ion gyrofrequency) in a dusty plasmas decay very slowly, and the turbulent cascade may be converted into a cascade of KAWs and allows the stability of the molecular cloud. As the molecular clouds have an excess of charged dust particles, the presence of magnetized dust particles introduce a cut-off in the Alfvén wave at dust cyclotron frequency (while in our investigations, the charged unmagnetized dust grains provides a cut-off at dust lower-hybrid frequency, which provides a limit to the propagation of electromagnetic mode). The streaming instabilities may be suppressed due to interaction with the background turbulence of interstellar medium and thus allow the wave damping in a nonlinear regime, which may be important when investigating wave-driven mass loss from evolved stars.

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DATA AVAILABILITY STATEMENT

Research data are not shared.

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