Shariah review of duration models: issues and future research directions

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Abstract: This research performs Shariah review of Macaulay's duration and all subsequently developed related models. It finds that there are twenty one models of duration that have been introduced so far but none of them is fully Shariah compliant for use in Islamic banks. The research identifies a variety of future research directions that include development of separate yield curves for assets and liabilities of Islamic banks; how book value based models are more compliant with Shariah; and need for incorporation of specific factors in an Islamic financial model to make them more Shariah effective in Islamic bank risk management.

Keywords: Islamic banks; duration models; Shariah review; interest rate risk; maturity gap risk management.

Reference to this paper should be made as follows: Shah, S.A.A., Sukmana, R. and Fianto, B.A. (2021) 'Shariah review of duration models: issues and future research directions', *Int. J. Accounting, Auditing and Performance Evaluation*, Vol. 17, Nos. 3/4, pp.336–360.

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1 Introduction

Macaulay (1938) extends the work of Lidstone (1895) who examined the relationship of changes in interest rates and asset values. Macaulay (1938) observes that long term bond prices fluctuate more than the prices of short term bonds except in a few specific circumstances. He proposes a new measure for bond tenure that can help understand variations in bond prices. A measure that is based on present values calculated on the basis of interest rates. Macaulay terms his measure as 'Duration' expressed mathematically as:

$$D = \sum_{t=1}^{N} w_t t \tag{1}$$

where

D: duration

w_t: present values of cash flows as weights, where $\sum_{t=1}^{N} w_i = 1$ and $0 \le w \le 1$

t: Relevant time period of cash flows

The function proposed by Macaulay is:

$$\left[\frac{1}{P(m)}\right] \times \sum_{t=1}^{m} C(t)t\left[1+R(t)\right]^{-t} \tag{2}$$

where

$P(m)$: price of bond

$C(t)$: stream of cash flows

t: timing of cash flows

$R(t)$: spot rates.

Investors use duration in making risk strategies. Risk taking investors use duration to exploit price volatilities whereas risk averse investors use it for immunisation. For institutions, it plays pivotal role in asset pricing and managing balance sheet stress (Gormsen and Lazarus, 2019).

Prominent research using duration includes Babbel et al. (1997) for the role of default risk; Acharya and Carpenter (2002) for valuing corporate bonds; Adrian and Shin (2010) for leverage and liquidity; Bajo et al. (2013) for estimating interest rate risk; Bessler and Wolff (2014) for government bond portfolios hedging; Beccacece et al. (2018) for fair value adjustment; Chattha and Alhabshi (2018) and Chattha et al. (2020) for benchmark and rate of return risk in Islamic banks; and Chattha et al. (2020) for duration gap risk management in Islamic banks. Recently, Shah et al. (2020a) proposed and Shah et al. (2021b, 2021c) tested Shariah compliant duration models. Historically, various other great works on duration modelling are led by Bierwag, Fooladi, Roberts, Cox and Kaufman for its applications in derivatives, immunisation and managing effects of interest rates respectively.

Ingersoll et al. (1978) and Bierwag and Fooladi (2006) provided two great reviews of literature on duration modelling followed by a more recent by Shah et al. (2020b) who

have consolidated all works on duration models published in the last 80 years. However, with recent expansions in Islamic banking the need for Shariah review of existing duration models has become need of the time Shah et al. (2020a, 2020b, 2021b). This research therefore, proceeds first by making an account of all duration models presented so far. Secondly, it performs Shariah review to evaluate how these models lack Shariah compliance, and finally by establishing future research directions in this regard.

1.1 Methodology

Research suggests that over and above the obvious difference of Shariah compliance, Islamic banks are different from conventional banks both in terms of financial structure (Chattha and Alhabshi, 2018; Chattha et al. (2020) and efficiency (Shah and Masood, 2017; Shah et al., 2021d). The focus of financial theory in asset and liabilities management is risk and return analysis. The Islamic equivalent of risk and return analysis can be regarded as analysis of gharar and Shariah compliant income. Therefore, investment management in Islamic context requires avoiding 'excessive gharar' and earning 'riba' free income. Its logical consequence is such a risk management mechanism that avoids 'excessive gharar' and 'riba'. With an intention to establish such an environment this research uses and extends the work of Shah et al. (2020a) to review various duration models. The literature in this regard has been collected according to the recommendations of Apriliyanti and Alon (2017), Falagas et al. (2008) and Alon et al. (2018) for use of ISI Web of Science, Scopus and Google Scholar. Lastly, Shariah review criteria of Shah et al. (2020a) have been augmented to evaluate all duration models that have identified deficiencies and established future research directions in duration modelling. The parameters of Shah et al. (2020) have been reproduced hereunder:

1.1.1 Parameters of a financial model for Islamic banks

- It should incorporate realised rates of returns earned and paid, benchmark rates, interbank offered rates and industry standards.
- Avoiding all future based transaction rule applies to a financial model as well. Accordingly, the financial model should avoid incorporating variables that can give rise to excessive gharar i.e., the model should not include all future value based variables. For the purpose of a model, this condition shall be applied in such a way that future based variables should not be more than 50% of the total variables used in the model and the composition of variables should not give rise to results of which more than 50% will be expected.
- The composition of variables in the model should not give rise to overall results that breach the 5,33,49 rule.
- As the returns earned and paid are determined at the end of the period, therefore model shall utilise only realised values not the expected values as are used in the case of Macaulay's duration model.
- The model shall function backwards i.e., it will calculate values from end of the year to beginning of the year. It is because the model uses realised values. The values so calculated shall be termed as 'Reversed Present Values'.

- Models should be proposed for intra-year and inter-year risk analysis and management.

Over and above the criteria of Shah et al. (2020a) this research profoundly explores the issue of 'excessive gharar' and takes stipulations from the works of Chapra (1986) and Farooq (2009) about 'riba alfadal' and 'riba alnasiah'. These have been included to derive guidelines from riba and gharar contexts about a Shariah compliant measure of risk management.

1.1.2 Gharar

Gharar has been primarily defined in the context of buying and selling of commodities. The most widely quoted definitions of gharar are "uncertainty or hazard caused by lack of clarity regarding the subject matter or the price in a contract or exchange" (Ayub, 2007, p.57); and gharar as 'trading in risk' (El-Gamal, 2006, p.61). Using these and many other similar definitions Muslim researchers have further evolved the concept of gharar into 'gharar el-kathir' (excessive gharar) that is prohibited and 'gharar qalil' that means business risk (El-Gamal, 2006; Ayub, 2007). Ayub (2007, p.61) narrates that in order to avoid 'gharar el-kathir' the following principles must be adhered:

i the financial contracts must not contain extreme uncertainty in relation to the subject matter and its counter value in exchanges

ii the product must be well defined, determined and distinctly identified in the contract to all parties

iii quality and quantity of the subject matter must be specified

iv a contract must not be unsure or indeterminate, as the entitlements and duties of the contracting parties have to be known to avoid any future abuse

v there should be no negligence (jahl) or uncertainty about obtainability, existence and deliverability of commodities and all parties should know their real conditions.

The above principles make gharar el-khatir(excessive gharar) arising only from two scenarios viz., the uncertainty about price and the uncertainty about commodity. However, if the hadith related to gharar is re-examined it widens the scope of gharar from merely sale and purchase of commodities to sale and purchase of values as well. In one, Ibn Majah narrated on the authority of Abu-Said al-Khudriy as quoted by Esore (2013) that:

> "The Prophet (pbuh) has forbidden the purchase of the unborn animal in the mother's womb, the sale of the milk in the udder without measurement, the purchase of spoils of war prior to their distribution, the purchase of charities prior to their receipt, and the purchase of the catch of a diver."

In the above "purchase of spoils of war prior to their distribution" and "purchase of charities prior to their receipt" are clearly the scenarios that involve distribution of values over and above physical objects. These two scenarios are examples of expected distribution of values the trading of which has been categorically prohibited on account of gharar el-kathir(excessive gharar) irrespective of the creditworthiness standing of the value distribution authority.

Extending the analogy to modern streams of inward cash flows, it can be observed that cash flow disbursing organisations themselves are subject to various credit ratings. These credit ratings predict the standing and ability of respective organisations to survive in future in such a way that even the highest rated organisations are not expected to survive with 100% surety, which has also been witnessed during the financial crises of 2008. In this regard, Hull (2012) devotes a complete Chapter 8 on dynamics of financial crisis 2008, which explains how false relying on guaranteed future cash flows triggered the whole crisis. Summing up, in the light of hadith quoted above and subsequent analytical discussions of credit rated institutions, it transpires that future cash flows always involve 'excessive gharar' irrespective of the commitments or sureties from disbursing organisations.

Another example in this regard can be quoted from Bay al-Dayn that is trading of 'debt' at par (Iqbal, 2020). However, Bay al-Dayn is firstly a consequence of a trading transaction and secondly cannot be sold at a discount despite the debt amount to be received at some future date. This also means that future receivable amounts cannot be discounted on account of conventional concept of time value of money in case of trading of debts (Iqbal, 2020).

1.1.3 Riba

About 'riba alnasiah' the hadith says: "The Prophet (peace be upon him) said: 'If anyone makes two transactions combined in one bargain, he should have the lesser of the two or it will involve usury (riba al-nasiah)" (Abu Dawud Book 23, Number 3454, narrated by Abu Hurayrah). On the other hand 'riba alfadal' actually relates to exchange (buy and selling). Al-Razi explains it in the words that "While the earning of profit is uncertain, the payment of interest is predetermined and certain. The profit may or may not be realised. Hence there can be no doubt that the payment of something definite in return for something uncertain inflicts a harm" (Tafsir al-Kabir, Chapra, 1985, p.63). In the words of Farooq (2009, p.112) "Here Al-Razi focuses on the nonequivalence of uncertain profit (from a loan) and set interest (increase on a loan). Without the delay inherent in a loan, the issue of uncertainty would not arise; hence, both these factors working together to create an unjust transaction in which one party benefits from the other's carrying the burden of 'uncertainty'. The same injustice is found in 'riba al-fadl' exchanges whose asymmetries are driven by waiting and uncertainty."

From the above explanations of the dynamics of riba it can be inferred that occurrence of 'riba alfadal' gives rise to 'excessive gharar'. This scenario gives rise to two further scenarios. First, the cost of commodity becomes uncertain; secondly, uncertain cost will lead to inaccurate profits, which amounts to injustices and exploitation. As was also narrated by Chapra (1985, p.61):

> "Because trade is allowed in principle, it does not mean that everything is allowed in trade. Since the injustice inflicted through 'riba' may also be perpetuated through business transactions, 'riba alfadl' refers to all such injustices or exploitations.. .. While 'riba alnasiah' was well-known in the Jahiliyyah (pre-Islam period) the concept of 'riba alfadl' was introduced by Islam and reflects the stamp of its own unflinching emphasis on socioeconomic justice." (Chapra, 1985, p.61).

Applying this analogy in the case of financial modelling it can be argued that assets and liabilities, their respective cash flows and returns emanate from separate buying and

selling transactions. As executing buying and selling transactions in one-bargain amounts to "riba al-nasiah", similarly using same benchmark rates and rates of return for valuation of assets and liabilities of Islamic banks gives rise to occurrence of 'riba al-nasiah' as well. This has to be dealt with using different benchmark rates and rates of return for assets and liabilities. This contrasts to the practices of conventional financial institutions that use interest rates and related benchmarks. Having developed the methodology of Shariah review, Shariah compliance analysis of various duration models has been given hereunder:

1.2 Re-explanation of duration

Shirvani and Wilbratte (2002) has provided an interesting and thought provoking definition of Macaulay's duration that has been reproduced hereunder:

> "Just as any physical object can be compressed into a single massive point at its center of gravity, the stream of cash payments of a coupon bond can be compressed into a single lump sum at its duration. Thus, any coupon bond with duration D can be represented as an equivalent zero coupon bond with maturity D. Furthermore, in the same way that more stable objects have lower centres of gravity, bonds with more stable values are those with lower durations."

They narrated *D* as a function of five factors:

$$D = f(t, n, C, M, k)$$

where

D: duration

t: time period

n: periods to maturity

C: coupon amounts

M: principal value

k: market interest rates.

From the list of these factors, principal value and time period remain fixed, while others are variable. Ceteris paribus, duration is positively related to period of maturity, and negatively to coupon amounts and market interest rates (Shirvani and Wilbratte, 2002).

Nivine et al. (2010) find that duration is a short term measure of changes in equity in response to changes in interest rate. For gauging the effect of larger changes in interest rates, however, they recommend convexity (Nivine et al., 2010).

2 Shariah review of duration models

2.1 Additive multiplicative models

Gultekin and Rogalski (1984) report similar results of all duration models. They examine the following models:

If Macaulay's duration can be defined by the following function:

$$D = \frac{1}{P(m)} \sum_{t=1}^{m} C(t)\, t (1+y)^{-t} \tag{3}$$

where P(m) means price of the security, m means maturity, $C(t)$ means cash flow stream from the bonds, t represents various intervals at which regular cash flows will be received and y being the yield to maturity.

Then Macaulay's model of duration along with subsequent models of Cooper (1977), Bierwag (1977), Bierwag and Kaufman (1978) and Khang (1979) can be presented in the following functional forms:

$$D1 = \left[\frac{1}{P(m)}\right] \int_{o}^{m} C(t)\, t\, exp\left[-R(t)t\right] \mathrm{d}t \tag{4}$$

$$D2 = \left[\frac{1}{P(m)}\right] \int_{o}^{m} C(t)\, tR(t)\, exp\left[-R(t)t\right] \mathrm{d}t \tag{5}$$

$$D3 = \left[\frac{1}{P(m)}\right] \int_{o}^{m} C(t)\, t^2 R(t)\, exp\left[-R(t)t\right] \mathrm{d}t \tag{6}$$

$$D4 = \left[\frac{1}{P(m)}\right] \int_{o}^{m} C(t)\, t \ln(t)\, R(t)\, exp\left[-R(t)t\right] \mathrm{d}t \tag{7}$$

$$D5 = \left[\frac{1}{P(m)}\right] \int_{o}^{m} C(t)\, t^2 exp\left[-R(t)t\right] \mathrm{d}t \tag{8}$$

$$D6 = \left(exp\left[\frac{1}{P(m)}\right] \times \int_{o}^{m} C(t)\, ln(1+\alpha t)\, exp\left[-R(t)t\right] \mathrm{d}t \right) - \frac{1}{\alpha} \tag{9}$$

In the above expressions $R(t)$ is the spot rates of respective cash flows and notation 'α' is the measure of variation in longer period yields with respect to shorter period yields. Where greater value of 'α' implies more changes in short terms rates relative to long term rates.

2.1.1 Shariah review

All of the models tested by Gultekin and Rogalski (1984) involve expected future values of cash flows, that is 'excessive gharar' and interest rates that is 'riba' prohibited in Islam in all its forms hence all the models are non compliant with Shariah.

2.2 Stochastic duration models

Cox et al. (1979) present duration models presently known as stochastic duration models. They base their proposition on the assumption that term structure of interest rates does not change in any predicted manner. The stochastic duration can be computed therefore using the following function:

$$D7 = G^{-1}\left[\frac{\sum C(t)P(t)G(t)}{C(t)P(t)}\right] \tag{10}$$

where:

$$G^{-1}x = \frac{2}{\gamma}coth^{-1}\left(\frac{2}{\gamma x} + \frac{\pi - \beta}{\gamma}\right) \tag{11}$$

$$P(t) = F(t)exp[-rG(t)] \tag{12}$$

$$F(t) = \left\{\frac{2\gamma exp\left[\frac{(\gamma + \beta - \pi)t}{2}\right]}{(\gamma + \beta - \pi)\left[exp(\gamma t) - 1\right] + 2\gamma}\right\}^{2\beta\mu/\sigma^2} \tag{13}$$

$$G(t) = \frac{2}{\left[\beta - \pi + \gamma coth(\gamma t/2)\right]} \tag{14}$$

where:

$$\gamma = \left[(\beta - \pi)^2 + 2\sigma^2\right]^{1/2}$$

$\pi = liquidity\ premium$

whereas duration for a pure discount bong is $G(t)$.

2.2.1 Shariah review

Stochastic duration models actually include interest rates that are to be generated based on some stochastic process in future. This is actually equivalent to adding 'excessive gharar' in 'riba', which means making something 'prohibited' as 'more prohibited'! Since the variable so created will be occurring in the future so its nature remains futuristic that involves 'excessive gharar'. In other words we can opine that stochastic measure amounts to 'excessive gharar' that is prohibited in Islam.

2.3 Duration and estimation of present values using Taylor expansion

Livingston and Zhou (2005) apply Taylor's Expansion to the logarithm of present value function of cash flows. They present the functions of duration of cash flows as:

$$D(i) = -(1+i)\frac{P'(i)}{P(i)} \tag{15}$$

and for approximated cash flows the duration has the following function:

$$D^i(i_o) = \frac{-M^2(i_o)}{1+i_o} = \frac{D^2(i_o) + D(i_o)}{1+i_o} - (1+i_o)C(i_o) \tag{16}$$

where M^2which actually measures dispersion equals

$$M^2\left(i_i\right)=\left(1+i_o\right)^2 C\left(i_o\right)-D^2\left(i_o\right)-D\left(i_o\right) \tag{17}$$

And $C(i_o)$ equals

$$C\left(i_o\right)=\frac{P^{''}\left(i_o\right)}{P\left(i_o\right)} \tag{18}$$

Dierkes and Ortmann (2015) base their duration model on estimated changes in present values of cash flows resulting from changing in interest rates and their respective curves. They base their approximation on linear differential equations that produce far more superior results than Taylor approximation. They argue that if we express the $D(i)$ function in terms of $P'(i)$ function, the function can be written as:

$$P^{'}\left(i\right)\approx-\frac{D\left(i_o\right)}{1+i}P\left(i\right) \tag{19}$$

Which can be solved by:

$$P\left(i\right)=c\left(1+i\right)^{-D\left(i_o\right)} \tag{20}$$

2.3.1 Shariah review

Estimating cash flows on the basis of a mathematical model adds to the expectations involved in the process of estimating risks as has been explained in the Shariah review 2.2.1 hereinabove. Excessive expectations mean 'excessive gharar' that render such models non Shariah compliant.

2.4 Effective duration

Leland (1994) and Leland and Toft (1996) while searching for optimal capital structure introduce a modification in Macaulay's duration, which they term as 'Effective Duration'. In their analysis of bonds, they find that bonds that default subsequently have shorter effective duration than that of Macaulay's, which in some cases was even negative.

They present the following function for effective duration:

$$Effective\,Duration=\frac{\%\,Change\,in\,Bond\,Price}{Changes\,in\,Interest\,Rates}=\frac{P_{Increase}-P_{decrease}}{2P_{Initial}\Delta r} \tag{21}$$

where

P_{Increase} = Increase in the price of bond

P_{decrease} = Decrease in the price of bond

P_{Initial} = Initially quoted bond price

Δr = Change in rate of interest in

2.4.1 Shariah review

The model of effective duration is based on changes in interest rates. The interest itself is actually riba the use of which is strictly prohibited in Islam. This model is therefore non Shariah compliant.

2.5 Use of duration in interest rate response of the banks

Bierwag and Kaufman (1985, 1992, and 1996) explore the idea of interest rate exposure of banks and other financial institutions. They argue that if we consider the net worth of a financial institution as $E(r) = A(r)–L(r)$ where $E(r)$ is the risk sensitive net worth, $A(r)$ is the value of risk sensitive assets and $L(r)$ is the value of risk sensitive liabilities, the effect of interest rate changes on risk sensitive net worth of a financial institution can be calculated using the following function:

$$\Delta E = -A\left(DGAP_E\right)\Delta r \tag{22}$$

where

$$DGAP_E\left(r\right) = \left[D_A\left(r\right) - \frac{L\left(r\right)}{A\left(r\right)}D_L\left(r\right)\right]\left(1+r\right)^{-1} \tag{23}$$

Bierwag and Fooladi (2006) argue that duration gaps can be devised for every item in the balance sheet and profit and loss statement as well. For instance, Bierwag and Kaufman (1992) working on the idea of Toevs (1983) develop a duration of interest income in terms of book value. Bierwag and Kaufman (1996) in their works show that such duration gaps can be helpful in determining the performance of financial institutions in various financial markets. In such cases, the duration gap of financial institutions become:

$$DGAP_E = DGAP_{EO} + DGAP_{EF} + DGAP_{ES} \tag{24}$$

where $DGAP_E$ is the duration gap of net worth of financial institutions, $DGAP_{EO}$ is the duration gap of on balance sheet activities, $DGAP_{EF}$ is the duration gap in future trading and $DGAP_{ES}$ is the duration gap in swap trading.

2.5.1 Shariah review

This model is again based on interest that is actually riba, making the model non Shariah compliant. Also the duration model for changes in net interest income is non Shariah compliant based on riba as well.

2.6 Use of duration in estimating returns on real estate investment trust (REIT)

Pattitoni et al. (2012) propose that returns of real estate investment trusts (REIT) can be estimated using a logarithmic price variation function between time t and time $t–1$ using a Taylor's expansion series. Hence, the modified duration and related convexity can be estimated using the function:

$$\delta_1 = \frac{\partial p_R\left(p_M, i\right)}{\partial i} = -MD \tag{25}$$

$$\delta_2 = \frac{1}{2} \times \frac{\partial^2 p_R\left(p_M, i\right)}{\partial i^2} \cong \frac{1}{2} CX = \delta_2 + \frac{\delta_1^2}{2}, \quad CX = 2\delta_2 + \delta_2 \tag{26}$$

Pattitoni et al. (2012) state that the relationship between price changes in REIT and interest rates is nonlinear after controlling the effects of the price of market portfolio. Also the results of modified duration appear positive showing interest rate increases negatively effects duration.

2.6.1 Shariah review

REIT duration model is based on mathematically calculated expected returns along with interest that is riba. This model is therefore non Shariah compliant on the basis of 'excessive gharar' and riba.

2.7 Key rate duration

Ho (1992) proposes 'Key rate duration' for the purpose of measuring the risk of interest rates. He defines key rate duration as a 'vector' that represents the price sensitivity of a security in response to change in every key rate of interest. According to him this leads to similar duration as under the case of effective duration. This estimation of yield curve shift leads us to key rate duration (KRD) which has a function in continuous environment as:

$$KRD_i = -\frac{1}{P} \frac{\partial P}{\partial y\left(t_i\right)} \tag{27}$$

2.7.1 Shariah review

This model involves interest, which is riba that is prohibited in Islam. It is further based on expected values based on a 'vector' that also introduces 'excessive gharar' in the model making the model non-Shariah compliant on the basis of riba and excessive gharar.

2.8 Principal component duration

Using the concept of KRD, Willner (1996) presents the concept of Principal Component Duration(PCD). They observe that three factors i.e., slope, height and convexity of yield curves are sufficient to explain almost all of their variations. Based on three factors the function of principal component duration can be expressed as:

$$PCD_{(v)} = \sum_{i=1}^{m} KRD\left(i\right) \times l_{in} \tag{28}$$

This suggests that principal component duration is the sum total of product of factor loading matrix and key rate durations of each bond with '*n*' representing the nature of component.

2.8.1 Shariah review

Since principal component duration is based on the concept of key rate duration that itself is non Shariah compliant, therefore PCD is non Shariah compliant as well.

2.9 Polynomial time value duration

Osborne (2005) and Osborne (2015) propose duration models based on approximate and exact present values. Dierkes and Ortmann (2015) extend their work by presenting various models for present value and duration estimation of various instruments. According to them, the duration of coupon bonds can be calculated using the following functions:

$$D(i) = \frac{1+i}{i} - \frac{n \times c(1+i-i \times n)F}{c(1+i)^n - c + i \times F} \tag{29}$$

In case of annuities the measure of duration has a function:

$$D(i) = \frac{1+i}{i} - \frac{n}{(1+i)^n - 1} \tag{30}$$

And, in case of perpetuities duration can be estimated using the function as:

$$D(i) = \frac{1+i}{i} \tag{31}$$

2.9.1 Shariah review

These models are again based on mathematically estimated present values alongside involvement of 'riba' that makes the model non-Shariah compliant on the basis of riba and excessive gharar.

2.10 Approximation of duration in non-flat yield curve environment

In the case of given spot rates Ho (1992) defines key rate duration as:

$$D_k(i) = -(1+i_k)\frac{\frac{\partial P(i)}{\partial i_k}}{P(i)} \tag{32}$$

2.10.1 Shariah review

This model is based on key rate duration that has been considered non-Shariah compliant hereinabove in Shariah review 2.7.1. Therefore, this model is non-Shariah compliant as well.

2.11 Dedicated duration

Extending the works of Macaulay (1938), Redington (1952) and Fisher and Weil (1971), Zaremba (2017) presents theorem about scenarios where Macaulay's duration can

perform a good measure of the sensitivity of bond by introducing the concepts of 'Dedicated Duration' and "Dedicated Convexity". According to him dedicated duration will have a function in case $a(q) \neq 0$ i.e., shifts at time q are nor zero:

$$D_v(BP) = \sum_{i=1}^{m} t_i \times \frac{\left(c_i^1 + c_i^2\right) exp - s(t_i) \times t_i}{PV(BP)} \times v_i \tag{33}$$

$$= \frac{PV\left(B^1\right)}{PV(PB)} \times \sum_{i=1}^{m} t_i \times \frac{c_i^1 exp - s(t_i) \times t_i}{PV\left(B^1\right)} \times v_i + \frac{PV\left(B^2\right)}{PV(PB)} \times \sum_{i=1}^{m} t_i \times \frac{c_i^2 exp - s(t_i) \times t_i}{PV\left(B^2\right)} \times v_i \tag{34}$$

$$= w_1 \times D_v^1 + w_2 \times D_v^2 \tag{35}$$

$$w_i = \frac{PV\left(B^k\right)}{PV(PB)}, 1 \leq k \leq 2 \tag{36}$$

2.11.1 Shariah review

Dedicated duration models are based on expected values of interest rates and bond portfolios. This involves 'riba' and 'excessive gharar' that are strictly prohibited in Islam making such duration models non-shariah compliant.

2.12 First-order, second-order durations and convexities

Alps (2017) extends the works on present value estimation of cash flows using the concept of duration. He refers earlier method of such estimation as first-order modified approximation that has the following function:

$$P(i) \approx P(i_o) \times \left(1 - (i - i_o) \times D_{mod}(i_o)\right) \tag{37}$$

He presents an alternative measure of approximation of present value of cash flows that he refers as first-order Macaulay approximation having the function:

$$P(i) \approx P(i_o) \times \left(\frac{1 + i_o}{1 + i}\right)^{D_{mac(i_o)}} \tag{38}$$

Similarly, concerning convexity the concept of modified convexity can be presented as:

$$C_{mod}(i) = P^n(i) / P(i) = \frac{\sum_{k \epsilon N} \left(t_k \times (t_k + 1) \times a_k \times (1 + i)^{-t_k - 2}\right)}{P(i)} \tag{39}$$

And Macaulay convexity has a function:

$$C_{mac}(i) = \frac{\sum_{k \epsilon N} \left(t_k^2 \times a_k \times (1 + i)^{-tk}\right)}{\sum_{k \epsilon N} \left(a_{k\times} (1 + i)^{-t_k}\right)} = \frac{\sum_{k \epsilon N} \left(t_k^2 \times a_k \times (1 + i)^{-t_k}\right)}{P(i)} \tag{40}$$

where a_k is the cash flows and t_k is are the time periods from *k* to *N*.

Alps (2017) narrates that C_{mod} can be computed using the following function:

$$C_{mod}(i) = \frac{C_{mac}(i) + D_{mac}(i)}{(1+i)^2} \tag{41}$$

With various numerical examples, Alps (2017) demonstrates that first order Macaulay's approximation of cash flows yields better results than first order modified acceleration of cash flows. However, second order Macaulay Approximation of net present values of cash flows yields better results only in case of newest interest rates.

2.12.1 Shariah review

This model involves interest and present values based on a mathematical model. This makes the model based on 'riba' and also involves 'excessive gharar' making the model non compliant with Shariah parameters.

2.13 Approximating duration using insurance risk management properties

Insurance companies have much larger duration of their liabilities as compared to their assets that is on average more than 10 years (Schlütter, 2017). Möhlmann (2017) working in the area of insurance risk management proposes an interest risk measure based on accounting data and the concept of duration. He proposes that before any interest rate change modified duration has the following function:

$$Dur_{ro} \approxeq -\frac{MV_{ro+\Delta r} - MV_{ro}}{MV_{ro}} \times \frac{1}{\Delta r} \tag{42}$$

Additionally, based on historical cost data taking $MV_{r0+\Delta r}$, $BV_{r0+\Delta r}$ i.e., market value of an insurer's assets and liabilities after the interest rate change the MV_{r0} i.e., the market value of an insurer's assets and liabilities before the interest rate change can be approximated using $BV_{ro+\Delta r}$ if and only if, before changes in interest rates the book value and market value was equal and the book value does not change with the change in interest rates, in which case the function of modified duration becomes:

$$Dur_{ro} \approxeq -\frac{MV_{ro+\Delta r} - BV_{ro}}{BV_{ro}} \times \frac{1}{\Delta r} \tag{43}$$

Subsequently, if we include the notion of time value of money the function becomes:

$$Dur_{ro,vo} \approxeq -\frac{MV_{ro+\Delta r,vo} - BV_{ro,vo}}{BV_{ro,vo}} \times \frac{1}{\Delta r} \tag{44}$$

That accounting for changes in market and book values will become:

$$Dur_{ro,vo} \approxeq \frac{\dfrac{MV_{ro+\Delta r,vo+\Delta v}}{\left(1+ln\left(1+r_o+\Delta r\right)\right)^{0.5\Delta v}} - \dfrac{BV_{ro,vo+\Delta v}}{\left(1+ln\left(1+r_o\right)\right)^{0.5\Delta v}}}{\dfrac{BV_{ro,vo+\Delta v}}{\left(1+ln\left(1+r_o\right)\right)^{0.5\Delta v}}} \times \frac{1}{\Delta r} \tag{45}$$

2.13.1 Shariah review

This model is based on interest, expected changes in interest, and expected changes in market values based on expected changes in riba. Therefore it involves 'riba' and 'excessive gharar' making the model completely non Shariah compliant.

2.14 Implied duration: a measure for equity duration

Dechow et al. (2004) propose a new measure of equity duration that they term as implied duration based on the concept of analysis of financial statements. They identify two key problems for extending the concept of duration to equity:

- A bond makes finite sum of cash payments, while the payments on equities are infinite.
- The amount and timing of cashflows on bonds are usually known in advance and are subject to little uncertainty that is not the case with timing of cashflows on equities.

They addressed these problems by bifurcating the duration function into two parts and also incorporating a function of level perpetuity. They propose the function of duration as:

$$D = \frac{\sum_{t=1}^{T} t \times CF_t / (1+r)^t}{P} + \left(T + \frac{1+r}{r}\right) \times \frac{\left(P - \sum_{t=1}^{T} CF_t / (1+r)^t\right)}{P} \tag{46}$$

where

P: market capitalisation of equity

CF: net cash flow distributions to shareholders

r: expected returns on equity

T: finite forecasting time horizon

t: respective periods of expected returns on equity.

2.14.1 Shariah review

Although a portion of the model is based on realised values thereby avoiding 'excessive gharar' to some extent, more than 50% of the model is based on interest and expected values. This makes the model non Shariah compliant based on 'excessive gharar' and 'riba'.

2.15 Orthogonalising the duration

Chu et al. (2017) work on the relationship of duration with various factors such as value and profitability. Based on the works of Chen (2014) and Weber (2017), Chu et al. (2017) present cash flow duration. This concept was originally forwarded by Dechow et al. (2004) who based duration on cash flows of the firm, current market prices and returns on equity. However, Dechow et al. (2004) model suffered from the notion that taking a specific rate does not solve the problem of cross sectional differences as there is an

inverse relation between equity returns and duration. In order to address this issue Chu et al. (2017) present the following model which they term as orthogonolised duration:

$$DUR_{i,t} = \frac{\sum_{s=1}^{T} s \times CF_{i,t+s} / (1+r)^{s}}{P_{i,t}} + \left(T + \frac{1+r}{r}\right) \frac{P_{i,t} - \sum_{s=1}^{T} CF_{i,t+s} / (1+r)^{s}}{P_{i,t}} \tag{47}$$

where '*t*' is fiscal year and '*T*' is forecasted maturity.

2.15.1 Shariah review

This model makes duration as a function of expected values of many factors including involvement of interest. This makes the model non Shariah compliant on the basis of 'riba' and 'excessive gharar' that are strictly prohibited in Islam.

2.16 Duration of an organisation

Weber (2018) also uses the work of Dechow et al. (2004) and argues that single period returns are actually returns from portfolios of different maturities. He engages the work of Campbell and Vuolteenaho (2004) and Hansen et al. (2008) who work on long run risk of portfolios that are meant for growth. Extending on the same Lettau and Wachter (2007) link the timing of cash flows to risk premium and Santos and Veronesi (2010) propose a portfolio with securities of cross section firms. Weber (2018) adopts the mechanism of Lettau and Wachter (2007) and modifies the model of Dechow et al. (2004) who propose the idea of negative correlations between higher cash flow duration and returns. Weber (2018) bisects duration function into 'finite detailed forecasting periods' and "infinite terminal values", assuming the payment of later as level perpetuity. He presents the following function of duration:

$$Dur_{i,t} = \frac{\sum_{s=1}^{T} s \times \dfrac{CF_{i,t+s}}{(1+r)^{s}}}{P_{i,t}} + \left(T + \frac{1+r}{r}\right) \times \frac{P_{i,t} - \sum_{s=1}^{T} \dfrac{CF_{i,t+s}}{(1+r)^{s}}}{P_{i,t}} \tag{48}$$

where

$Dur_{i,t}$: Duration of a firm i at the end of a year t

$CF_{i,t+s}$: Cash flows of firm i at time $t+s$

$P_{i,t}$: is the current price

and

r : Expected retun on equity.

2.16.1 Shariah review

This model also makes duration as a function of expected values of different factors including involvement of interest. This makes the model non Shariah compliant on the basis of 'riba' and 'excessive gharar' that are strictly prohibited in Islam.

2.17 Equity duration

Besides autoregressive factors Mohrschladt and Nolte (2018) extend the works of Merton (1973), Dechow et al. (2004), Lettau and Wachter (2007), van Binsbergen et al. (2012), Schröder and Esterer (2012) and Weber (2018) in the area of equity duration and present a new model of duration based on a new factor.

$$D_t^k = \sum_{i=t+1}^{\infty} \frac{E_t\left(CF_i^k\right) \times \left(d_t^k\right)^{t-i} \times (i-t)}{P_t^k} - \sum_{i=t+1}^{\infty} \frac{\partial E_t\left(CF_i^k\right)}{\partial d_i^k} \times \frac{\left(d_i^k\right)^{t-i+1}}{P_t^k} \tag{49}$$

2.17.1 Shariah review

Firstly, the model itself is based on the previous models that are non Shariah compliant on the basis of 'riba' and 'excessive gharar'. Secondly, adding a new factor based on expected values of timing of cash flows merely adds to the expectations about cash flows that further adds to 'excessive gharar' making the model non-Shariah compliant.

2.18 Book value based measures of duration

Mohrschladt and Nolte (2018) present a book value based measure of duration where the duration of equity can be calculated as a difference between the duration of assets and liabilities. The duration of assets can be estimated using the following functions:

$$D(A) = D(CS) \times \frac{CS}{A} + D(CA) \times \frac{CA}{A} + D(TA) \times \frac{TA}{A} + D(IA)\frac{IA}{A} \tag{50}$$

where

CS = Cash, CA = Current Assets, TA = Tangible Long term assets A(Total Assets), IA = Intangible long term assets.

The duration of liabilities is computed as:

$$D(L) = D(CL) \times \frac{CL}{L} + D(LL) \times \frac{LL}{L} \tag{51}$$

where L = Liabilities, CL = Current Liabilities, LL = Long Term Liabilities.

And Duration of Equity can be computed as:

$$D(BE) = D(A) \times \frac{A}{BE} - D(L) \times \frac{L}{BE} \tag{52}$$

with BE = A-L, called Book Equity.

In the above functions duration is calculated in terms of Stohs and Mauer (1996) as the average time until the remaining assets or liabilities are converted into cash i.e., current assets divided by cost of goods sold for duration of current assets and current liabilities divided by cost of goods sold for duration of current liabilities. Duration of tangible assets is calculated in terms of Guedes and Opler (1996). This is achieved firstly by dividing the median value of gross property plant and equipment by annual depreciation plus the product of the first ratio and a ratio of net property plant and

equipment to gross property plant and equipment. The duration of intangible assets is calculated using the same procedure as for tangible assets.

2.18.1 Shariah review

The book value based measures of true duration are the most Shariah compliant measures that have been proposed so far. These do not involve expected variables avoiding gharar; and also do not involve interest rates, hence avoiding riba. However, as the model also involves some portion that is expected values of cash flows involving excessive gharar therefore the results of the model also have to be examined for Shariah compliance. Furthermore, such models are meant for organisations dealing physical goods making them unpractical for financial institutions.

2.19 Duration model of accounts receivable

Xu and Ma (2018) forward a model for duration of accounts receivable while pricing accounts receivables. They argue that if there is no default risk in an organisation, it can avoid over dues. Furthermore, as risk free rates do not vary with market trends, the duration concept may become meaningless. However, since all these scenarios do not exist in reality, the pricing of accounts receivable is also possible based on the concept of duration on the basis of 'expiration time' measurement. The function of duration in such a case can be expressed as:

$$E\left(D_{AR_t}\right) = \frac{AR_t \times e^{-R_{f\times N}}}{B} \tag{53}$$

where

$$R_f \times N = \left(\sum_{n=1}^{i} \frac{r_{f_n}}{n}\right) \times 365 \times \left(\frac{n}{365}\right) \tag{54}$$

where R_f is the risk free rate, being one year average of 30 Day rates and N being $\frac{n}{365}.AR_t$ equals book values of accounts receivable, n being the days left before final payment and B is the market value of accounts receivable.

The application of the concept can be made in 'Distance to Default', 'Probability to Default', 'Loss Given Default' and 'Exposure at Default'.

2.19.1 Shariah review

As the duration model of account receivable is based on interest that is 'riba' therefore this is also non Shariah compliant, although more than 50% of the variables involved in the variables are at book values and market values hence avoiding "excessive gharar". Furthermore, it also suffers from the similar shortcoming that accounts receivables relate with physical goods trading that does not happen in financial institutions.

2.20 Duration of assets and liabilities of insurance company

Fernándeza et al. (2018) in their study on management of assets and liabilities in insurance companies utilise the following functions for duration:

Duration of liabilities in an insurance company has a function:

$$L_k^D = \frac{\frac{1}{12}\sum_{i=1}^{\max(n,M-k)} i \times d_i \times L_{k-1+i}^F}{\sum_{i=1}^{\max(n,M-k)} d_i \times L_{k-1+i}^F} \tag{55}$$

where

L_k^D: Duration of liabilities

$$L_k^F = \sum_{m_i \epsilon \rho} d_k l_i^F(t_k) \tag{56}$$

In which case if we let m_i be the cash flow liability at time t_k, $l_i^F(t_k)$ is the amount payable by the insurance company to the policy holder, and 'i' denotes the period.

Similarly, the duration of the assets of an insurance company can be estimated using the function:

$$A_k^D = \frac{\frac{1}{12}\left[\sum_{i=0}^{n} i \times N_i \times d_i + \sum_{i=0}^{n} i \times c_i \times d_i\right]}{B_k^v} \tag{57}$$

where

A_k^D: duration of assets

N_i: nominal principal amount of the asset a maturity

c_i: coupons associated with the assets

d_i: monthly discount rate

B_k^v: value of asset portfolio:

$$B_k^v = \tilde{N}_k + \hat{c}_k \tag{58}$$

With $\tilde{N}_k, \hat{c}_k$ being the present values of bond principal and maturity respectively. The duration Gap of an insurance institutions can be computed by $A_k^D - L_k^D$.

2.20.1 Shariah review

The model of duration presented for insurance companies is based on expected values of cash flows, expected time intervals and interest. This makes the model subject to 'excessive gharar' and 'riba' making it non Shariah compliant.

2.21 Duration measures for corporate project valuation

Arnold and North (2008) extend the work of Macaulay (1938) to evaluate corporate projects. They state that after cash flows from a project can be stated in the form of a function, then after taking its negative partial derivatives and multiplying this negative partial derivative with $(1 + k)$, and subsequently dividing it by the value of the firm V, the function of duration of the project can be produced as under:

$$D_p = \frac{\left\{\sum_{i=0}^{N-1}\left[\frac{i \times CF_i}{(1+k)^i}\right] + \left[\frac{N \times CF_N}{(1+k)^N}\right] \times \left[1 + \frac{(1+g)}{(k-g)} + \frac{(1+k)\times(1+g)}{N \times (k-g)^2}\right]\right\}}{\left\{\sum_{i=0}^{N-1}\left[\frac{CF_i}{(1+k)^i}\right] + \left[\frac{CF_N}{(1+k)^N}\right] \times \left[1 + \frac{(1+g)}{(k-g)}\right]\right\}} \tag{59}$$

where

V: value of the project

CF: cash flows from the project

g: growth rate in cash flow of the project

k: discount rate of cash flows

N: total number of cash flows

Δ: rate of change in cash flows

D_p: duration of the project.

2.21.1 Review

This model of duration is simply an extension of Macaulay's duration in corporate project evaluation based on expected values of cash flows and interest. This makes the model subject to 'excessive gharar' and 'riba' making it non shariah compliant.

3 Conclusion

This research performs Shariah compliance review of 21 models of duration presented so far by various leading researchers. The review suggests that most of the models presented so far suffer from involvement of 'excessive gharar' and 'riba' making them non compliant with Shariah. The models that somewhat comply with Shariah are the models proposed by Mohrschladt and Nolte (2018) that are the models based on book values of assets and liabilities, and the model proposed by Xu and Ma (2018) that is also a book value based model but only for accounts receivable. These models do not involve variables used in banking operations making them not practicable for Islamic banks.

The major reason that renders most of the models non Shariah compliant is involvement of "excessive gharar". This is because all duration models are based on expected values of cash flows and interest rates occurring in future. This research also finds that there are two kinds of expected values occurring in future. First are original

expected values, for instance, cash flows that are committed from issuers of bonds/ financial securities. There are firm commitments behind such cash flows but the issuers of bonds themselves suffer from varying default risks. It has been witnessed during the global financial crisis 2008 that even AAA rated organisations ran into cash flow problems. It leads to form an opinion that even firmly committed cash flows do not have 100% surety and therefore are subject to default risk.

The second type of expected cash flows are mathematically calculated cash flows based on certain models such as Taylor's expansion, first-order/second-order, simulation etc. Both these types of cash flows involve expectations and therefore are subject to 'excessive gharar' making them non Shariah compliant. Similar is the case with interest, the originally expected and the mathematically estimated, where interest also suffers from "excessive gharar". As interest is "riba" that is strictly prohibited in Shariah context therefore it does not matter if is originally expected or mathematically calculated, it is still a non Shariah compliant variable.

4 Future research directions

This research forms a huge basis of future research directions in the field of Islamic financial modelling. Firstly, it extends the scope of Shah et al.'s (2020a) parameters of Islamic financial modelling by including 'riba' parameters along with profound explanations on "excessive gharar". It forms a solid ground on how an Islamic financial model is to be developed. That is to say an Islamic financial model should be based on book values of assets and liabilities and the risk management should be based on expected variations in the book values rather than calculating an expected value of an asset and then calculating variations that involve "excessive gharar". An Islamic financial model should be based on specific factors transformed into variables that are affecting organisations. In this regard, rigorous studies need to be performed to identify such factors according to specific requirements.

In case of Islamic banks, IFSB and other major researches have established that they are vulnerable to benchmark and rate of return risks. Keeping in view the scope of this study we recommend that duration models in Islamic banks should be based on book values of their assets and liabilities, rate of returns, benchmarks and separate set of other variables that affect their assets and liabilities separately. This is because assets and liabilities of an Islamic bank are under separate contracts under different circumstances. Their respective rates of return cannot relate with each other as it will give rise to "riba alnasiah". This also leads to form an opinion that yield curves of returns on assets and liabilities of an Islamic bank should be different. This also forms the need to work on developing separate yield curves for assets and liabilities of Islamic banks.

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