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Macaulay's theory of duration: 80-year thematic bibliometric review of the literature

The application of the concept of durations

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Abstract

Purpose – The purpose of this research is to propose a framework for research on Macaulay duration and establish future research directions.

Design/methodology/approach – Thematic, bibliometric and content analyses have been used to review 168 research papers published between 1938 and 2019 taken from ISI Web of Science and Scopus contributed by leading authors, journals and regulatory bodies.

Findings – Identification and integration of themes of duration theory, duration model development and duration model implementation leading to unattended research gaps, and framework for research on Macaulay duration.

Research limitations/implications – The study is based on an extensive review of the literature to extract important themes, research gaps and frameworks. It does not empirically investigate significance of Macaulay duration and various sectors.

Practical implications – This research has several aspects that are helpful for practitioners. Macaulay duration has been the subject of empirical research only without any guiding framework. This research provides a platform to initiate profound researches in various areas of finance. Various proposed models are required to be tested under holistic approach in conventional and emerging fields, especially in Islamic settings.

Originality/value – This research highlights, research themes leading to framework, research gaps and factors that are crucial in developing, extending and testing duration models leading to enhancement of theoretical base of Macaulay duration.

Keywords Gap, Theoretical framework, Duration, Duration gap, Interest rate risk

Paper type Literature review

1. Introduction

Lidstone (1895) works on the relationship between asset value and interest rate changes with respect to maturity and predicts some relationship in between them. However, in 1938 Frederick R. Macaulay while examining the bond prices in his work on movement of bond prices finds that prices of long-term bonds fluctuate more than their short-term counterparts with a few exceptions. Inferred from his observations he proposes a new measure for tenure of bonds, which can explain bond prices more accurately. He bases his proposition on the ground that as a bond has many cash flows, therefore it is quite unjustified to consider the price of a bond based solely on the basis of its terminal payment. He proposes a new measure of the period of bonds taking into account all cash flows of a bond. For the purpose of weights, he had two options, i.e. the present values or future values. However, as the bond prices are calculated using present values, therefore he considers it more appropriate to use present values of related cash flows as weights.

Going forward in his work Macaulay narrates that if yield to maturity is also regarded as a function of term to maturity, it can also lead to further developments in the proposed measure of the model. This is the concept that was later addressed by Fisher and Weil (1971) after which the measure of duration was addressed as Fisher and Weil duration.

Hicks (1939) addresses changes in value of assets inferred from changing interest rates. His observations conclude that asset values depend on maturity and time pattern of relevant cash flows. Hicks (1939) standardizes the observed cash flows as annuity and calculates its

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elasticity terming it as average period. Samuelson (1945) uses duration in order to address the concern of bank profitability against rising interest rates in post-World War II scenario. He narrates that net worth of financial institutions equals principal or maturity value of their bond investments. The institutions not having equal current earnings on assets with equal payments on liabilities can have adverse or beneficial effects depending upon future inflows/outflows on their commitments in case of interest rate variations.

Samuelson (1945) contends that financial institutions have very large cash in/outflows and depositors do not respond to interest rates rise by withdrawing their deposits taking into considerations the benefits for financial institutions. This phenomenon was later revised with the phenomenon that average inflows could be in fact small relative to average outflows leading to possible losses of financial institutions in case of interest rate rise.

Research using the concept of duration has expanded in many dimensions recently. Bliss (1996) investigates various types of models for differences between observed yields and expected forward rates where he regards duration models as more superior. An account of recent studies includes default risk by Babbel *et al.* (1997), corporate bond valuation by Acharya and Carpenter (2002), liquidity and leverage by Adrian and Shin (2010), interest rate risk estimation by Bajo *et al.* (2013), hedging government bond portfolios by Bessler and Wolff (2014), adjustment in fair value by Beccacece *et al.* (2018), Islamic banking by Chattha and Alhabshi (2018) and other various great works lead by Bierwag, Fooladi, Roberts, Cox and Kaufman in the area of immunization, managing interest rates and derivatives. These enhanced applications require a comprehensive review of the theory of duration since its evolution in 1938 to gather an overall picture and to develop a framework for generation and application of duration models.

Previously, there had been two great review of literature on duration modeling that are Ingersoll *et al.* (1978) and Bierwag and Fooladi (2006). However, keeping in view the financial crisis of 2008, recent growth, expansion and diversification in financial industry along with enhanced footprint of Islamic financial industry their scope has become limited. Therefore, there is a need to reevaluate the literature of duration modeling to identify enhanced parameters and principles of extended applications. The objectives of this research is to identify a framework for development and implementation of duration models taking into account the historical perspective of various models developed and applied.

2. Methodology

Keeping in view the research objective recommendations of Apriliyanti and Alon (2017) and Alon *et al.* (2018) have been followed to apply bibliometric analysis. Bibliometric analysis is a tool to search quantitative and qualitative growth in a particular research topic (Apriliyanti and Alon, 2017). It is in fact a tool of meta-analysis by Fetscherin and Heinrich (2015) to identify linked research in well reputed journals on the basis of citations, authors and research topic development (Alon *et al.*, 2018). This research uses ISI Web of Science and Scopus primarily because of their leading nature for similar works (Falagas *et al.*, 2008). To meet the objectives of this research a thematic approach has been followed to collect 169 research papers published in the last 80 years addressing theory, development and use of various financial models based on the concept of duration. For the purpose of this research, three major themes have been followed, i.e. theory of Macaulay duration, development of duration models and application of duration models that have been presented in the following Figure 1:

3. Review of the literature

3.1 Theme 1: theory of Macaulay duration

3.1.1 Re-explanation of Macaulay duration theory. Shirvani and Wilbratte (2002) forwarded some interesting explanations to the concept of duration. According to them:

> Just as any physical object can be compressed into a single massive point at its center of gravity, the stream of cash payments of a coupon bond can be compressed into a single lump sum at its duration. Thus, any coupon bond with duration D can be represented as an equivalent zero coupon bond with maturity D. Furthermore, in the same way that more stable objects have lower centers of gravity, bonds with more stable values are those with lower durations.

They narrated D as a function of five factors:

$$D = f(t, n, C, M, k)$$

Where:

D = Duration

t = Time period

n = periods to maturity

C = Coupon amounts

M = Principal value

k = Market interest rates

Out of these factors, two remain fixed while others change. The factors that remain fixed are principal value and time period, while the factors, which vary, are periods to maturity, coupon amounts and market interest rates. Other things remain the same, and duration of bonds has positive functional relationship with "n" and negatively related to C and k (see Figure 2).

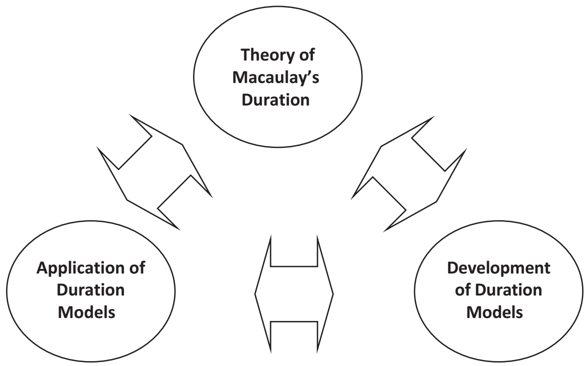

Figure 1.
Proposed conceptual framework of research on Macaulay duration

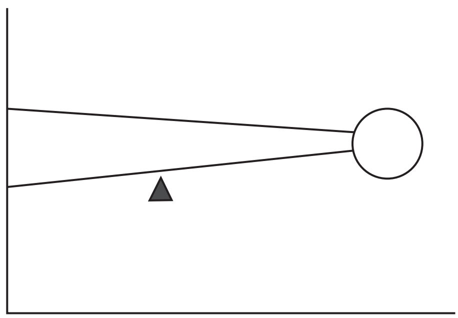

Figure 2.
Flagpole diagram explaining the concept of duration

Shirvani and Wilbratte (2002) explain these relationships with the help of flagpole diagrams. They state that if maturity of a security is represented by a flagpole's length, coupon by diameter and spherical head representing par value, the market rate effect on duration, which reduces future values to par values, can be explained by reduced flagpole diameter at the rate of *k* while moving from base to end of the pole. This relationship can be represented in the form of a diagram as under:

In this figure, time has been measured along the pole and duration is the center of gravity. Furthermore, like the center of gravity which is a function of shape, length and diameter of the pole, duration is also a function of maturity, market rates and coupons.

The variations in duration could be explained with the help of following two further scenarios and diagrams. Consider the following diagram first for the effect of maturity:

If we consider this diagram as two flagpoles representing two bonds, with bond A having a maturity of ten years and bond B having a maturity of 20 years where all else remains the same, the definition forwarded by Macaulay states that the bond with 20 years of maturity will be having longer duration as compared with bond with ten years of maturity. Furthermore, the concept of duration has been represented by placing a triangle in this diagram representing the center of gravity, which elaborates that the longer pole has less stability similar to higher volatility in longer duration bonds (see Figures 3–5).

Now consider the following diagram for the effect of size of cash flows:

If we consider two flagpoles *C* and *D* as bonds *C* and *D* with identical maturity. Bond *C* having larger coupon payments than *D*, the reduction in the size of coupon is represented by thinner flagpole *D*. In this case, according to the definition of Macaulay, the maturity of bond *D* is longer than that of bond *C* that pushes the triangle of center of gravity, i.e. duration further with decrease in strength. This is exactly our intuition from this diagram that small coupon payments weaken a security and relevant duration.

In order to gauge the effect of changing interest rates on duration let's consider the following diagram:

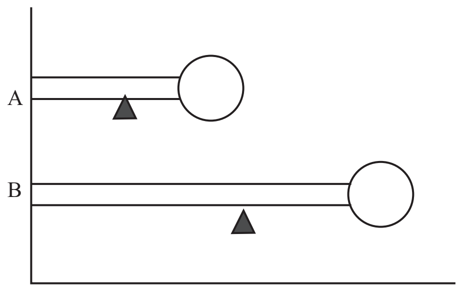

Figure 3.
Flagpole diagram explaining changes in duration

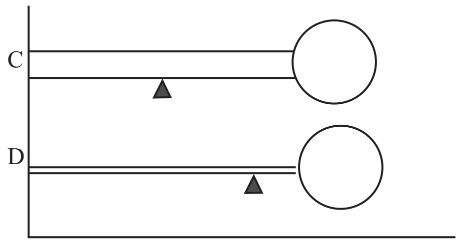

Figure 4.
Flagpole diagram explaining the effect of changes in cash flows on duration

Again in this diagram if we consider two flagpoles E and F as two bonds E and F where flagpole E representing higher interest rates than flagpole F. In this case as we move further into future flagpole F reduces at slower pace than flagpole E showing the effect of lower interest rates. In addition, the triangle in case of flagpole F is placed farther than the triangle in case of flagpole E showing increased duration in case of lower interest rates. However, in this case the farther triangle entails healthier flagpole, meaning thereby securities with reduced interest rates have healthier duration than the securities with higher interest rates.

The flagpole algorithm also explains an interesting analogy between with coupon and zero-coupon bond, i.e. why coupon bonds with greater duration depict higher volatility than zero-coupon bonds. This case specifically arises when coupon bonds having duration more than maturity of zero-coupon bonds. It is because volatility of coupon bonds that have duration equal to maturity of zero-coupon bonds will be similar to volatility of zero-coupon bonds as well.

Extending the scope into the realm of immunization, an investor wishing to fix its yield over a period of time will certainly fail to achieve his objective due to coupon reinvestment rate risk if he purchases a security whose maturity equals his intended time horizon instead of duration. The best strategy to lock in a given yield is therefore to invest in securities whose duration equals intended time horizon of the investor be it zero-coupon or coupon security.

Khachatryan (2019) find that if serial and conventional bond parameters are equal, i.e. the coupon rate and number of periods, then under the assumptions of flat curve the durations of conventional bonds will be greater than the duration of serial bonds in case coupon rate equals discount rate, exceeds discount rate or remains lower than discount rates. Similar is the case with modified duration of conventional and serial bonds.

Nivine *et al.* (2010) find that duration is a short-term measure of changes in equity in response to changes in interest rates. For gauging the effect of larger changes in interest rates, however, convexity analysis is recommended (Nivine *et al.*, 2010).

3.1.2 Mid-term developments in the concept of duration. In 1984, Gultekin and Rogalski (1984) examined several duration models and reported that the results of all duration models are almost similar.

In accordance with the works of Hicks (1939) and Hopewell and Kaufman (1973), the use of duration as a measure of volatility, or as in terms of Cox *et al.* (1979) who termed volatility as "basis risk," can be made in understanding the notion that price of a bond has linear relationship with duration with respect to smaller changes in interest rates. However, Cooper (1977) and Ingersoll *et al.* (1978) indicate that such a relationship is somehow static which helps explain the relationship if the movement of yield curves is parallel.

Gultekin and Rogalski (1984) indicate two possible ways for handling the issue of stasis. The first being to develop measures of duration for specific kinds of yield curves. While the

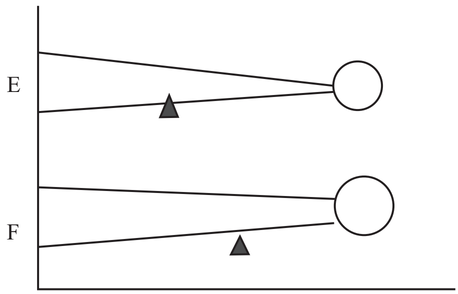

Figure 5. Flagpole diagram explaining the concept of changes in interest rates on duration

second is to theoretically incorporate natural yield curves in the duration models to develop models that are more robust.

3.2 Theme 2: development of duration models

3.2.1 Stochastic duration models. Cox *et al.* (1979) utilize second approach and present duration models presently known as stochastic duration models. They based their proposition on the assumption that term structure of interest rates does not change in any predicted manner. They presented functions as the basis of three hypotheses for duration measures that they labeled D1 to D7:

(1) Security price changes and duration have linear relationship. (Linearity hypothesis)

(2) Duration is a measure of risk that gauges the effect of changes in maturity and coupon differences on volatility of prices. In this way, duration is a complete measure of risk. (Completeness hypothesis)

(3) Bonds have efficient capital markets. (Efficient bond market hypotheses).

Gultekin and Rogalski (1984) use test procedures of Fama and MacBeth (1973). The tests consist of *t*-statistics for hypothesis testing. This is calculated by computing the ratio of relevant stochastic term to the square root of the eligible number of periods for which the returns have been computed. Eligible number of period here means the number of periods for which standard deviation has been computed. Along with the Fama tests the degree of freedom adjusted mean values of average R^2 and $S\ (R^2)$ values.

Application of these tests on the Macaulay's model of duration reveal that duration has linear relationship with returns however; this measure overestimates the risk of longer-term securities.

In their analysis, Gultekin and Rogalski (1984) have two points in focus:

(1) The works on duration conducted so far do not specify any minimum or maximum period in order to establish linear relationship between return and duration.

(2) The robustness of duration model across multiple periods.

Concerning the first point, they observe that the length of holding period does effect relationship between return and duration because price volatility is not measured accurately for longer-term securities. Concerning second point they observe different holding periods affect the relationship between duration and price volatility, hence the relationship between holding periods and relationship is not clear.

In the analysis of non-stochastic duration models, Gultekin and Rogalski (1984) report that, the results do not differ from the basic model of Macaulay to a greater extent. Also in some cases the results of original Macaulay' model was better as other models overstate the level of riskiness of securities. This means all models fail to establish the linearity between price volatility and duration and are not an appropriate measure of risk against unexpected price volatility.

Analyzing the results of stochastic duration Gultekin and Rogalski (1984) indicate that the relationship between stochastic duration and return are linear. Furthermore, the estimation of risk in case of stochastic duration is far much lower than other measures of duration, which indicates that the stochastic model is although a better measure of risk but is not complete.

In a nutshell, stochastic duration is not significantly superior to other measures of risk, which negates the claim of the proposer because of three reasons:

(1) Cox *et al.* (1979) assume that liquidity risk premium is zero and does not affect duration. However, results indicate that due to their existence they must be included in stochastic duration particularly in the short end of the yield curve.

(2) There may be an error in interest rate forecasting. This is because stochastic duration has very much dependence on true spot rates and if there is some misspecification in rate determination process, it will also affect duration.

(3) The estimation procedure for stochastic spot rates suffers from inadequate statistical procedures.

Gultekin and Rogalski (1984) also negated the proposition of Fisher and Weil (1971) that a target yield can be achieved by buying a portfolio of bonds whose duration equals the investment horizon of the investor. It is because the number of securities in a portfolio also found to be a factor that affects results.

Regarding the number of factors that should have been incorporated in duration, results by Gultekin and Rogalski (1984) confirmed the proposition of Ingersoll (1981) and suggested that increasing the number of factors in a duration model better explains the variability in holding period returns than a single factor model.

In a nutshell, Gultekin and Rogalski (1984) argue that all measures of duration produce valid results only in the case of short term. In the case of long-term analysis all the duration models need to be implemented with caution for want of interest rate shocks, misspecifications and unexpected movement of yield curves.

3.2.1.1 Classification of stochastic duration models. Bierwag *et al.* (1982) report that for holding period returns stochastic processes are mostly additive leading to non-convex functional relationship with interest rates. Working on the same Bierwag (1987) further narrates that the relationship between duration measures and stochastic process is not always consistent therefore traditional measures of duration are also consistent. Bierwag and Roberts (1990) present some examples confirming the arguments of Bierwag *et al.* (1982) and Bierwag (1987). Whatever might be the dynamics of equilibrium and disequilibrium, Bierwag and Fooladi (2006) argue that the aspect of convexity has significance in risk-less profit making and needs to be addressed accordingly.

Goodman and Vijayaraghavan (1987) present two-factor interest rate risk model where they use convexity of cash flows as a second factor along with duration. Brennan and Schwartz (1983), Nelson and Schaefer (1983) and Hull (1993) all work on continuous finance duration models. Their arguments focus on correlation between factors surrounding duration models. Similarly, Bierwag *et al.* (1983), Brennan and Schwartz (1983), Bierwag *et al.* (1987) and Bierwag *et al.* (1993) all work on duration models in non-flat yield curve environments. The question remains under criticism that which of the model, i.e. discrete or continuous, performs better and under what circumstances. The discussion about discrete or continuous models apart from the link between discrete and continuous interest rates cannot be overlooked and ignoring this relation may lead to stochastic process risk (Bierwag and Fooladi, 2006).

Nawalkha and Chambers (1996) propose another mechanism of managing "stochastic process risk" which they termed as "M-Absolute" based on weighted average of absolute distances of cash flows from the date of liquidation. They argue that minimization of bond risk could be achieved by minimizing the M^A function. Lately, Nawalkha and Chambers (1997) utilize the concept of M^2. Fong and Fabozzi (1985) works on fifth order on the Taylor expansion and propose a new model for management of stochastic process risk termed as "M-Vector". They claim to address 95% of stochastic process risk using their proposed model.

Bierwag and Khang (1979), Khang (1983) and Bierwag (1987) using a return function of $V_o(r_O) = (1 + r^*) = V_o(r)(1 + r)^q$, where r is the initial rate at the time of investment and r_o is the subsequent rate, showed that immunization strategy produces maximum results during some part of immunization process and therefore term it maximin strategy. Prisman (1986) argue that whatever immunization model an investor might choose it always yields minimum possible results, which is the current observed rate.

3.2.2 Macaulay duration using Taylor expansion. Fisher (1966) computed Macaulay duration taking first derivative of bond price with respect to interest rates as suggested by Macaulay (1938). Bierwag and Fooladi (2006) applied Redington (1952) concept of using Taylor expansion series for estimating interest rate risk and resultant sensitivities of securities.

Using modified duration they show that larger the value of duration the larger with be the impact of sensitivity, i.e. securities with higher duration will have higher risk sensitivities. Furthermore using convexity in case of falling interest rates, percentage rise in bond price exceeds percentage fall in interest rates and vice versa, a phenomenon that favors long investors and works against short investors.

Fooladi and Roberts (2004) argue that this situation encourages financial institutions to maintain certain relationship between the convexities of their assets and liabilities in order to ensure "on balance sheet macro hedging," that too under the assumptions of parallel shifts of relevant yield curves. This follows that management of convexity relationship is a desirable function in all circumstances.

3.2.2.1 Duration and estimation of present values using Taylor expansion. Livingston and Zhou (2005) apply Taylor expansion to the logarithm of present value function of cash flows. Tchuindjo (2008) extend the work of Livingston and Zhou (2005) and argue that it overestimates the present values in case of increasing interest rates, a feature not desirable by risk-averse investors.

Dierkes and Ortmann (2015) work on duration model is based on a new mechanism of estimating changes in present values of cash flows inferred from changing in interest rates and respective curves. They base their approximation on linear differential equations that produced far more superior results than Taylor approximation.

3.2.3 Effective duration. Leland (1994) and Leland and Toft (1996) while searching for optimal capital structure introduce a modification in Macaulay duration which they term as "effective duration". In their analysis of bonds, they find that bonds that default subsequently have shorter effective duration than that of Macaulay's, which in some cases was even negative. Babbel *et al.* (1997) argue that effective duration differs from Macaulay duration not only on the basis of default risk alone but also there exists differing basis about choice and behavior of interest rates.

3.2.4 Key rate duration. Ho (1992) proposes a new measure of duration that he termed as "key rate duration (KRD)" for measuring the risk of interest rates. He defines KRD as a "vector" which represents the price sensitivity of a security in response to change in every key rate of interest. According to him, this would lead to similar duration as under the case of effective duration. Zeballos (2013) explains this concept with the help of the following figure:

According to him if we denote key rates by $t(i)$ where "t" is the time period and "i" is the rate of interest, $S[t(i)]$ being the key rate shifts where $t(i-1)$ and $t(i+1)$ are various key interest rates around the key rate $t(i)$ then the linear interpolation of movements in key rates can be used to estimate shifts in yield curve along the maturity. Where the estimation of first shift in basic yield curve does not require any calibration of shift in second key rate, which is the peak point of shifts in first interest rates. In other words the shift in "ith" interest rate is a zero shift for interest rates "$i-1$" and "$i+1$," which are actually the upper and lower ranges of shifts of "ith" interest rates represented by the two triangles along the line "ith" line which Zeballos (2013) represented as:

In other words, KRD is the linear decomposition of effective duration. Therefore, in terms of KRD effective duration may be expressed as $D = \sum \text{KRD}$. Zeballos (2013) narrates that KRD can identify sensitivity of security prices based on each portion of the yield curve. In this way this model acknowledges that yield curve changes are driven by different market forces, which have different impact on different portions of the yield curve. Furthermore, it is also helpful in replicating bond portfolios with embedded options (see Figures 6–8)

3.2.5 Principal component duration. Using the concept of KRD Willner (1996) presents the concept of principal component duration. Over the period of time research has revealed that three factors, i.e. slope, height and convexity of the yield curve, are sufficient to explain almost all the variations in the yield curve. Nawalkha and Soto (2009) show the curvature of three factors using the following diagram:

In this diagram C_h represents "level or height factor" i.e. parallel yield curve changes, C_s represents "twist factor" i.e. extent of movements in short- and long-term rates in opposite directions, C_c and represents curvature factor, i.e. a factor representing direction of short- and long-term rates opposite to the medium term rates. This suggests that principal component (PC) duration is the sum total of product of factor loading matrix and KRDs of each bond with "n" representing the nature of component. The sensitivity factors of the change in bond prices in percentages due to three factors are calculated by PC duration.

The research studies over the period have shown that durations of PCs represent 80%–95% of bond return differentials depending upon the period of time for which the sample has been chosen.

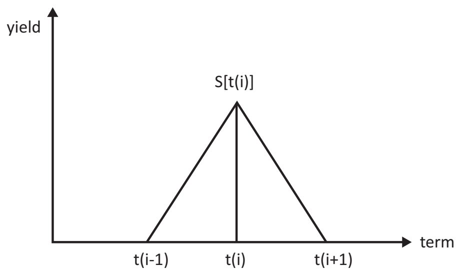

Figure 6.
Linear interpolation of a shift of the spot curve

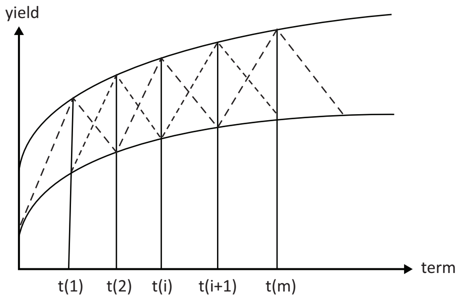

Figure 7.
Yield curve shift as the sum of key rate duration

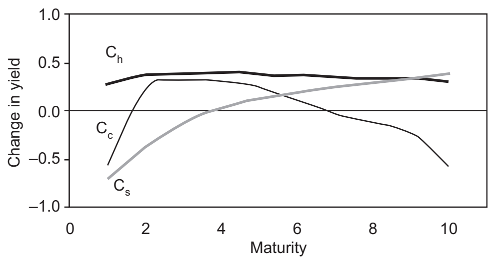

Figure 8.
Curvature of slope height and convexity of yield curve in response to change in yield

3.2.6 Polynomial time-value duration. Osborne (2005) proposes a duration model based on the notion that time value of money has polynomial properties, i.e. having more than one roots, which helps in solving inaccuracy problems. He based his model on the assumption that because a Bond's price is polynomial therefore, it is a function of all of its roots. This follows that a bond price after change in interest rates can be calculated as we calculate all of its roots. However, in his later work Osborne (2014) proposes a new mechanism for predicting yield on interest rates which is based on a mark-up rate of $1 + m = \frac{1+i_o}{1+i}$ and all cash flows.

In their work presenting the models to approximate an exact present values, Dierkes and Ortmann (2015) present various models for present value and duration estimation of various instruments, such as present value of coupon bonds, present values of equities and present values of perpetuities.

3.2.7 Approximation of duration in non-flat yield curve environment. Given the present value of a stream of cash flows Ho (1992) gives functions of KRD that calculate present values in case of sudden shifts in exchange rates and sudden shifts in yield curve. In such cases, duration can be calculated by summing up the components of key rates of duration.

3.2.8 Dedicated duration. Zaremba (2017) presents theorem about scenarios where the Macaulay duration can perform as a good measure of the sensitivity of bond by introducing the concepts of "dedicated duration" and "dedicated convexity". His extends the works of Macaulay (1938), Redington (1952) and Fisher and Weil (1971) showing the shifts in term structure of interest rates.

3.2.9 First-order, second-order durations and convexities. Alps (2017) extends the works on present value estimation of cash flows using the concept of duration. With various numerical examples, Alps (2017) demonstrates that first order Macaulay approximation of cash flows yields better results than first order modified acceleration of cash flows. However, second order Macaulay approximation of net present values of cash flows yields better results only in case of newest interest rates.

3.2.10 Approximating duration using insurance risk management properties. Insurance companies have much larger duration of their liabilities as compared to their assets that is on the average more than ten years (Schlütter, 2017). Möhlmann (2017) working in the area of insurance risk management proposes an interest risk measure based on accounting data and the concept of duration.

Additionally, based on historical cost data and market value of an insurer's assets and liabilities after the interest rate change the market value of an insurer's assets and liabilities before the interest rate change can be approximated if before changes in interest rates the book value and market value was equal and the book value does not change with the change in interest rates.

In order to utilize the function to calculate the possible change in book and market value due to time Möhlmann (2017) develops a single example of valuing a zero-coupon bond at some different time, having a certain face value and time to maturity. In such a case by discounting market value with interest rates after the change and discounting book values with interest rate before the change, and assuming they are not sensitive to interest rates but are sensitive to time passage.

Furthermore, as changes in market value after change in interest rates, book value before change in interest rates do not involve reinvestment assumption, therefore Möhlmann (2017) in order to incorporate change with respect to earlier payments in coupon bonds introduces a reduction in value change in response to small change in time. It can be interpreted as "the time structure is that interest rates changed from r0 to r0 +Δr just after the item was recognized on the balance sheet and then, from time v0, a time Δv passed while interest rates remained constant". However, due to unknown nature of exact time structure, Möhlmann (2017) recommends to include only ½ of Δv to account for the time insensitive present value of relevant net cash flows. The duration model of such a stream of cash flows can simultaneously be used to calculate the durations of assets and liabilities. This duration is actually based on comparison of sensitivities and not of values. Therefore, it is applicable in situations where the investment managers maintain large duration gaps as investment strategy.

Zaremba (2017) argues that if payments on maturity are dependent solely on continuity of interest rates where time is defined as a period at the end of which payments are to be matured. This will provide a function of dedicated duration.

3.2.11 Orthogonalizing the duration. Chu *et al.* (2017) work on the relationship of duration with various factors such as value and profitability. They argue that the relationship between duration and stock return could be presented as $R_{i,t} = \alpha + b_1 \text{ DUR}_{i,t} + \varepsilon_{i,t}$. Where $R_{i,t}$ stands for stock return and $\text{DUR}_{i,t}$ stands for duration. Based on the works of Chen (2014) and Weber (2017), Chu *et al.* (2017) present the function $R_{i,t} = \alpha + b_1\text{BM}_{i,t} + b_2F_{i,t} + e_{i,t}$.

Where $\text{BM}_{i,t}$ is the book to market value ratio and $F_{i,t}$ is an orthogonal factor, upon which Chu *et al.* (2017) forward the concept of orthogonalizing the duration. A concept which is based on the notion that the relationship between duration and other factors might move in two dimensions, i.e. time series and cross section. In order to address these two types of dimensions Chu *et al.* (2017) conduct two-stage orthogonalization.

3.2.12 Duration of an organization. Weber (2018) argues that single period returns are actually returns from portfolios of different maturities. They extend the work of Campbell and Vuolteenaho (2004) and Hansen *et al.* (2008) who work on long run risk of portfolios that are meant for growth. Extending the same Lettau and Wachter (2007) link the timing of cash flows to risk premium and Santos and Veronesi (2010) propose a portfolio with securities of cross section firms. Working on the same Weber adopts the mechanism of Lettau and Wachter (2007) and modifies the model of Dechow *et al.* (2004) who propose the idea of negative correlation between higher cash flow duration and returns. Weber (2018) bisected the duration function into "finite detailed forecasting period" and "infinite terminal value" assuming the payment of later as level perpetuity. For the purpose of this duration models, returns on equity and growth on equity have been used in accordance with Dechow *et al.* (2004).

3.2.13 Equity duration. Mohrschladt and Nolte (2018) extend the works of Merton (1973), Leibowitz (1986), Kadiyala and Subrahmanyam (2000), Dechow *et al.* (2004) work of implied duration, Lettau and Wachter (2007), van Binsbergen *et al.* (2012), Schröder and Esterer (2012) and Weber (2018) in the area of equity duration and present a new model of duration based on a new factor of estimating unexpected stock returns. However, this approach suffers from long-term instability of durations against slow movements in interest rates (Sweeney and Warga, 1986). The second approach upon which Weber (2018) bases his latest work is based on future timing of cash flows where first term is the Macaulay duration and second term is its sensitivity. Furthermore using the findings of Nissim and Penman (2001) that accounting based information provides the most effective information for forecasting, the net cash flow distribution has been developed. Mohrschladt and Nolte (2018) state the reason of their proposed factor that, investment opportunities increase with higher expected returns that changes the discount rate. Hence, future cash flows and changes in discount rates are positively related in order for conventional Macaulay duration is close to the true duration.

3.2.13.1 Duration of negative book value of equity. Luo *et al.* (2019) calculate and compare durations of healthy negative book value of equity and other negative book value of equity firms. They narrate that durations of both types of firms increase over time; however, healthy negative book value of equity firms stay in negative status over a longer period of time.

3.2.14 Book value based measure of duration. Mohrschladt and Nolte (2018) also suggest having a measure of duration related to cash flows of only those assets and liabilities that exist at the time when the duration is calculated as other cash flows shall be dependent on other opportunities that might not exist actually in future. Consequently, in place of the first term of duration model, which is simply an explanation of Macaulay duration, they suggest a balance sheet book value based measure of duration where the duration of equity can be calculated as the difference between the duration of assets and duration of liabilities.

Stohs and Mauer (1996) calculate duration as the average time until the remaining assets or liabilities are converted into cash, i.e. current assets divided by cost of goods sold for duration of current assets and current liabilities divided by cost of goods sold for duration of current liabilities. Duration of tangible assets is calculated in terms of Guedes and Opler (1996). This is achieved first by dividing the median value of gross property plant and equipment by annual depreciation plus the product of the first ratio and a ratio of net property plant and equipment to gross property plant and equipment. The duration of intangible assets is calculated using the same procedure as for tangible assets.

The duration of liabilities for maturity, buckets are calculated for mean period of the bucket. For instance, for 1–2 years, duration is calculated for 1.5 years, for 2–3 years maturity is calculated for 2.5 years. The maturity of last bucket that is usually over five years or ten years category, is calculated on the basis of an assumption that each of the following year has the same proportion of debt as the immediately preceding bucket till 100% of the remaining amounts are finished. That is to stay if the last category is over five years and immediately preceding category 4–5 years have 9% of the total liabilities then 9% of the amount of liabilities in over 5-years category will be assigned to each of the subsequent year till the full amounts of over 5-years category have been utilized. Mohrschladt and Nolte (2018) further winsorized their values in order to deal with the effect of any unexpected outlying factors.

3.2.15 Duration model of accounts receivable. Xu and Ma (2018) forwarded a model for duration of accounts receivable in their work on pricing the accounts receivables. They argue that if there is no default risk in an organization and it can also avoid overdue and also risk free rates that do not vary with market trends, the duration concept will become meaningless. However, since all these scenarios do not exist in reality, the pricing of accounts receivable is also possible based on the concept of duration as a basis of "expiration time" measurement.

The application of the concept can be made in "distance to default," "probability to default," "loss given default" and "exposure at default".

3.2.16 Duration of assets and liabilities of insurance company. Fernándeza *et al.* (2018) in their study on the management of assets and liabilities management in insurance companies argue that balance sheet of an insurance company has a different structure and therefore requires separate set of measures for risk management. They argue that average duration of liabilities of insurance companies is more than ten years that is well above all other organizations.

3.2.17 Duration measures for corporate project valuation. Arnold and North (2008) extend the work of Macaulay (1938) to evaluate corporate projects. They state that their measure of duration provides a single value for evaluation of cash flows from a project in response to changes in discount rates.

3.3 Theme 3: application of duration models

3.3.1 Duration of surplus in financial institutions. Messmore (1990) uses accounting equation to explain the use of duration in assets and liabilities management. He introduces a new term for the purpose that he terms as duration of surplus. He expressed surplus as Surplus = Assets−Liabilities. Since in terms of accounting equation equity is also the difference between assets and liabilities, therefore Messmore (1990) uses the term surplus actually for the term equity.

Messmore (1990) draws the following theorems from the D_S functions:

(1) When the assets match with liabilities, the duration of surplus is not zero i.e. $D_S \neq 0$ rather $D_S = D_A = D_L$.

(2) To compute the duration of assets that eliminates the interest rate risk, D_S equation needs o be set equal to zero and then solving it for D_A yields the function DA – DL = –D /(A/S).

Which implies negative duration of assets in case of zero duration of surplus being $L < A$.

(3) An increase in leverage increases duration of assets.

(4) In case of negative surplus, assets duration remains shorter than liability duration that cannot be maintained for immunization.

(5) When the surplus is zero duration of assets must be maintained equal to duration of liabilities.

3.3.2 Application of Macaulay duration model using IFRS. Beccacece *et al.* (2018) utilize Macaulay' duration in the application of IFRS-13, 25 and26. They narrate that if we calculate Macaulay duration and net present values using free market interest rate in compliance with Fabozzi (1999), two types of risk adjusted economic present values as described in IFRS-13, B25 and B26 can be computed using risk adjusted economic present values.

Through rigorous calculations Beccacece *et al.* (2018) prove that risk adjusted Expected Present Value (EVP) computed using methods elaborated in IFRS13, B25, B26 is approximately proportional to the expected inflows of Macaulay duration and not proportional to project life. This further leads to the opinion that project with similar cash flows, EPV and expiry may depict similar risk profiles.

3.3.3 Effect of climate change on duration. Hellmich (2015) in a report on investment strategies in climate change regime report that climate change elongates the duration of banks and insurance companies.

3.3.4 Duration of pension funds. Schrager (2019) observe that in a pension fund management strategy an optimized position of credit risk and duration is necessary. The interaction of interest rate with duration reveals the quantum of impact of interest rate sensitivity (Schrager, 2019). Due to unique longevity risk in assets and liabilities of a pension fund Schrager (2019) argues that its duration should be determined using affine Makeham parameters

3.3.5 Bond duration in managing transmission of monetary policy. Darmouni *et al.* (2019) observe that firms relying on bonds respond to monetary policies less quickly due to longer bond durations amongst other factors. However, the duration mismatch in a bond-financed firm is smaller as compared to a bank-financed firm. The effect of monetary policy on stock price also transmits through equity duration, which is actually the duration of the difference between assets and liabilities (Darmouni *et al.*, 2019). The larger the equity duration, the larger will be the effect on stock prices. The effects of monetary policy can in fact be modeled following a change in present value due to policy rate. This happens in two dimensions, which are change in duration of new projects, and change in duration of old projects. In both cases the quantum of returns might be different but effect is reduction in investment due to rise in duration (Darmouni *et al.*, 2019). Darmouni *et al.* (2019) also report that a monetary policy effect on default risk is its reduction through reduction in duration.

3.3.6 Use of duration to manage interest rate response of banks. Bierwag and Kaufman (1985, 1992 and 1996) explore the idea of interest rate exposure of banks and other financial institutions. They argue that if we consider net worth of a financial institution as $E(r) = A(r)$-$L(r)$ where $E(r)$ is the risk sensitive net worth, $A(r)$ is the value of risk sensitive assets and $L(r)$

is the value of risk sensitive liabilities, the effect of interest rate changes on risk sensitive net worth of a financial institution can be calculated using duration gap.

Bierwag and Fooladi (2006) argue that duration gaps can be devised for every item in the balance sheet and profit and loss statement. For instance, Bierwag and Kaufman (1992) working on the idea of Toevs (1983) develop a duration of interest income in terms of book value.

3.3.6.1 Duration response of banks. Gomez *et al.* (2016) use Mishkin and Stanly (2009) concept of income gap to measure interest rate risk exposure of financial institution. Income gap has some limitations such as difference in roll over rates, negative correlations between interest income and interest rates in case interest rate remains stagnant while hedging against interest rate risk exposure. However, it has superiority over duration gap based on the fact that income gap remains positive even when duration gap becomes negative, and also, income gap is a cash flow concept while duration gap is a value concept (Gomez *et al.*, 2016). In simple words income gap measures the effect on net income of a financial institution in short term in response to changes in interest rates whereas duration gap measures the extent of sensitivity of equity to changes in short-term interest rates. Therefore, in short term, income gap is a better measure of sensitivity in net income of a financial institution (Gomez *et al.*, 2016).

Liviello (2018) examines duration of banks in low interest rate regime and finds that low interest rates elongate their durations. In order to manage duration of maturity gaps Liviello (2018) argues that there is a need to introduce modeling of non-maturing financial assets and liabilities in addition to conventional management of duration gaps.

Hoffman *et al.* (2018) find that two type of managerial decisions help in reducing duration gaps. Those are decision to issue variable rate loans and decision to accept non-commercial deposits, e.g. deposits from households that are rather insensitive to interest rates.

Du (2019) in their work on securitized banking find that even in the case of securitized banking higher duration gaps lead to higher interest rate risk making negative beta relationship between interest rate sensitivity and liquidity creation. Ferrero *et al.* (2019) observe that in case of steepening of yield curve bank respond by increasing duration amongst other steps. However, activity of reducing ex-ante risk taking on new loans is independent of duration gap (Ferrero *et al.*, 2019).

3.3.6.2 Duration of Islamic Banks. Salman (2006) uses duration analysis of Islamic financial institutions in Turkey and finds that maturity mismatch was the primary reason of collapse. Chattha and Bacha (2010) explain the vulnerability of Islamic banks against benchmark and rate of return risk exposure and recommend analysis based on duration. Chattha and Archer (2016) and Chattha and Alhabshi (2017) all recommend the use of duration based management of Islamic banks to gauge their vulnerability. Chattha and Alhabshi (2018) apply duration models consistent with the works of Koch and MacDonald (2009) and Chattha and Bacha (2010) using four point criteria as under:

(1) Determine Macaulay duration for each class of assets and liabilities.

(2) Use appropriate weights based on market value of respective asset/liability divided by total market value of all assets/liabilities.

(3) Calculating weighted average duration

(4) Calculate weighted average duration gap by deducting weighted average duration of liabilities from assets.

3.3.7 Duration and default risk. Default risk is another factor that is affected by duration. Bierwag and Kaufman (1988) identify many patterns of cash flow that can trigger default risk. They argue that postponement of cash flows increases duration and vice versa but their relationship with default risk needs to be addressed in details. Fooladi *et al.* (1997) propose a single factor duration model with an adjustment of probability of default that was extended by Jacoby (2003) with the addition of log-utility function to include preference of bondholders.

Bierwag and Fooladi (2006) argue that most of the duration models do not include default risk adjustment because of an implicit assumption of no relationship between duration and default risk. They argue that this assumption is only valid if default risk premium and cash flow patterns off set each other with almost absolute certainty. Merton (1974) and Chance (1990), however, do not agree to the implicit assumption and propose a mechanism to account for default risk. They argue that duration of a bond encompassing default risk can be calculated in two steps. First, the value of a bond with high default probability can be expressed as price of a risk free bond as reduced by the price of a put option on the assets of the firms. Second, the duration of such a bond can be calculated by taking first derivative of the log value of bond with respect to risk free rate of interest. Chance (1990), Acharya and Carpenter (2002) and Jacoby and Roberts (2003) all explore the relationships between duration and default risk and emphasize adjustment by considering the similarity between default and call risk.

An investor constructs his portfolio using different securities of various maturities and various issuers therefore it faces various default risks of various respective term structures (Bierwag and Fooladi, 2006). This scenario urges investors to construct such a portfolio for any specific target return wherein the weighted average of the duration of securities to be included in the portfolio equals liquidation period of the portfolio (Bierwag and Fooladi, 2006).

Bierwag *et al.* (1990, 1992) argue that for securities having similar maturities, coupon rates and credit or default risk is represented by differences in respective yield curves. Leibowitz *et al.* (1990) propose a framework for analyzing changes in yields and its effects on spread. They propose that changes in prices of bonds due to yield can be analyzed by decomposing the effect into "changes in underlying yield" and "changes in spread". Linking the spread on the bonds with the rating and introducing new notions of "spread beta" for sector of bonds and "broad index spread" for overall spread on index, they propose that changes in value of a portfolio resulting from change in interest rates can be computed using their proposed notions more accurately.

Duration matching by its basic instinct favors securities with lower credit or default risk and stable cash flows extending over longer periods such as governmental bonds (Domanski *et al.*, 2017). Adrian and Shin (2010) argue that the values of assets and liabilities of a financial institutions fall in case of interest rate rise. However, the magnitude of the change depends on convexity, such as negative convexity gives sharper rise to respective asset or liability. Consequently, an institution whose value of assets decreases endeavors to increase its assets and an institution whose value of liabilities increases endeavors to decrease them. In other word it is the demand for interest risk bearing assets that increase with the rise in price, i.e. the demand of interest bearing assets is upward sloping.

The return on any portfolio can also be defined as return of a cash flow portfolio having varying maturities. Lettau and Wachter (2007) model the cash flow timing with an objective to gauge cash flow timing risk premium. They use a set of firms having different dividend patterns. They argue that growing firms disburse dividends in distant futures that cause increase in the duration of their cash flows. Their model suggests that dividend of growing firms covary with discount rate shocks and value firms with similar effects of cash flows. They suggest this pattern as basis of value premium of cash flow schedule. Hansen *et al.* (2008) extend their work by examining the cash flow patterns after sorting out the portfolios from "book-to-market". Wojakowski *et al.* (2019) observe that duration gap between loan default and asset realization time also need to be modeled to arrive at actual cost of funds in a defaulted loan transaction.

3.3.8 Use of duration in cross sectional asset pricing. Lettau and Wachter (2007) present duration model for value premium to address the anomaly of the cross sectional asset pricing literature. They argue that investors perceive short duration assets as sensitive to cash flow shocks and long-term assets sensitive to interest rate shocks. Consequently, investors focusing short-term durations are compensated with higher risk premium than long terms. Such models however do not clear the profit anomaly (Fama and French, 2006; Novy-Marx, 2013).

In their function for price of a security, they argue that quantum of price response with respect to change in interest rates increase with increase in maturity to which interest rates apply, which they term as duration effect. Combining the analysis of bonds with equity securities, they opine that duration operates in cases. They reason of which they regarded as the interaction between duration and risk premium.

Using this function Dierkes and Ortmann (2015) observe that duration decreases with rise in interest rates, which means the derivate of exact present value, will be higher than the derivative of approximated function with negative values. Dierkes and Ortmann (2015) further compare classical functions with their proposed functions and term them as P^{cla} and $P^{cla'}$ respectively for exact present value function. The findings suggested that $P^{cla'}(i) < P^{app'}(i)$ for $i > i_o$ whereas the exact present value works out to be $P^{cla}(i) \leq P^{app}(i) \leq P(i)$ in case of $i < i_o$ opposite argument holds, whereas equality exists only in case of $i = i_o$.

Bajo *et al.* (2013) present function for estimation of cash flows where they prove that their model outperforms the model of Livingston and Zhou (2005). However, Dierkes and Ortmann (2015) point that the model by Bajo *et al.* (2013) lacks strong theoretical reasoning. They further point that the model by Bajo *et al.* (2013) produces inferior zero-coupon bond estimation results with similar approximation error when compared with their model.

3.3.9 Use of duration in estimating returns on real estate investment trust (REIT). Pattitoni *et al.* (2012) propose that returns of real estate investment trusts (REITs) can be estimated using a logarithmic price variation function between time t and time $t-1$ using a Taylor expansion series. Pattitoni *et al.* (2012) state that the relationship between price changes in REIT and interest rates is nonlinear after controlling the effects of the price of market portfolio. In addition, the results of modified duration appear positive showing interest rate increases negatively effects interest rates.

3.3.10 Use of duration in determining interest rate risk. Negative duration gap means duration of assets is greater than duration of liabilities David (1995). Banks normally choose to manage their interest rate risk through a combination of factors that affect their duration gaps (Duan *et al.*, 1999; Buetow Jr *et al.*, 2003). Such factors include decisions about choosing assets and liabilities of differing profiles that can provide better resilience to changes in variations to interest rates Bierwag and Kaufman (1985), Belongia and Santoni (1984) and Duan *et al.* (1999).

Market value of equity, net worth and interest income is also a function of duration amongst other factors (Koch and MacDonald, 2009). The impact of changes in interest rates on net worth, net income and market value of equity can be better analyzed by gauging the duration impact using appropriate stress tests.

3.3.11 Immunization using duration. Extending the work of Samuelson (1945), Redington (1952) utilizes the concept of duration for immunization of financial institution balance sheet against interest rate movements by introducing the concept of "immunization". He contends that by choosing appropriate streams of cash flows, with positive spreads so that the duration of assets equals duration of liabilities the net duration of assets and liabilities of a financial institutions can be brought down to zero. Their works were extended by Durand (1957) and Wallas (1960) who merely reviewed and tested the models presented by Macaulay, Hicks, Samuelson and Redington. Summing up, these four persons are regarded as pioneers of the concept of duration (Bierwag, 1977).

Fisher and Weil (1971) utilize the concept of duration in bond portfolio management leading to bond investment theory and investment risks. An increase in the level of interest rates decreases the market value of assets and liabilities and vice versa thereby keeping the market value of net worth unaffected if asset duration equals that of liabilities (Grove, 1974). This strategy in fact leads to the process termed as immunization and can be applied to various derivative positions as well. This process of immunization was explored by many researchers that included Bierwag (1977 and 1979), Bierwag and Kaufman (1977), Bierwag *et al.* (1978), Kaufman (1978), Reilly and Sidhu (1980) and Fooladi and Roberts (1989 and 1992). Besides Little (1984) and Kalotay (1984) also showed that immunization strategies fail in case of negative cash flows and sometimes lead to even longer duration than expected. Negative cash flow is a case which arises sometimes due to tax adjustments and called after-tax duration strategies. The impact of tax adjustment was also addressed by Hessel and Huffman (1981,1981a) who opine that ignoring tax adjustment leads to erroneous duration.

Taylor expression of duration model laid foundation for analysis of duration in terms of interest rates, coupons, yield to maturity and maturity. It explained the concept of higher variations in values of longer maturity bonds with respect to interest rate changes (Hopewell and Kaufman, 1973; Haugen and Wichern, 1974). Boquist *et al.* (1975) establish the relationship between beta, a measure of risk, and duration. They argue that since duration also expresses relationship between movement in prices and interest rates, therefore there must also be a relationship between duration and another measure of risk, i.e. the beta. Accordingly, they argue that change in risk must be related to change in duration. Elton *et al.* (1990) while testing duration theory report that higher duration portfolios have higher return volatilities that are related up to 80%.

Since the introduction of duration models in 1938, it did not throw any light on the mechanism for changes in interest rates until the mid-1970s. However, Bierwag (1977) demonstrate that as the movements in interest rates take random pattern therefore duration is only an implied measure. He argues that shifts in yield curve involve many random variables. Chambers *et al.* (1988) develop multiple factor duration models, where the number of factors affecting interest rates and duration were the same, which is later extended by Nawalkha and Lacey (1990) making the model workable between coupon dates. They further argue that perfect immunization of portfolios may be almost impossible to achieve because of prevalence of stochastic factors. Vasicek (1977) and Boyle (1978) used stochastic measures for Redington (1952) for immunization process.

Ingersoll *et al.* (1978) criticize the immunization processes on account of risk-less arbitrage profit making opportunities due to convex relationship between interest rates and portfolios. They argue that an investor can make risk-less arbitrage profits by selling a discount bond short and subsequently taking a long position from proceeds on immunized coupon bond portfolio with the same maturity. In this way the investor will make profits if interest rates change, and will break even in case interest rates remain stagnant. Given the scenario of arbitrage opportunity, Bierwag and Fooladi (2006) opine that such an opportunity is hypothetical given difficulty in taking large short positions in bonds. Cox *et al.* (1979) develop a duration model based on stochastic processes that is more consistent than its contemporary measures. Their duration model also serves the purpose of measurement of implied basis risk avoiding risk free profit making opportunities. Although their models receive some criticism for instance, Brennan and Schwartz (1983), yet both types of models, i.e. continuous and discrete are in practice at present.

Fong and Vasicek (1983,1984) propose an immunization model addressing a portion of stochastic process risk. They base their argument on Fisher and Weil (1971) duration and state that the return on a portfolio is equal to some initially promised return and some amounts that is multiplication of a variable M^2.

They suggest that immunization is to be achieved in such a way so as to minimize the value of M^2 They argue that such a process constructs portfolios termed as "bullet" portfolios with either maturity clustered around some points or around the liquidation date. They argue that by minimizing M^2 stochastic process risk could be minimized. Working on the same Bierwag *et al.* (1987) and Fooladi and Roberts (1992) argue that portfolio immunization is better when portfolio of bonds have most of the bonds with maturity close to liquidation date and duration of overall portfolio equal to maturity. Later, Bierwag *et al.* (1993) compare this approach with earlier M^2 approach and argue that a portfolio with two bonds having maturity of one bond equal to target liquidation period has similar performance to a portfolio constructed as M^2 approach.

Hauser *et al.* (2001) address the impact of international exchange rates on immunization using duration strategies and found that immunization in case of assets and liabilities using foreign currencies can be achieved by holding such foreign assets and liabilities having matching duration. Fooladi *et al.* (2005) also discuss foreign currency risk in case of holding foreign portfolios and opine that foreign exchange adjusted durations are significantly different from unadjusted durations, hence should be accounted for while considering the stochastic process of interest rates and exchange rates.

Kolb and Chiang (1981, 1982) propose using durations in hedging of bond portfolios. They argue that given the definition of duration of asset as D_A, duration of futures as D_F and the value assets as $A(r) = V(r) + h[F(r) - F(r_o)]$ where $V(r)$ is value of interest sensitive securities h is the number of contracts an investor may acquire and $F(r)$ is the interest sensitive future contract.

Chance (1982) finds that joint execution of immunization and "hedging using future" strategy yields better results than any of the strategy alone. Gay and Kolb (1983) argue that keeping the investment horizon equal to duration involves continuous up gradation and rebalancing of bond portfolios which also allow holding the securities with no active market. Jeffery and Schaefer (1983) and Yawitz and Marshall (1985) also state that this kind of joint strategy enables investors to construct bond portfolios with duration of their choice. Kolb and Overdahl (1991) extend the scope to almost all securities where interest rate sensitivities apply.

Bierwag and Fooladi (2006) regard bond immunization strategy as "dynamic passive investment" which means the investor only has to follow a set of prescribed rules for management of bond portfolios, i.e. adjustment of bond durations. An alternative strategy in this case is active strategy that involves continuous buying and selling of bonds for excessive gains. Given these two strategies Leibowitz and Weinberger (1981) suggest another "contingent immunization strategy" whereby in normal cases an investor follows active strategy and moves to immunization strategy in case of adverse movement of interest rates. They argue that investors usually specify a minimum rate of return after which he shifts to immunization strategy, in which case this strategy may be termed as stop loss strategy.

Bierwag and Fooladi (2006) argue that by keeping the duration of interest risk sensitive assets equal to duration of interest risk sensitive liabilities so that the duration gap becomes zero a financial institution may be protected from adverse movements of interest rate. Furthermore, the duration gaps of on balance sheet and off balance sheet items are not mutually exclusive, i.e. management of either of the gap can be done against the other to achieve an overall objective of immunizing the financial institution as a whole. In this regard, Fooladi and Roberts (2004) present the concept of "convexity gap" in addition to duration gap. They propose that in order to immunize the net worth of financial institutions it is advisable to maintain positive duration gap alongside zero duration gap.

Goodman and Vijayaraghavan (1987) extend the scope of hedging using future contracts in case of larger interest rates changes. They argue that in case of large interest rate changes, the use of two different interest rate futures yield better results. This argument is also supported by Daigler and Copper (1998).

3.3.11.1 Use of duration in estimating changes in short and future positions. Bessler and Wolff (2014) in their works on government bond portfolios utilize the concept of matching the sensitivities of spot and future positions using the concepts of duration and convexity as proposed by Kolb and Chiang (1981, 1982) and Booth *et al.* (1989). They segregate the models for small and parallel shifts to larger and non-parallel shifts. However, as computing the exact duration of a future position could be difficult because of the option at the hand of future position holder that he has the "option" to take delivery of the bond, Daigler and Copper (1998) suggest that using hypothetical bonds is the best option because it entails minimum variations for the original delivery.

Yield changes are not identical because yield curves do not change in parallel fashion. Further equal changes in spot and future positions will imply nonexistence of default risk. Therefore in order to address the observations of Booth *et al.* (1989), who report that simple durations based hedges do not hold in case of sovereign debts a beta showing the riskiness between changes in spot and future positions need to be computed by regressing the changes in Yield to Maturity (YTM) of future positions on spot positions. This hedging strategy suffers from the fact that they provide linear approximations only in case of short-term movement of interest rates (Goodman and Vijayaraghavan, 1987; Daigler and Copper, 1998; Chen and Zhao., 2009). In order to account for larger movement of interest rates KRD measure is suggested to be included in the hedging measure (Ho, 1992; Falkenstein *et al.*, 1996).

3.3.11.2 Immunization using principal component duration. As the PC duration models are based on contribution of various factors toward total interest rate variance, it is expected that they gain efficiency of hedging. Furthermore, they produce better results than KRD in cases where taking short and long positions are not allowed due to no-possibility of key zero-immunization. However, even in the case of PC durations, error in hedging is possible due to stationary covariance of interest rate assumption (Nawalkha and Soto, 2009).

Commenting on this model Dierkes and Ortmann (2015) argue that the model of Tchuindjo (2008) produces inaccurate results with zero-coupon bonds although yields similar approximation when compared with their model.

3.3.11.3 Immunization using dedicated duration. Zaremba (2017) approaches immunization by making a subset of all possible shifts in yield curve into many classes and estimating duration for each of the class separately using the assumption that since all shifts in term structures are continuous functions. This assumption is based on Zaremba and Rzadkowski (2016) because theoretically there is no portfolio with such a present value of cash flows exists whose future value would be less than face value of the portfolio. Such a portfolio is immunized in linear subspace in m-dimensional similar space of continuous shifts, where m stands for number of times cash payments will be certain. Another assumption for their models is the fact that "all zero-coupon bonds with maturity t_i, $1 \leq i \leq m$, depict different dedicated durations."

Based on the underlying assumptions and functions Zaremba (2017) propose the following theorem:

(1) If a bond portfolio having highly dedicated convexity in a specific class, it is immunized against all possible shifts,

(2) There exists a tradable bond in that portfolio and,

(3) The immunized portfolio will be such a portfolio that consists of bonds with the minimum and maximum dedicated durations, with the proportions of investments in both types of bonds.

It follows that what matters in an immunized portfolio are two components of the immunized portfolio with minimal and maximal dedicated durations. However, this does not apply if an investor does not follow immunization strategy using our duration model and instead makes barbell portfolios.

4. Conclusion and future research directions

Application of the concept of duration depends upon strategy of the investors and their risk tolerance. For risk taking investors, it helps in making decisions in accordance with price volatility of securities. On the other hand, for risk-averse investors it provides a mechanism for immunization of portfolios. Measure of duration is the key to asset pricing and in managing stress on balance sheets of financial institutions (Gormsen and Lazarus, 2019). This review of the literature shows that over a period of time three major themes have emerged from research on duration modeling out of which the theory of Macaulay duration is the major theme whereas development of duration models and implementation of duration models are two subthemes that are firmly integrated and overlapped with each other that has been explained in Figure 9 hereunder:

Models have been developed based on the theory that has been subsequently applied leading to the augmentation of theory. Initially Macaulay (1938) argues that interest rates, yield to maturity, cash flows and its timing were the only factors affecting duration. However, later works show that there are a range of factors that are responsible for changes in duration. These factors include controllable factors, i.e. the factors that are local to financial institutions, e.g. decisions about choice of maturity, choice of interest rates, choice of quality of loans, quantum of assets and liabilities, etc.; regulatory factors, i.e. the factors that stem from actions taken by the regulatory authorities, e.g. changes in monetary policy, etc.; market factors, that stem from impact of market forces on market variables, e.g. changes in yield to maturity, market risk, etc. and lastly the environmental factors, i.e. the factors stemming from changes in climate in a particular area that impact the demand and supply of financial services. This has been explained in Figure 10 hereunder:

Our research shows that although great work has been done by leading researchers and many different types of duration models have been developed but there is a lot to be done in implementing and testing all such models. For instance every duration model needs to be tested at least on all four types of factors to gauge its strength and vulnerability in all circumstances not only mostly against two types of factors, i.e. controllable and market. This will be very helpful in taking a holistic managerial decision in assets and liabilities. Such models will be in accordance with the recommendations of great researchers of duration modeling for instance, Ingersoll (1981) and Gultekin and Rogalski (1984) who confirm that increasing the number of factors in a decision model provides more reliable results as

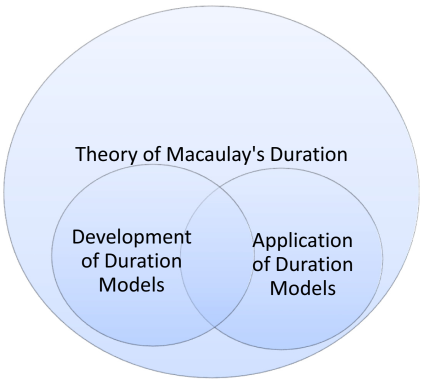

Figure 9.
Pattern of research on Macaulay duration

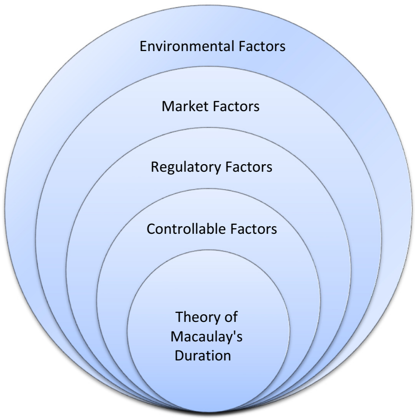

Figure 10.
Step-wise effects of various factors on duration

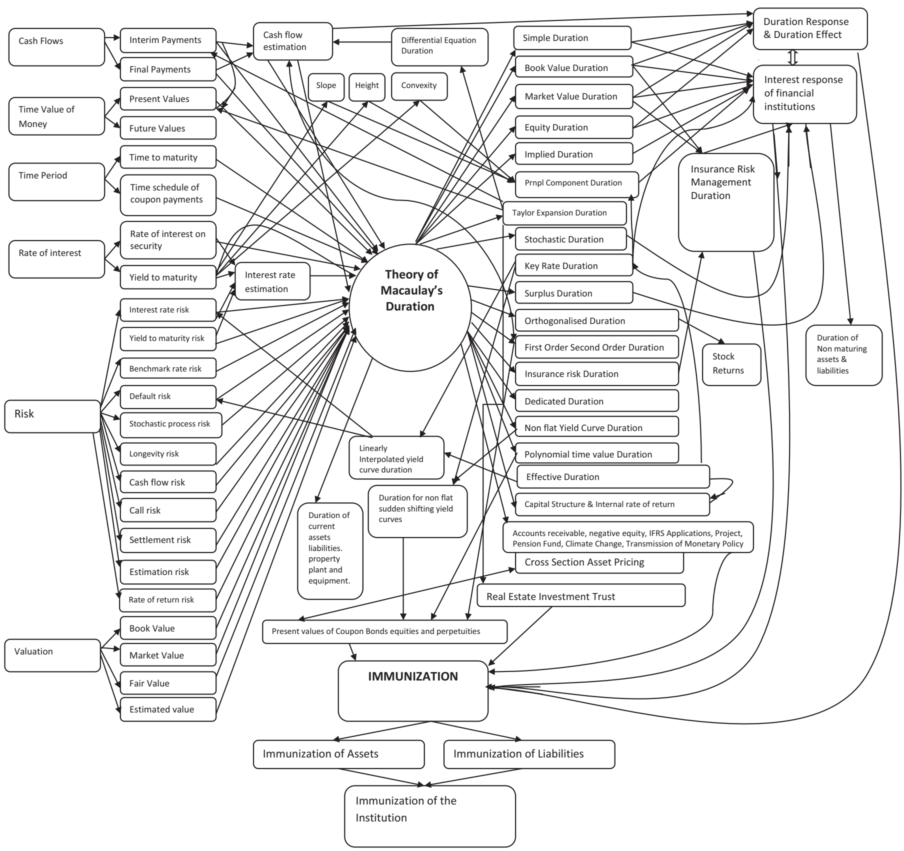

Figure 11.
Framework for development and application of models under Macaulay's theory of duration

compared to single factor models. Having explored the literature, the framework for research on Macaulay duration has been extracted hereunder:

Further works are also required to be done in the area of duration modeling for Islamic financial institutions. This is because though duration models perform better in managing mismatch of duration but these suffers from the aspect of shariah compliance Salman (2006). Islamic financial institutions have unique risk characteristics (Chattha and Archer, 2016; Chattha and Alhabshi, 2017). Current works involve application of Koch and MacDonald (2009) models that are although market based but lack discussion about shariah compliance (see Figure 11).

There are also a range of duration models that require further testing in dynamic environments to determine their strengths, suitability and mechanism for improvement. These models include key rate duration, equity duration, effective duration, implied duration, dedicated duration, principal component duration, duration using Taylor expansion, impact of duration on default rate, polynomial time-value duration and finally use of duration in capital budgeting and Internal Rate of Return (IRR).

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