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# Collective Mode Interactions in Lorentzian Space Plasma

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## Abstract

Plasmas exhibit a vast variety of waves and oscillations in which moving charged particles produce fields which ultimately give rise to particle motion. These wave-particle effects are used in the acceleration heating methods of plasma particles, and in wave generation as well. Plasmas are often manipulated with EM waves, e.g., Alfvén waves are long-wavelength modes (drift-waves) where fluid theory is most reliable, while for short wavelength modes (e.g., Kinetic Alfvén waves), collisionless effects become important. In this chapter, the properties of kinetic Alfvén waves are aimed to study by employing two potential theories by taking particle streaming and Weibel instability with temperature anisotropy in a Lorentzian plasma.

**Keywords:** KAWs, Lorentzian distribution, streaming and temperature anisotropy, dusty plasma

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## 1. Introduction

This chapter addresses one of the intriguing topics of Astrophysics—the existence of kinetic Alfvén wave (KAW) and the important consequences for astrophysical and space science to explore and investigate the new avenues. Due to the fact that KAWs have non-zero electric field  $E_{\parallel}$  which is parallel to background magnetic field and possess anisotropic polarized and spatial structures which contribute to particle energization. It is an interesting mechanism that KAWs can accelerate the field-aligned charged particles and has been applied in the dissipation of solar wind turbulence, the acceleration, and heating of charged particles in both the field-aligned and perpendicular directions and is anticipated to play a vital role in the particle energization in laboratory, space and astrophysical plasma. The progress reported here would have immense impact and hence a small step in particular direction.

The solar wind plasma is hot and weakly collisional, existing in a state far from thermal equilibrium [1] as observed in situ in the solar wind through its nonthermal characteristics of

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velocity distribution function (VDF). The electron VDFs measured at 1 AU have been used as boundary condition to determine the VDFs at different altitudes. It has been confirmed that for several solar radii, the suprathermal population of particles is present in the corona [2]. For low collision rates in such plasmas, the particles can develop temperature anisotropy and the VDFs become slanted and build up high energy tails and heat fluxes along the magnetic field direction especially in fast winds and energetic interplanetary shocks. Various processes in a collisionless solar wind plasma lead to the development of particle temperature anisotropy to generate plasma instabilities which are often kinetic in nature. The free energy sources associated with the deviation from the thermodynamical equilibrium distribution function could also excite plasma waves [3–11].

In general, the study of plasma waves and micro-instabilities in the solar wind shows that proton VDFs are prone to anisotropic instability and originate to be stable or marginally stable. Marsch [12] has discussed four significant electrostatic and electromagnetic wave modes and free energy sources to make them unstable. For example, the electrostatic ion acoustic wave may be destabilized by the ion beams and electrons and electron heat flux, [13] the electromagnetic ion Alfvén-cyclotron wave needs proton beam and temperature anisotropy, magnetosonic wave requires proton beam and ion differential streaming and whistler-mode and lower-hybrid wave [14] unstable solutions. Among several electromagnetic instabilities, the kinetic Alfvén wave instability is the most important one.

The satellite missions in space and astrophysical plasmas have confirmed the presence of non-Maxwellian high energy and velocity tails in the particle distribution function and found in the magnetosphere of Saturn, Mercury, Uranus and Earth [2, 15–17]. The non-Maxwellian distribution of charged particles has been observed to give a better fit to the thermal and superthermal part by employing kappa distribution, since it fits both thermal and suprathermal parts in the energy velocity spectra.

The subject area of this chapter involves the basic research of space plasma physics and in particular, focuses the investigations of electrostatic and electromagnetic waves in a multi-component dusty (complex) Maxwellian and non-Maxwellian plasmas. In the last few years, various power-law distribution functions (in velocity space), i.e., kappa and  $(r, q)$  have been used to investigate collective phenomena and associated instabilities, such as dust-acoustic waves, kinetic Alfvén waves, Weibel instabilities, dust charging processes (in linear and nonlinear regimes) in space and astrophysical situations for better fitting the observational data in comparison to Maxwell distribution. These distributions have relevance to space plasmas containing solar wind, interstellar medium, ionosphere, magnetosphere, auroral zones, mesosphere, lower thermosphere, etc.

When the intense radiations interact with plasmas, it ends up with many applications like instabilities, inertial confinement fusion [18], and pulsar emissions [19]. These instabilities further generate turbulent electromagnetic fields in plasma regimes. We can characterize instabilities as electrostatic as well as electromagnetic according to the conditions provided by nature [20]. In this chapter, we shall also discuss electromagnetic instability called Weibel instability in a Lorentzian plasma. The free energy source available for Weibel instability is temperature anisotropy and can be developed in magnetically confined and magnetic free plasma environment as well. First time Weibel [21] came up with the calculations of impulsive

growing transverse waves with anisotropic velocity distribution function in 1958. This instability developed when the electrons in the fluctuating magnetic field generates momentum flux, this flux sequentially effects velocity  $\langle \mathbf{v} \rangle$  (and ultimately current density  $\langle \mathbf{J} \rangle$ ) as to increase the fluctuating field [22]. The property of Weibel instability is that it is different from normal resonant wave-particle instabilities because it depends on effects in bulk plasma without any resonant particle contributions [23]. The particle distribution functions in kinetic model adequately describe a physical phenomenon in terms of time and phase space configurations providing more information to investigate plasma waves, instabilities, plasma equilibrium, Landau damping phenomena, etc. In this chapter, we shall review the kinetic/Inertial Alfvén waves and instabilities, the effects of dust grain charging as well as field aligned/cross field currents, streaming velocity and the non-Maxwellian power-law distribution and its effect on various electromagnetic modes. We intend to show that the presence of dust grains introduces a new cutoff frequency  $\Omega_{dih}$  which is associated with the motion of mobile charged particles. Moreover, an interesting feature is to show that the employed model inhibits the temperature anisotropy and supports the velocity anisotropy. Further, we shall calculate the linear dispersion relation for Weibel instability in Lorentzian plasma ( $B_0 = 0, B_0 \neq 0$ ) by using linearized, nonrelativistic Vlasov equation. We shall solve  $Z_\kappa^*(\alpha)$  by assuming  $\alpha < 1$  or  $\alpha > 1$  for  $\kappa = 3, 5, 7$ .

## 2. Model and methodology

In long-wavelength modes the fluid theory is most reliable, while for short wavelength modes (like KAWs), collisionless effects are important, for example, Landau damping due to finite ion Larmor radius explains observed damping rate and in dusty plasmas and charge fluctuations. Kinetic Alfvén waves (KAWs) are small scale dispersive Alfvén waves (AWs) which plays a significant role in particle acceleration and plasma heating. A coupling mechanism between small-scale KAWs and large-scale AWs in the presence of superthermal particles has been discussed which in turns giving rise to the excitation of KAWS in a solar/stellar wind plasma have been studied in the past. In this chapter, we intend to show the relationship between the growth rates of excited anisotropic KAWs and perpendicular wavelength by taking charge fluctuation and Landau damping variations into account. Moreover, when the perpendicular component of the wavelength, when comparable to the ion gyroradius, a magnetic field aligned electric field plays a significant role in the plasma acceleration/heating. Utilizing a two potential theory along with kinetic description, the properties of kinetic Alfvén waves are aimed to investigate different modes in low beta plasmas by incorporating the streaming effects. We present overview of electromagnetic KAW streaming instability in a collisionless dusty magnetoplasma, whose constituents are the electrons, ions and negatively charged dust particles. The interaction between monochromatic electron/ion beam with plasma is also discussed under various conditions. Further, to calculate the linear dispersion relation for Weibel instability in unmagnetized Lorentzian plasma, we shall employ linearized, nonrelativistic Vlasov equation.

### 2.1. Two potential theory

In a low beta plasma,  $\beta < 1$ , the electric field can be described by two potential theory or fields expressing the electromagnetic perturbations with shear perturbations only in the magnetic

field. We may neglect the electromagnetic wave compression along the direction of magnetic field ( $B_{1z} = 0$ ), which leads to the coupling of Alfvén-acoustic mode. Thus, we adopt a two potential theory which represents both the transverse and parallel components of the electric field as  $E_{\perp} = -\nabla_{\perp}\varphi$  and  $E_{\parallel} = -\partial\psi/\partial z$ , with  $\varphi \neq \psi$  [23]. We shall also consider the charge on the dust grain which may fluctuate according to the plasma conditions. At equilibrium, the charge neutrality imposes the condition  $n_{e0} - n_{i0} + Z_{d0}n_{d0} = 0$ , where  $n_{e0}(n_{i0})$  is the electron (ion) number density and  $Z_{d0}$  is the equilibrium dust charging state.

The linearized Poisson and Maxwell equations in terms of parallel and perpendicular operators can be expressed as

$$\nabla_{\perp}^2\varphi + \partial_z^2\psi = \frac{1}{\epsilon_0}(n_{e1} + Z_{d0}n_{d1} + n_{d0}Z_{d1} - n_{i1}), \quad (1)$$

and

$$\partial_z\nabla_{\perp}^2(\varphi - \psi) = \mu_0\partial_t(J_{e1z} + J_{i1z} + J_{d1z}), \quad (2)$$

where  $\epsilon_0(\mu_0)$  is the permittivity (permeability) of the free space and  $J_{j1z}$  represents the field aligned current density for  $j$ th species ( $j = e$  for electrons,  $i$  for ions and  $d$  for dust grains). In obtaining Eq. (2), we have ignored the factor  $\nabla_{\perp}\nabla_z E_z - \nabla^2 E_{\perp}$ . The main idea is to decouple the compressional Alfvén mode under the assumption  $(\nabla \times E)_z = -(\partial B/\partial t)_z = 0$ , i.e., to highlight bending of line of force and minimize any change in field strength due to wave compression. Moreover,  $(\nabla \times E)_z$  engross only  $E_{\perp}$  and the least restrictive assumption for  $(\nabla \times E)_z$  to vanish is  $E_{\perp} = -\nabla_{\perp}\varphi$  in which the perpendicular electric field  $E_{\perp}$  is electrostatic, leaving an incompressible mode. When  $\varphi = \psi$ , the twist of the magnetic field lines vanishes, therefore, the incompressible shear modes have  $\nabla \cdot u_{\perp} = 0 = B_{1z}$ , and  $E_{\perp} = -\nabla_{\perp}\varphi$ .

$$(\partial_t + \mathbf{v} \cdot \nabla)f_{j1} + \frac{q_j}{m_j}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_{\mathbf{v}} f_{j0} = 0, \quad (3)$$

$$f_{j1} = \frac{q_j k_z \psi}{m_j(\omega - k_z v_z)} \frac{\partial f_{j0}}{\partial v_z}, \quad (4)$$

where  $f_{j0}$  is the equilibrium distribution function. The dynamics of cold and magnetized dust is governed by set of fluid equations, i.e.,

$$\partial_t v_d = \frac{Z_d e}{m_d} \mathbf{E} + \mathbf{V}_d \times \boldsymbol{\omega}_{cd} \quad (5)$$

and

$$\partial_t n_d + \text{div}(n_d \mathbf{V}_d) = 0 \quad (6)$$

## 2.2. Number density and current density perturbations

Here, we may define the number density and current density as

$$n_{j1} = n_0 \int f_{j1} d\mathbf{v}, \quad j = e, i, d$$

$$J_{j1} = q_j n_0 \int \mathbf{v} f_{j1} d\mathbf{v},$$
(7)

### 3. Dispersion and damping of kinetic Alfvén waves (KAW)

Kinetic Alfvén waves (KAWs) are small scale dispersive Alfvén waves (AWs) which plays a significant role in particle acceleration, turbulence, wave particle interaction and plasma heating. Kinetic processes prevail in the regimes where plasma is dilute, multi-component, and non-uniform. A coupling mechanism between small-scale KAWs and large-scale AWs with superthermal plasma species which in turns gives rise to the excitation of KAWS in a solar/stellar wind plasma has proved dispersive Alfvén waves responsible for the solar wind turbulence especially when the turbulence cascade of these electromagnetic waves transfer from larger to smaller scale as compared to proton gyro radius. Moreover, from spacecraft observations in ionospheric plasma, it is evident that Alfvénic Poynting flux is responsible to transfer the energy for particle acceleration. All the energized auroral particles accelerate in ionosphere, initiate Joule heating phenomenon and stream out into the magnetosphere [25–28].

There are number of studies to show the relationship between the growth rates of excited anisotropic KAWs and perpendicular wavelength by taking charge fluctuation and Landau damping variations into account. Moreover, the perpendicular component of wavelength, when comparable to ion gyroradius, a magnetic field aligned electric field plays a significant role in plasma acceleration/heating.

One of the important features in astrophysical plasma is the transportation of electromagnetic energy through the wave interaction with thermal plasma ions [29–31]. The KAW plays a vital role to transfer the wave energy through Landau damping (when thermal electrons travel along the magnetic field lines), which is regarded as collisionless damping of low-frequency waves and during this process the particles gain kinetic energy from the wave. This process can only happen when the distribution function has a negative slope which results in the heating of plasmas or acceleration of electrons along the magnetic field direction [24]. Recent studies also suggest the impact of non-Maxwellian distribution functions on the dynamics of solar wind and auroral plasma [32]. This study shows that the plateau formation in the parallel electron distribution functions minimize the Landau damping rate significantly.

In this chapter, the properties of kinetic Alfvén waves would be discussed by employing two potential theory, Maxwell equations and Vlasov model to study different plasma modes and by taking streaming of charged particles along and across the field direction in a Maxwellian and Lorentzian plasma.

#### 3.1. Kinetic Alfvén waves in Maxwellian plasma

The propagation of kinetic Alfvén waves in a dusty plasma with finite Larmor radius effects will be discussed using a fluid-kinetic formulation by taking charge variations of dust

particles. The coupling of Alfvén-acoustic mode results in the formations of kinetic Alfvén wave which would be discussed in forth coming subsections. In a magnetized plasma, we shall consider the electrons are thermal and strongly magnetized obeying an equilibrium Maxwellian distribution, while ions are hot and magnetized so that finite Larmor radius can be taken into account. For ions, we may employ Vlasov equation by utilizing guiding center approach to obtain the perturbed distribution function for an electromagnetic wave when the electric field and the wave vector  $\mathbf{k}$  lie in the  $xz$  plane and,  $\mathbf{B}_0 = (0, 0, \mathbf{B}_0)$ ,  $\mathbf{k} = (k_\perp, 0, k_z)$ ,

$$f_{i1} = -\left(\frac{n_{i0}e}{T_i}\right) \sum_l \sum_n \frac{k_z v_z \psi + n\Omega_{ci}\varphi}{\omega - n\Omega_{ci} - k_z v_z} \exp(i(n-l)\theta) J_n\left(\frac{k_\perp v_\perp}{\Omega_{ci}}\right) J_l\left(\frac{k_\perp v_\perp}{\Omega_{ci}}\right) f_{i0} \quad (8)$$

where  $J_n$  is the Bessel function of first kind, having order  $n$  and  $f_{i0}$  is the equilibrium distribution function. On using Eq. (7), we obtain the modified number and current densities for hot and magnetized ions and thermal electrons, i.e.,

$$n_{i1} = -\frac{en_{i0}}{k_z m_i v_{ti}^2} \sum_n [k_z v_{ti} \psi (1 + \xi_{in} Z(\xi_{in})) + n\Omega_{ci} \varphi Z(\xi_{in})] I_n(\vartheta_i) e^{-\vartheta_i}, \quad (9)$$

$$J_{i1z} = -\frac{n_{i0}e^2}{T_i k_z} \sum_n [(1 + \xi_{in} Z(\xi_{in})) (k_z v_{ti} \xi_{in} \psi + n\Omega_{ci} \varphi)] I_n(\vartheta_i) e^{-\vartheta_i}, \quad (10)$$

and

$$n_{e1} = \frac{en_{e0}}{T_e} \psi (1 + \xi_e Z(\xi_e)), \quad (11)$$

$$J_{e1} = \frac{e^2 n_{e0} \psi}{m_e v_{te}} \xi_e Z'(\xi_e), \quad (12)$$

where  $I_n$  is the modified Bessel function with argument  $\vartheta_{i,e} = k_\perp v_{ti,e}^2 / 2\Omega_{ci,e}$  and  $Z(\xi_{in})$  is the usual dispersion function for a Maxwellian plasma with  $\xi_{in} = (\omega - n\Omega_{ci}) / k_z v_{ti}$  and  $Z'$  is the derivative of  $Z$  with respect to its argument.

The dust component is considered to be cold and unmagnetized such that  $\omega \ll \omega_{ci}$ ,  $k_z v_{te} \gg \omega$  and  $k_z v_{ti} \ll \omega$ , therefore, we use hydrodynamical model with momentum balance equation and continuity equation For cold and unmagnetized dust and thus we obtain

$$n_{d1} = -\frac{n_{d0} Z_{d0} e}{m_d \omega^2} [k_\perp^2 \varphi + k_z^2 \psi], \quad (13)$$

and

$$J_{d1z} = \frac{n_{d0} Q_{d0}^2}{m_d \omega} k_z \psi. \quad (14)$$

To find the relation between  $\varphi$  and  $\psi$ , the expressions of  $n_{e1}$ ,  $n_{i1}$  and  $n_{d1}$  are used into Eq. (1) and  $J_{i1z}$ ,  $J_{e1z}$ , and  $J_{d1z}$  into Eq. (2) to obtain the following coupled equations:

$$\begin{aligned} A\varphi + B\psi &= 0, \\ C\varphi + D\psi &= 0, \end{aligned} \tag{15}$$

The coefficients in Eq. (15) are given by

$$\begin{aligned} A &= \frac{k_{\perp} F_i}{\omega^2} \omega' \left( 1 - \frac{3}{4} \vartheta_i \right), \\ B &= \lambda_{De}^{-2} - \left( k_z^2 \omega_{pd}^2 \right) / \omega^2, \\ C &= c^2 k_z^2 k_{\perp}^2, \\ D &= \frac{1}{k_z} \frac{2\omega^2}{\lambda_{De}^2} - k_z \epsilon_{\perp}, \end{aligned} \tag{16}$$

where  $F_i = \omega_{pi}^2 / \omega_{ci}^2$ ,  $\omega' = \omega^2 - \Omega_{(dlh)i}^2$ ,  $\epsilon_{\perp} = \omega_{pd}^2 + c^2 k_{\perp}^2$  and  $\Omega_{(dlh)i}^2 = \omega_{pd}^2 / F_i$ . The solution of homogeneous Eq. (15) in the form of a biquadratic equation, i.e.,

$$p\omega^4 + Q\omega^2 + R = 0 \tag{17}$$

where,

$$\begin{aligned} p &= \frac{2\omega_{pe}^2}{k_z v_{te}^2}, \\ Q &= -\frac{2\omega_{pd}^2}{k_z v_{te}^2} \Omega_{(dlh)i}^2 - k_z \epsilon_{\perp} - \frac{k_z v_{Ai}^2}{\lambda_{De}^2} \left( 1 + \frac{3}{4} \vartheta_i \right), \\ R &= \Omega_{(dlh)i}^2 k_z \epsilon_{\perp} + k_z^3 v_{Ai}^2 \omega_{pd}^2 \left( 1 + \frac{3}{4} \vartheta_i \right) \end{aligned} \tag{18}$$

where  $V_{Ai} = c\omega_{ci} / \omega_{pi}$  is the Alfvén velocity of ions. The solution of biquadratic equation in the form of kinetic Alfvén wave is as follows,

$$\omega^2 = \Omega_{(dlh)i}^2 + k_z^2 V_{Ai}^2 \left[ 1 + \left\{ \frac{3}{4} + T' \Lambda_i \frac{\epsilon_{\perp}}{c^2 k_{\perp}^2} \right\} \vartheta_i \right] \tag{19}$$

where,  $T' = \frac{T}{T_i}$  and  $\Lambda_i = n_{i0} / n_{e0}$ . This shows the dispersion relation of kinetic Alfvén waves in the presence of mobile dust that are the extension of shear Alfvén waves in the range of small perpendicular wavelength. The first term on the R. H. S appears due to dust dynamics, i.e., a new cut off frequency due to the hybrid dynamics of cold dust and magnetized ions which provides a limit to the propagation of electromagnetic wave. In a dustless plasma, i.e.,  $\omega_{pd} = 0$ , we obtain usual dispersion relation in electron-ion plasma. Expressing  $\omega$  in terms of real and imaginary part,  $\omega = \omega_r + i\gamma$ , with  $\omega_r \gg \gamma$ , we either obtain growth or damping of KAW satisfying the condition,  $\omega/k_z = v_A \leq v_z$  through wave particle interactions [33, 34]. In a dusty

plasma with dust charge fluctuation effect, the main mechanism of wave damping is associated with dust charge fluctuation effects as compared to Landau damping [34]. It is a well-known fact that if the particle thermal velocity exceeds the Alfvén velocity, then the particles interact with Alfvén wave as the result of wave particle interaction/resonance, the linear Landau damping prevails. In a dusty plasma, the massive dust grains move slowly as compared to Alfvén velocity, therefore they may interact with Alfvén wave through linear Landau damping (which is negligible in case of dust species) or charge fluctuation effects.

### 3.2. Lorentzian distribution function

A number of processes in a space based plasma lead to the development of particle anisotropy through streaming or temperature and are responsible for plasma instabilities in collision-free plasma which are frequently kinetic in nature and their persistent features have been confirmed by many spacecraft measurements, e.g., the electron energy spectra and the near-earth environment observations have witnessed the presence of superthermal populations. It is a well-known fact that the equilibrium Maxwell-Boltzmann distributions are associated with the Boltzmann collision term, but on the large scale Fokker-Plank model is not appropriate due to strong interaction and correlation in a collisionless plasma. The kinetic foundations of generalized Lorentzian statistical mechanics has been remarkably established by [35] with the generalization of Boltzmann collision term that is not based on binary collisions. The long range correlation between particles vindicates that power law distributions possess a particular thermodynamical equilibrium state. The mathematical form on isotropic Lorentzian distribution function is given by

$$f_{j0}^\kappa = A_\kappa \left[ 1 + \frac{1}{\kappa v_{ij\kappa}^2} (v_z^2 + v_\perp^2) \right]^{-\kappa-1}; \quad \kappa > 3/2, \quad (20)$$

$$\text{where } A_\kappa = n_{j0} \left( \frac{1}{\pi \kappa v_{ij\kappa}^2} \right)^{\frac{3}{2}} \frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)}$$

Due to the stated fact, the deviation from the Maxwellian equilibrium distribution function could also excite plasma waves by using free energy sources. Such distributions are frequently observed in solar and terrestrial environments and can be represented by anisotropy in temperature and velocity, i.e., [36]

$$f_{j\kappa}(v_z, v_\perp) = A_\kappa \left[ 1 + \frac{1}{\kappa} \left( \frac{(v_z - v_0)^2}{v_{ij\kappa z}^2} + \frac{(v_\perp - v_0)^2}{v_{ij\kappa \perp}^2} \right) \right]^{-\kappa-1}, \quad (21)$$

where  $v_{ij\kappa}^2 = \left(\frac{2\kappa-3}{\kappa}\right)v_{ij}$ ,  $v_{ij} = (\sqrt{k_B T/m_j})$  is the thermal speed of  $j$ th plasma component, the number densities are represented by  $n$  and anisotropic temperatures components are represented as moment of second order

$$T_z = \frac{m}{n_c k_B} \int f_\kappa v_z^2 d^3 v, \quad T_\perp = \frac{m}{n_c k_B} \int f_\kappa v_\perp^2 d^3 v, \quad (22)$$

In the limit  $\kappa \rightarrow \infty$ , the bi-Lorentzian function is reduced to bi-Maxwellian,  $f_\kappa(v) \rightarrow f_M(v)$ .



### 3.3. Lorentzian current and number density perturbations

Many space and astrophysical plasmas have been found to have generalized Lorentzian particle distribution functions. It is of some interest to observe the impact of the high energy tail on the current and number densities of plasma species. By using Eqs. (4), (7) and (20), we get the modified expressions of number and current densities based on kappa distribution function, i.e.,

$$n_{j1} = \pm \frac{2e\psi n_{j0}}{m_j v_{thj}^2} [\kappa' + \xi_{j0} Z_\kappa(\xi_{j0})], \quad (23)$$

and

$$J_{j1z} = -\frac{2e^2\psi n_{j0}}{m_j v_{thj}^2} [\kappa' \xi_{j0} + \xi_{j0}^2 Z(\xi_{j0})], \quad (24)$$

where  $Z_\kappa(\xi_{j0}) = \frac{1}{\pi^{1/2}} \frac{\Gamma(\kappa+1)}{\kappa^{3/2} \Gamma(\kappa-1/2)} \int_{-\infty}^{+\infty} \frac{x dx}{(x-\xi_{j0})(1+x^2/\kappa)^{\kappa+1}}$ , is the plasma dispersion function and  $\kappa' = (2\kappa - 1)/2\kappa$ .

### 3.4. KAW and instability in Lorentzian plasma

In a low  $\beta$  plasma, the kinetic Alfvén wave instability driven by field aligned currents has dependence on plasma  $\beta$  and streaming velocity of current carrying species which can be responsible for particle energization. In this subsection, we extend the above scenario of electromagnetic kinetic Alfvén wave by introducing the streaming of Lorentzian ions along an external magnetic field ( $\mathbf{B}_0 \hat{z}$ ) with constant ion drift velocity ( $V_0 \parallel \mathbf{B}_0$ ), strongly magnetized and hot electrons to be Maxwellian and cold unmagnetized dust. The plasma beta  $\beta_e$  is assumed to be very small. The electric field and the wave vector  $\mathbf{k}$  lie in the  $xz$  plane, i.e.,  $\mathbf{B}_0 = (0, 0, B_0 \hat{z})$ ,  $\mathbf{V}_0 = (0, 0, V_0 \hat{z})$ ,  $\mathbf{k} = (k_\perp \hat{x}, 0, k_z \hat{z})$ . We again solve the Vlasov equation For hot and magnetized electrons [33] to get the number and current density of electrons as obtained in the previous section. Making use of Eqs. (4), (7) and (21), we get the perturbed number density of Lorentzian type streaming ions,

$$n_{i1} = -\frac{2en_{i0}\psi}{m_i v_{thi}^2} [\kappa' + \eta Z_\kappa(\eta)], \quad (25)$$

The longitudinal components of current density perturbation [7, 19, 37] is given by

$$J_{i1z} = -\frac{2e^2 n_{i0} \psi}{m_i v_{thi}^2} [\kappa' \eta + \eta^2 Z_\kappa(\eta)], \quad (26)$$

where  $\eta = (\omega - k_z V_0)/k_z v_{thi}$ .

By incorporating the values of  $n_{i1}$  and  $J_{i1}$  in Eqs. (1), (2) and using (15), the dispersion relation of KAW streaming instability in a Lorentzian dusty plasma is obtained as

$$1 + \frac{2\omega_{pe}^2}{k_z^2 v_{te}^2} \delta_1 + \frac{2\omega_{pi}^2}{k_z^2 v_{tik}^2} \delta_2 + \frac{\omega_{pd}^2}{k_z^2 c^2} + K - (1 + K) \frac{\omega_{pd}^2}{\omega^2} = 0, \quad (27)$$

where  $K = k_{\perp}^2/k_z^2$  and

$$\begin{aligned} \delta_1 &= \sum_n \left[ \epsilon_{\perp} \frac{n\Omega_{ce}}{k_{\perp}^2 c^2 k_z v_{te}} Z(\xi_{en}) I_n e^{-\vartheta_e} + \left( 1 - \frac{\omega v_{te} \xi_{en}}{c^2 k_z} + \frac{n\omega\Omega_{ce}}{k_{\perp}^2 c^2} \right) (1 + \xi_{en} Z(\xi_{en})) I_n e^{-\vartheta_e} \right] \\ \delta_2 &= \left( 1 - \frac{2v_{tik}\omega\eta}{c^2 k_z} \right) \left( 1 - \frac{\omega_{pd}^2}{\omega^2} \right) (\kappa' + \eta Z_{\kappa}(\eta)). \end{aligned} \quad (28)$$

A visible modification can be noticed by the effect of superthermality via the kappa-modified plasma dispersion function and the appearance of dust lower hybrid frequency due to dust effects on the dispersion characteristics. Numerous standard wave modes can originate from the above dispersion equation by applying particular limits, i.e.,

(i) ( $\mathbf{k} \parallel \mathbf{B}_0, V_0$ ): For  $n = 0$ ,  $\vartheta_e \ll 1$ , a dispersion relation two stream instability (TSI) in unmagnetized plasma is obtained [37], i.e.,

$$1 + \frac{2\omega_{pe}^2}{k_z^2 v_{te}^2} (1 + \xi_{en} Z(\xi_{en})) + \frac{2\omega_{pi}^2}{k_z^2 v_{tik}^2} (\kappa' + \eta Z_{\kappa}(\eta)) - \frac{\omega_{pd}^2}{\omega^2} = 0. \quad (29)$$

In the limit  $\kappa \rightarrow \infty$ , our results approach to its classical Maxwellian counterpart in a dustless plasma environment [38].

(ii) ( $\mathbf{k} \parallel \mathbf{B}_0, V_0 = 0, (\Omega_{ci} \ll \omega \ll \Omega_{ce})$ ): In a dustless plasma, we get whistler-like mode whose frequency is below the electron cyclotron frequency, i.e.,

$$\omega = k_z v_{ph},$$

where  $v_{ph} = c^2 k_z \Omega_{ce} / \omega_{pe}^2$  is the phase velocity of whistler waves which is obviously not susceptible to the Lorentzian index  $\kappa$ . Again, in the limiting case  $\omega \ll \Omega_{ce}$ ,  $\vartheta_e \ll 1$ , and expanding plasma dispersion function Eq. (15) depicts the coupling of electromagnetic and electrostatic mode, i.e., shear Alfvén-acoustic mode due to thermal kinetic effects due to which shear Alfvén wave builds a longitudinal component, e.g.,

$$\left( \omega^2 - \Omega_{(dlh)e}^2 \right) \left[ \kappa' \frac{\omega_{pi}^2}{k_z^2 v_{tik}^2} (\omega^2 - \omega k_z V_0) - k_z \epsilon_{\perp} \right] - \frac{k_z V_{Ae}^2}{\lambda_{Dik}^2} \left[ \omega^2 - k_z^2 \omega_{pd}^2 \lambda_{Dik}^2 \right] = 0, \quad (30)$$

where  $\Omega_{(dlh)e}^2 = \omega_{pd}^2 / F_e$ ,  $F_e = \omega_{pe}^2 / \Omega_{ce}^2$ ,  $\lambda_{Dik} = \sqrt{\frac{T_i}{4\pi n_{i0} e^2} \frac{1}{c_{\kappa}}}$  and  $c_{\kappa} = (2\kappa - 1)/(2\kappa - 3)$ . In the limit  $\omega^2 / k_z^2 V_A^2 \rightarrow 0$ ,  $V_0 = 0$ ,  $k_{\perp}^2 \rho_e^2 \ll 1$  we get the dispersion relation of Lorentzian dust-acoustic waves,  $\omega^2 = (k_z^2 C_D^2) / c_{\kappa}$ , where  $C_D^2 = Z_{d0} T_i / m_d$  and  $V_{Ae} = B_0 / (4\pi n_{e0} m_e)^{1/2}$  is the electron Alfvén speed with electron mass density. For a low beta plasma, the coupling between dust-acoustic and shear Alfvén wave becomes weak and two modes would decouple. In the limit  $\kappa \rightarrow \infty$ , we approach to a Maxwellian DAW [40]. It is worthy to mention here that due to the

contribution of Lorentzian particles, the KAW instability suppresses. As a matter of fact, the coupling mechanism enhances the unstable regions as the wave exchanges the energy, and we can deduce that in case of generalized Lorentzian plasma, the coupling between two modes becomes weak to some extent. Moreover for non-zero streaming velocity of ions, the unstable regions tend to grow. After simplifying Eq. (30), we get the mixed shear Alfvén-acoustic mode, i.e.,

$$\omega^2 = \Omega_{(dlh)e}^2 + k_z^2 V_{Ae}^2 \left[ 1 + \frac{\epsilon_{\perp} \Lambda_i^{-1} \rho_e^2}{c^2 c_{\kappa}} \right], \quad (31)$$

where  $\rho_e^2 = T_i/m_e \Omega_{ce}^2$ , and  $V_{Ae} = c \Omega_{ce} / \omega_{pe}$ . In the limit  $\beta_i \ll 1$  and for  $k_{\perp}^2 \rho_e^2 \ll 1$ , the two modes decouple and we get,

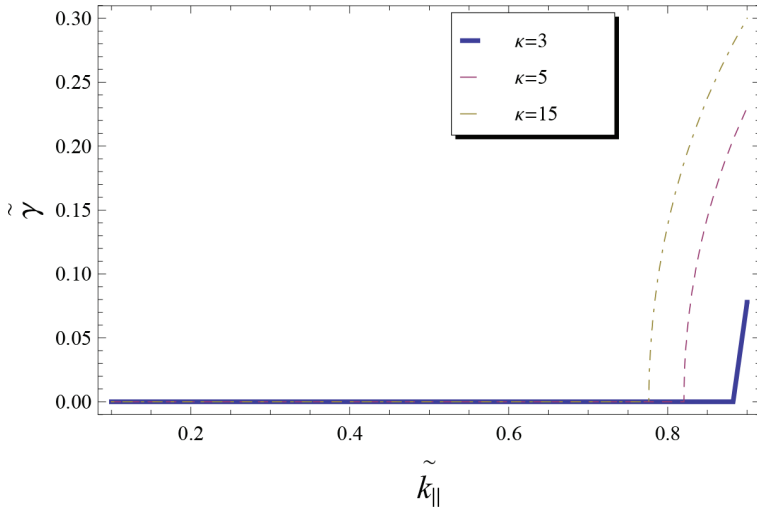
$$\omega^2 = \epsilon_z \frac{\Omega_{(dlh)e}^2}{\omega_{pd}^2}, \quad (32)$$

where  $\epsilon_z = \omega_{pd}^2 + k_z^2 c^2$  and  $\Omega_{(dlh)e}^2 = \omega_{pd}^2 / F_e$  is the dust lower hybrid frequency which arises due to the hybrid motion of magnetized electrons and unmagnetized dust grains and is referred as a cutoff frequency which gives rise to a limit for the propagation of electromagnetic waves in the presence of dust grains. For graphical representation, we have chosen parameters typical to space dusty environment, for example, we consider  $n_{i0} = 10 - 10^4 \text{ cm}^{-3}$ ,  $n_{d0} = 10 - 10^{-2} \text{ cm}^{-3}$ ,  $Z_{d0} = 10 - 10^4$ ,  $m_d = 10^5 - 10^8 m_i$ . For computational convenience, we introduce dimensionless parameters which are as follows:  $\omega = \Omega_c \tilde{\omega}$ ,  $k_z = \Omega_c \tilde{k}_z / V_A$ ,  $V_0 = V_A \tilde{V}_0$ . It has been observed that the growth rates of KAW instability are significantly affected by the presence of superthermal population, i.e., instability suppresses due to energetic particles possessed by kappa distribution when compared to its Maxwellian counterpart as shown in **Figure 1**. Similarly, the effect of streaming velocity, dust number density and charge on the growth rates is depicted in **Figures 2–4** respectively. The free energy is associated with the drift motion of ions along the field direction which is responsible for the excitation of KAW. In a streaming plasma the velocity of ions is directly coupled to dust-acoustic waves and through this coupling the maximum growth rate is obtained when the wave exchanges energy through the streaming of ions. Moreover, the presence of dust particles has a noticeable effect on the wave dynamics through dust charge  $Z_d$  and number density  $n_d$  i.e., it modifies the wave propagation and excitation. We can observe that  $Z_d$  and  $n_d$  enhances the growth rates of KAW due to the reason that when dust concentration in plasma is introduced they attach the plasma electrons toward them and the electron loss rate increases which in particular enhances the drift velocity to facilitate the unstable wave structure.

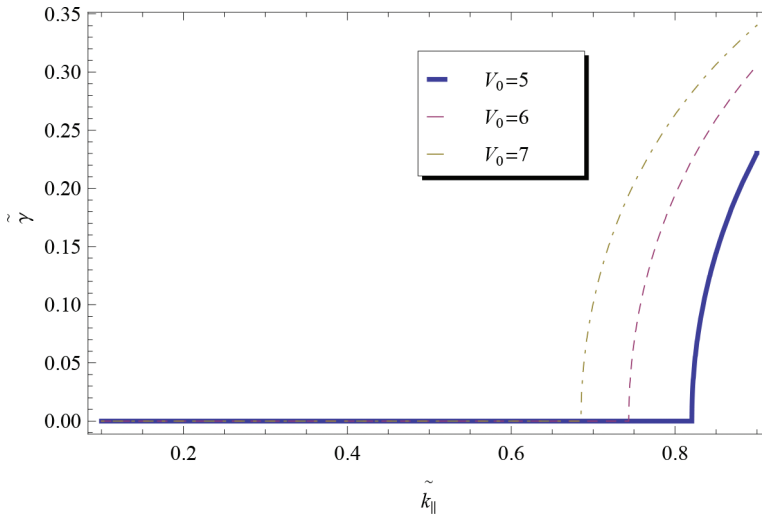
### 3.5. Dust kinetic Alfvén waves (DKAWs)

DKAWs arise when the dispersion relation of ordinary Alfvén waves is modified by the finite Larmor radius effect of dust. This process is dominated by the collective dynamics of magnetized dust particles. We have investigated shear Alfvén waves and their coupling with dust-acoustic wave by considering magnetized dust and Lorentzian electrons and ions.

The perturbed current and number densities of cold and magnetized dust are obtained by using Eqs. (5) and (6)



**Figure 1.** Effect of  $\kappa$  on the imaginary part ( $\tilde{\gamma} = \gamma/\Omega_{ce}$ ) of the dispersion relation.



**Figure 2.** Effect of  $V_0$  on the imaginary part of dispersion relation.

$$n_{d1} = -\frac{n_{d0}Q_{d0}}{m_d} \left[ \frac{1}{\omega^2 - \omega_{cd}^2} k_{\perp}^2 \varphi + (k_z^2 \psi)/\omega^2 \right] \quad (33)$$

The parallel component of perturbed dust current density turns out to be from Eq. (14)  $J_{d1z} = (\omega_{pd}^2/\omega) \epsilon_0 k_z^2 \psi$  and the dispersion relation of kinetic Alfvén wave in the presence of magnetized dust is given by

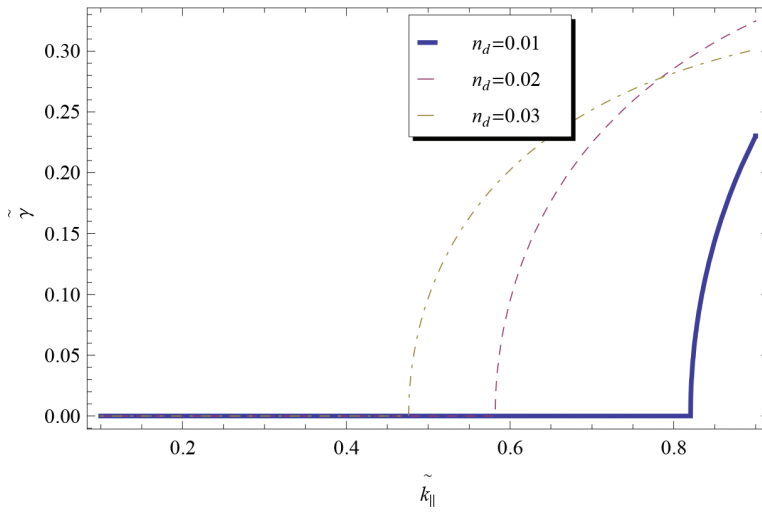


Figure 3. Role of dust number density  $n_{d0}$  on the growth rates.

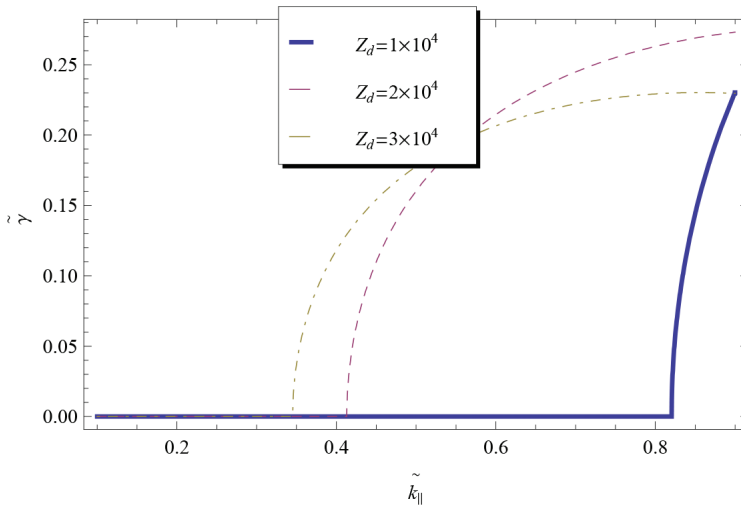


Figure 4. Role of dust charge  $Z_{d0}$  on the growth rates.

$$\omega^2 = k_z^2 V_{DA}^2 \left[ 1 + \frac{\epsilon_{\perp} \rho_d^2}{c^2 c_{\kappa}} \right] \quad (34)$$

In the limit  $\kappa \rightarrow \infty$ , we obtain classical results in a Maxwellian plasma.

### 3.5.1. Lorentzian-type charging currents

The charging equation containing Lorentzian electron and ion currents is

$$\frac{\partial Q_{d1}}{\partial t} = \sum I_{e1}^\kappa + I_{i1}^\kappa \quad (35)$$

where the electron and ion currents are calculated using a surface integral through the dust grain surface of radius  $r_d$  having potential  $\varphi_d$  are given as

$$I_{e1}^\kappa = -\tau_1 [\xi_{e0}(\kappa' + \xi_{e0}Z(\xi_{e0}))m_e\theta_{e\parallel} + 2e\varphi_d Z_\kappa(\xi_{e0})], \quad (36)$$

and

$$I_{i1}^\kappa = -\tau_2 [\xi_{i0}(\kappa' + \xi_{i0}Z(\xi_{i0}))m_i\theta_{i\parallel} - 2e\varphi_d Z_\kappa(\xi_{i0})], \quad (37)$$

where  $\tau_1 = (a_d^2\psi)/(4m_i\theta_{i\parallel}(\lambda_{Di}^\kappa)^2\kappa')$ ,  $\tau_2 = (a_d^2\psi)/(4m_i\theta_{i\parallel}(\lambda_{Di}^\kappa)^2\kappa')$  and  $\lambda_{e,i}^\kappa = \left(\frac{1}{c_\kappa} \frac{T_{e,i}}{4\pi n_{e,0}e^2}\right)^{\frac{1}{2}}$  is the Debye wavelength in superthermal plasma which is much smaller than found for a Maxwellian plasma and has been shown by [39, 41] and  $a_d$  is the radius of dust grain.

The Lorentzian charging currents are derived by using Vlasov-kinetic model whose fluid version by Rubab and Murtaza [41] and in the limit  $\kappa \rightarrow \infty$ , our results matched with Das et al., [32]. Now, by putting the value of perturbed dust grain charge,  $Q_{d1} = \pm \frac{i}{\omega} \Omega\psi$ , in Eq. (1), the dispersion equation of DKAW becomes,

$$\omega^2 = k_z^2 V_{DA}^2 \left[ 1 + \frac{\epsilon_\perp \rho_d^2}{c^2 c_\kappa} \pm i\pi n_{d0} \frac{a_d^2}{k_\parallel} \right], \quad (38)$$

which clearly shows that charge fluctuation effects are insensitive to the form of the distribution function.

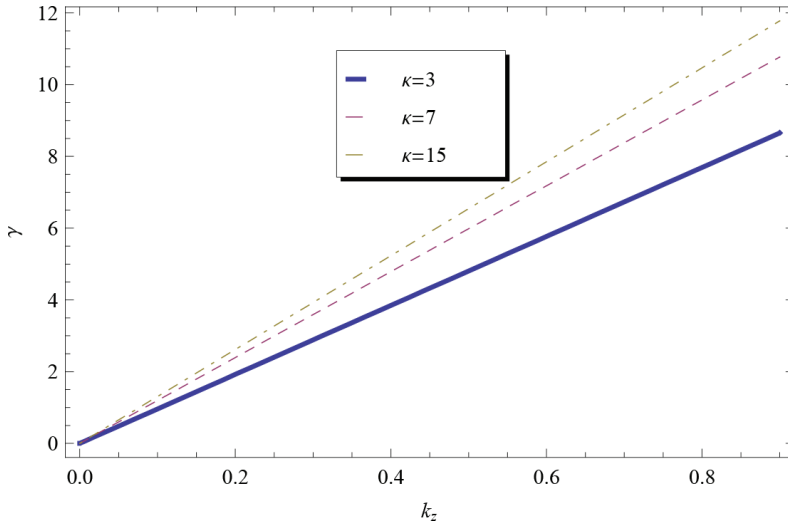
### 3.5.2. Modified dust-acoustic wave

In the limit  $\omega^2/k_z^2 V_A^2 \rightarrow 0$ , the Eq. (16) after simplification turns out to be

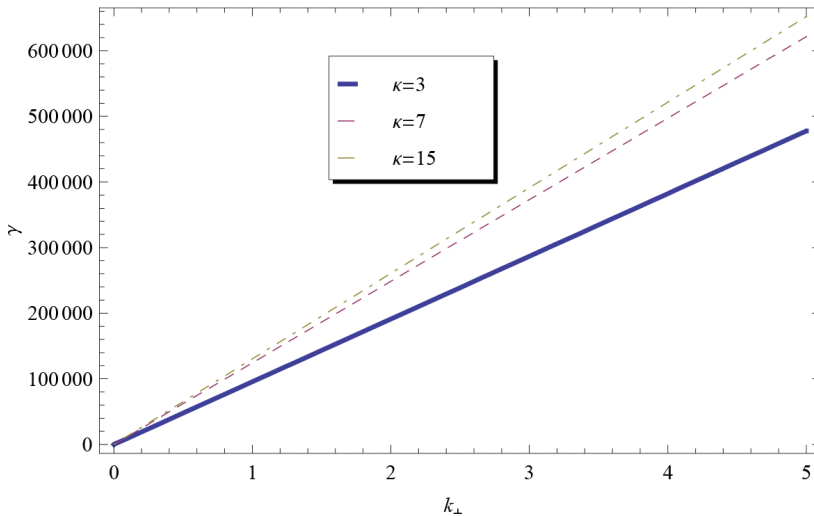
$$\omega^2 = \frac{c_\kappa^{-1} k_z^2 C_D^2}{\left( c_\kappa^{-1} + k_\perp^2 \rho_d^2 + \pm i\pi n_{d0} \frac{r_d^2}{k_z} \right)}, \quad (39)$$

where  $\rho_d^2 = C_D^2/\omega_{cd}^2$ ,  $C_D = (T_{eff}/m_d)^{\frac{1}{2}}$  and  $T_{eff} = n_{d0} Z_d^2 (T_e n_{i0} + T_i n_{e0})/n_{i0} n_{e0}$ . Eq. (39) is the dispersion relation of dust-acoustic wave in a magnetized plasma whose Maxwellian version without dust charge fluctuation effects is given by Mahmood and Saleem [42]. It could be seen that the component of dust velocity in the direction of magnetic field ( $V_{dz}$ ), which finally turns out to be dust gyroradius, is responsible for the coupling of Lorentzian type DKAW with DAW. When the dust-acoustic wave frequency is very large compared to the dust gyroradius, then the dust is considered to be unmagnetized. In an unmagnetized plasma ( $B_0 = 0$ ) with  $T_d = 0$ , we get the dispersion relation of Lorentzian dust-acoustic wave (without dust charge

fluctuation effects) which is exactly equal as discussed by [40]. The effect of Lorentzian index when growth rates are plotted as function of parallel and perpendicular wave number are depicted through graphical representation in **Figure 5** and **Figure 6** and shows that Maxwellian distribution functions are supportive to enhance the wave frequency.



**Figure 5.** Growth rates ( $\bar{\gamma} = \gamma/\Omega_{cd}$ ) as a function of  $k_z$  for different values of  $\kappa = 3, 5, 7$ .



**Figure 6.** Growth rates ( $\bar{\gamma} = \gamma/\Omega_{cd}$ ) as a function of  $k_{\perp}$  for different values of  $\kappa = 3, 5, 7$ .

### 3.5.3. DKAW: Perpendicular streaming

We consider an electromagnetic dust kinetic Alfvén wave streaming instability in a collisionless electron-ion dusty magnetoplasma. The motion of DKAW is followed by considering thermal and magnetized Lorentzian electrons to be Maxwellian and Lorentzian ions drifting across the external magnetic field ( $B_0 \parallel \hat{z}$ ) with a constant drift velocity  $V_0 \hat{x}$ , i.e., ( $V_0 \perp B_0$ ). The dust is considered to be cold and magnetized ( $\omega \ll \omega_{cd}$ ) and the charge on the dust grain surface is taken to be constant. The wave vector associated with the electromagnetic wave lies within  $xz$  plane

$$\mathbf{B}_0 = (0, 0, B_0), \quad \mathbf{V}_0 = (V_0, 0, 0), \quad \mathbf{k} = (k \sin \theta, 0, k \cos \theta)$$

where

$$k_z = k \cos \theta, \quad k_\perp = k \sin \theta$$

The distribution function of Lorentzian ions where ions are streaming perpendicular to the field direction is given as,

$$f_{i0}^\kappa = A_\kappa \left[ 1 + \frac{1}{\kappa v_{th\kappa}^2} \left( v_z^2 + (v_\perp - V_0)^2 \right) \right]^{-\kappa-1}; \quad \kappa > 3/2, \quad (40)$$

$$n_{i1} = -\frac{2e\psi n_{i0}}{m_i v_{th\kappa}^2} [\kappa' + \eta' Z(\eta')]. \quad (41)$$

where  $\eta' = (\omega - k_\perp V_0) k_\perp v_{th\kappa}$ . As there are no ions along the field direction due to perpendicular streaming, therefore we may neglect the ions current density  $J_{iz} = 0$ . In the limit  $\kappa \rightarrow \infty$ , our results reduce to Maxwellian distribution.

The dispersion relation with the aid of Eq. (15) is obtained by using Eqs. (23), (24), (33) and (41) in Eqs. (1) and (2), i.e.,

$$1 + \frac{2\omega_{pi}^2}{k_z^2 v_{th\kappa}^2} \left[ (\kappa' + \eta' Z_\kappa(\eta')) \chi \right] + \frac{2\omega_{pe}^2}{k_z^2 v_{te\kappa}^2} \left[ (\kappa' + \xi_{e0} Z_\kappa(\xi_{e0})) + \frac{\omega v_{te\kappa}}{c^2 k_\perp^3} \xi(\kappa' + \xi_{e0} Z_\kappa(\xi_{e0})) \right] + \frac{k_\perp^2}{k_z^2} \chi (1 + F_D) - \frac{\omega_{pd}^2}{\omega^2} = 0, \quad (42)$$

which is the general dispersion relation of kinetic Alfvén waves in the presence of perpendicular streaming ions and cold and magnetized dust. In the above equation,  $F_D = \omega_{pd}^2 / \omega_{cd}^2$  is responsible for the magnetized dust part.

For parallel propagation and in the limit  $\omega_{pd}^2 / c^2 k_\perp^2 \ll 1$ ,  $F_D \ll 1$ , we get dispersion relation of two stream instability (TSI) in an unmagnetized dusty plasma. In a dust free plasma ( $\omega_{pd}^2 = 0$ ), we get the classical well know relation of TSI, while in the absence of streaming ions, i.e.,  $V_0 = 0$ , we obtain the dispersion relation of dust kinetic Alfvén waves



$$\omega^2 = \Omega_{(dlh)d}^2 + k_z^2 V_{AD}^2 \left[ 1 + \frac{\epsilon_{\perp} \rho_d^2}{c^2 c_{\kappa}} \Lambda_d \right]. \quad (43)$$

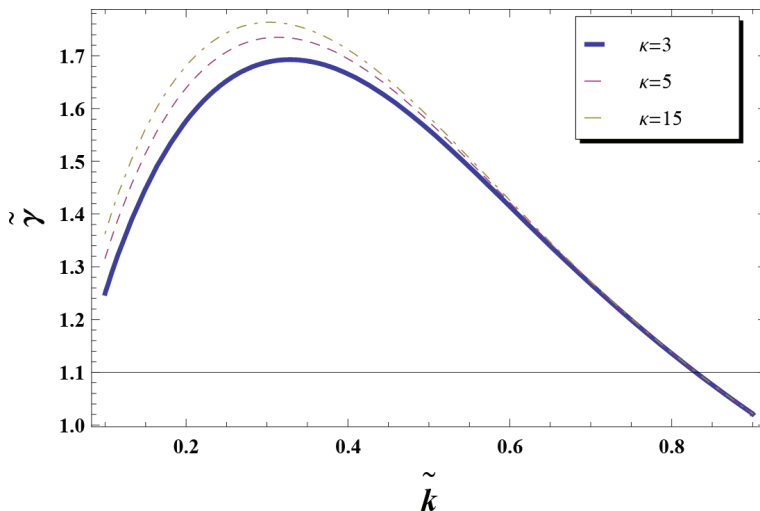
where  $\Omega_{(dlh)d}^2 = \omega_{pi}/F_D$ ,  $\Lambda_d = n_{d0} z_{d0}/n_{e0}$  and  $\rho_d = C_D/\omega_{cd}$ . In the limit  $k_{\perp}^2 \rho_d^2 \ll 1$ , we obtain modified shear Alfvén wave associated with the hybrid dynamics of the ions and magnetized dust through  $\Omega_{(dlh)d}^2$  which provides a cut-off for the EM wave propagation, i.e.,

$$\omega^2 = \Omega_{(dlh)d}^2 (1 + \lambda_i^2 k_z^2) \quad (44)$$

where  $\lambda_i = c/\omega_{pi}$ .

The dispersion relation for the DKAW instability is found to be dependent on the spectral index  $\kappa$  which means Lorentzian plasma is able to support a number of unstable branches. Lorentzian index is found to be more effective in large wave length limit as compared to small wavelength where the tail of unstable region remains independent of  $\kappa$ . When a large number of dust grains are introduced, it will enhance the loss rate of electrons by attachment on a dust grain surface which reduces the wave activity. At the same time the electron loss rate increases the drift velocity which in turns helps to excite the DKAWs and a further increase will help to stabilize the system. Due to particular choice of equations which involves parallel current density, the ions electromagnetic response cant not take part which limits the existence of ions Weibel instability.

By using the same parameters as above, we have plotted the growth rates as the function of propagation vector for different values of kappa. We have seen that the cross-field streaming of superthermal ions inhibit the growth rate of instability as shown in **Figure 7**. Similarly,  $\beta_d$  is found to support the unstable structure and the instability increases with the value of  $\beta_d$  as shown in **Figure 8**.



**Figure 7.** Growth rates ( $\tilde{\gamma}$ ) for perpendicular streaming as a function of wave vector  $\mathbf{k}$  for  $\kappa = 3, 5, 15$ .

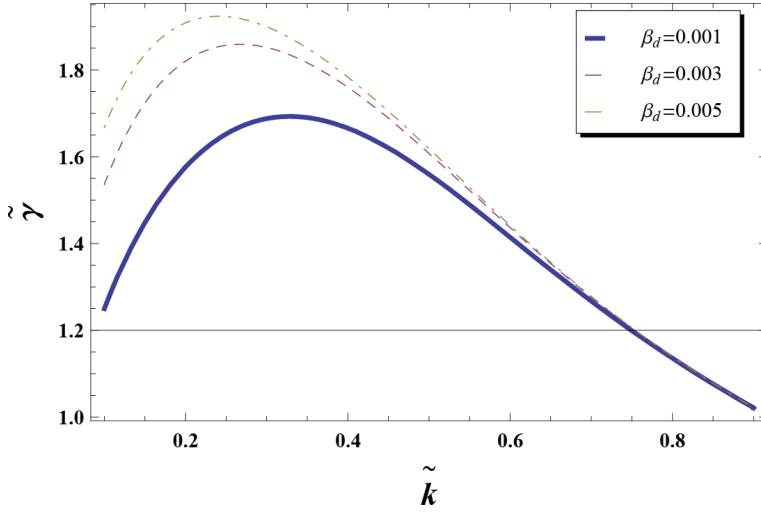


Figure 8. Growth rates ( $\tilde{\gamma}$ ) for perpendicular streaming as a function of wave vector  $k$  for  $\beta_d = 0.001, 0.003,$  and  $0.005$ .

#### 4. Weibel instability in a Lorentzian plasma

The Weibel plasma instability has so many applications in astrophysical [43], and in laboratory plasmas as well [44]. The generation of magnetic field can be explained in the domain of gamma ray burst, galactic cosmic rays and supernovae [45, 48]. For the case of unmagnetized plasma, the Weibel instability [20] has been widely discussed in relativistic and nonrelativistic regimes. In 1989, Yoon [46, 47] generalized his work by using relativistic bi-Maxwellian plasma. Later, Schaerfer [48] have discussed this instability in relativistic regimes of plasma with arbitrary distributions and presented comparison with his previous works which was based on bi-Gaussian distribution functions. The Weibel instability was investigated by Califano [49, 50] with temperature anisotropy, produced by two counterstreaming electron populations. Davidson probed the multi species Weibel instability for the charged beam and intense ions in plasma [51].

In our work, we have derived the analytical expressions and compared the results numerically for the real and imaginary parts of the dielectric constant with the Maxwellian and kappa ( $\kappa$ ) distributions under two conditions i.e.,  $\alpha = \frac{\omega_0}{kz} \gg 1$  and  $\ll 1$ .

By using kinetic model, the linear dispersion relation for Weibel instability in unmagnetized plasma has been derived after solving the linearized, nonrelativistic Vlasov equation as below [52],

$$\omega^2 - c^2 k^2 - \omega_{pe}^2 + \pi \omega_{pe}^2 \left( \frac{k}{m} \right) \int_{-\infty}^{\infty} \frac{m^3 dv_z}{(\omega - kv_z)} \int_0^{\infty} v_{\perp}^3 dv_{\perp} \times \left( \frac{\partial f_{0\kappa}}{\partial v_z} \right) = 0, \quad (45)$$

where  $f_0$  is the distribution function and here we will discuss the different velocity distributions, i.e., Maxwellian distribution and  $\kappa$ - distribution functions.

To calculate Weibel instability in a Lorentzian plasma, we use Eq. (21), for zero streaming velocity of particle, i.e.,  $V_0 = 0$  and using  $\left(\frac{\partial f_0}{\partial v_z}\right)$  in Eq. (45), and performing perpendicular integration, we are left with parallel integral which is called modified plasma dispersion function for kappa distribution.

$$\omega^2 - c^2 k^2 - \omega_{pe}^2 \left(1 - \frac{T_\perp}{T_z}\right) + \frac{\omega_{pe}^2}{\sqrt{\pi}} \left(\frac{T_\perp}{T_z}\right) \left(\frac{\Gamma(\kappa)}{\kappa^{\frac{1}{2}} \Gamma(\kappa - \frac{1}{2})}\right) (\alpha) \int_{-\infty}^{\infty} \frac{\left(1 + \frac{x^2}{\kappa}\right)^{-\kappa}}{(x - \alpha)} dx = 0, \quad (46)$$

where  $x = \Theta_z^{-1} v_z$ ,  $\alpha = \frac{\omega \Theta_z}{kz}$  and  $\Theta_{z,\perp} = \frac{(2\kappa-3)T_{z,\perp}}{\kappa m}$ .

Applying same procedure as above and again using Plemelj's formula,

$$\int_{-\infty}^{\infty} \frac{\left(1 + \frac{x^2}{\kappa}\right)^{-\kappa}}{(x - \alpha)} dx = P \int_{-\infty}^{\infty} \frac{\left(1 + \frac{x^2}{\kappa}\right)^{-\kappa}}{(x - \alpha)} dx + i\pi \left(1 + \frac{\alpha^2}{\kappa}\right)^{-\kappa}, \quad (47)$$

the integration of principal part yields

$$P \int_{-\infty}^{\infty} \frac{\left(1 + \frac{x^2}{\kappa}\right)^{-\kappa}}{(x - \alpha)} dx = \frac{\sqrt{\pi} \kappa^{1/2} \Gamma(\kappa - \frac{1}{2})}{\Gamma(\kappa)} \left(1 + \frac{\kappa \Gamma(\kappa - \frac{3}{2})}{\Gamma(\kappa - \frac{1}{2})}\right). \quad (48)$$

The dispersion relation will be solved under two following conditions

For  $(\alpha > 1)$ :

$$\omega^4 - \left(c^2 k^2 + \omega_{pe}^2\right) \omega^2 - \omega_{pe}^2 \left(\frac{T_\perp}{T_z}\right) (v_{tz}^2 k_z^2) = 0, \quad (49)$$

which shows the real part of Weibel instability is insensitive to the value of Lorentzian index and the imaginary part  $i\pi \left(1 + \frac{\alpha^2}{\kappa}\right)^{-\kappa} \rightarrow 0$ .

For  $(\alpha < 1)$ :

The dispersion relation takes the form

$$\omega^2 - c^2 k^2 - \omega_{pe}^2 \left(1 - \frac{T_\perp}{T_z}\right) + \frac{\omega_{pe}^2}{\sqrt{\pi}} \left(\frac{T_\perp}{T_z}\right) \left(\frac{\Gamma(\kappa)}{\kappa^{\frac{1}{2}} \Gamma(\kappa - \frac{1}{2})}\right) (\alpha) \int_{-\infty}^{\infty} \frac{\left(1 + \frac{x^2}{\kappa}\right)^{-\kappa}}{(x - \alpha)} dx = 0. \quad (50)$$

Now, we define a new plasma dispersion function, i.e.,

$$Z_\kappa^*(\alpha) = \frac{1}{\sqrt{\pi}} \left(\frac{\Gamma(\kappa)}{\kappa^{\frac{1}{2}} \Gamma(\kappa - \frac{1}{2})}\right) \int_{-\infty}^{\infty} \frac{\left(1 + \frac{x^2}{\kappa}\right)^{-\kappa}}{(x - \alpha)} dx, \quad (51)$$

the corresponding dispersion relation can be expressed as

$$\omega^2 - c^2k^2 - \omega_{pe}^2 \left(1 - \frac{T_{\perp}}{T_z}\right) + \omega_{pe}^2 \left(\frac{T_{\perp}}{T_z}\right) (\alpha) Z_{\kappa}^*(\alpha) = 0. \quad (52)$$

We can Solve  $Z_{\kappa}^*(\alpha)$  by taking  $\kappa$  an integer and assuming  $\alpha < 1$ . So for  $\kappa = 3, 5, 7$  we get the following three  $Z$ - functions respectively.

$$\begin{aligned} Z_3^*(\alpha) &= \alpha(-1.66 - 0.370\alpha^2 - \dots) + i(1.539 - 1.539\alpha^2 + \dots) \\ Z_5^*(\alpha) &= \alpha(-1.8 - 0.48\alpha^2 - \dots) + i(1.635 - 1.635\alpha^2 + \dots) \\ Z_7^*(\alpha) &= \alpha(-1.98 - 0.59\alpha^2 - \dots) + i(1.732 - 1.736\alpha^2 + \dots) \end{aligned} \quad (53)$$

So the three dispersion relations for the above three corresponding  $Z$ -functions are.

For  $\kappa = 3$ , we get

$$\begin{aligned} c^2k^2 + \omega_{pe}^2 - \left(\frac{T_{\perp}}{T_z}\right) \omega_{pe}^2 (1 + 1.66\alpha^2) - i\omega_{pe}^2 \left(\frac{T_{\perp}}{T_z}\right) 1.539\alpha &= 0 \\ \gamma = \text{Im}\omega &= -(0.649) \frac{k_z v_{Tz}}{\omega_{pe}^2} \left(\frac{T_z}{T_{\perp}}\right) \left(c^2k^2 + \omega_{pe}^2 \left(1 - \frac{T_{\perp}}{T_z}\right)\right) \end{aligned} \quad (54)$$

Similarly, for  $\kappa = 5$  and 7 we obtain the followings

$$\gamma = \text{Im}\omega = -(0.7324) \frac{k_z v_{Tz}}{\omega_{pe}^2} \left(\frac{T_z}{T_{\perp}}\right) \left(c^2k^2 + \omega_{pe}^2 \left(1 - \frac{T_{\perp}}{T_z}\right)\right) \quad (55)$$

and

$$\gamma = \text{Im}\omega = -(0.8152) \frac{k_z v_{Tz}}{\omega_{pe}^2} \left(\frac{T_z}{T_{\perp}}\right) \left(c^2k^2 + \omega_{pe}^2 \left(1 - \frac{T_{\perp}}{T_z}\right)\right) \quad (56)$$

Using the Vlasov model, we have derived new dispersion relations based on  $\kappa$ - distribution function in an unmagnetized plasma. The analytical expressions for the dielectric constant have been obtained under two conditions i.e., ( $\alpha \gg 1$ ) and ( $\alpha \ll 1$ ), which finally give real and imaginary parts respectively. The real part is found to be insensitive to the value of Lorentzian index while imaginary part shows strong dependence on  $\kappa$ . A graphical representation has also been added for the comparison of non-Maxwellian distributions with that of the Maxwellian. The imaginary parts of the dispersion relation obtained above have been plotted for different values of  $\kappa$ - showing the variation of the normalized frequencies, i.e.,  $\frac{\text{Im}\omega}{\omega_{pe}}$  against  $\frac{ck}{\omega_{pe}}$ . Figure exhibits the comparison of the result of kappa distribution with that of the Maxwellian. For small  $\kappa$ , the growth rate also reduces but on other hand on increasing the  $\kappa$  value, the growth rate enhances and finally approaches the Maxwellian results which is shown in **Figure 9**.

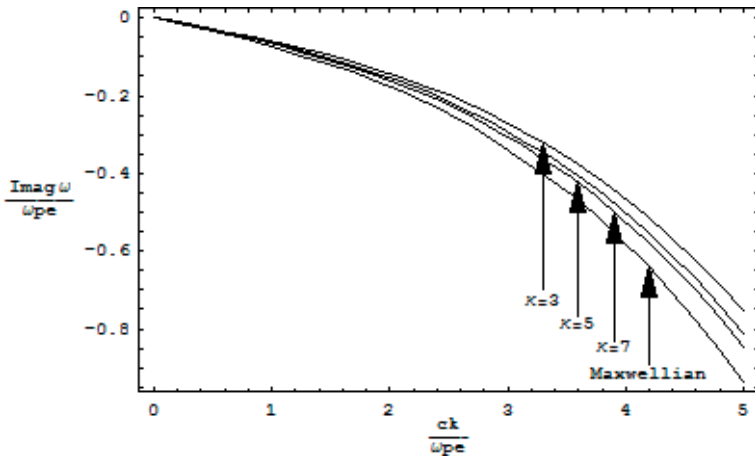


Figure 9. Growth rates of Weibel instability for  $\kappa = 3, 5, 7$  and the comparison of results with Maxwellian.

### 5. Collisional Weibel instability with non-zero magnetic field

The dispersion relation of Weibel instability for transverse waves propagating parallel to magnetic field is obtained as

$$\omega^2 - c^2k^2 - \omega_{pe}^2 \left(1 - \frac{\Theta_{\perp}^2}{\Theta_z^2}\right) + \left(\frac{\omega_{pe}^2}{k\Theta_z}\right) \left(\omega - \left(1 - \frac{\Theta_{\perp}^2}{\Theta_z^2}\right)(\omega \pm \Omega)\right) Z_{\kappa}^*(\alpha) = 0 \tag{57}$$

where  $\Theta_{z,\perp} = \left(\frac{2\kappa-3}{\kappa}\right) \frac{T_{z,\perp}}{m}$  taking the limit  $\alpha > 1$ , we obtain

$$\omega^2 - c^2k^2 - \omega_{pe}^2 \frac{\omega}{(\omega \pm \Omega)} + \omega_{pe}^2 \left(1 - \frac{T_{\perp}^2}{T_z^2}\right) \left(\frac{k^2 v_{Tz}^2}{(\omega \pm \Omega)^2}\right) = 0 \tag{58}$$

We notice that the final expression becomes independent of the spectral index  $\kappa$ .

However, for  $\alpha$  small, the dispersion function  $Z_{\kappa}^*(\alpha)$  is obtained by choosing specific values of  $\kappa$ .

For  $\kappa = 5, 7$  we get

$$\begin{aligned} Z_5^*(\alpha) &= \alpha(-1.86 - 0.560\alpha^2 - \dots) + i(1.73 - 1.79\alpha^2 + \dots) \\ Z_7^*(\alpha) &= \alpha(-2.05 - 0.734\alpha^2 - \dots) + i(1.9 - 1.92\alpha^2 + \dots) \end{aligned} \tag{59}$$

The imaginary  $\omega$  therefore becomes

$$\omega = -i(0.79) \left(\frac{kv_{Tz}}{\omega_{pe}^2}\right) \left(\frac{T_z}{T_{\perp}}\right) \left[c^2k^2 - \omega_{pe}^2 \left(1 - \frac{T_{\perp}}{T_z}\right)\right] \mp \left(1 - \frac{T_z}{T_{\perp}}\right) \Omega \tag{60}$$

and

$$\omega = -i(0.88) \left( \frac{kv_{Tz}}{\omega_{pe}^2} \right) \left( \frac{T_z}{T_\perp} \right) \left[ c^2 k^2 - \omega_{pe}^2 \left( 1 - \frac{T_\perp}{T_z} \right) \right] \mp \left( 1 - \frac{T_z}{T_\perp} \right) \Omega \quad (61)$$

for  $\kappa = 5$  and  $7$  respectively.

Considering

$$\omega = \omega_r + i(\omega_i + \nu_e)$$

$$\omega = -i(0.79) \left( \frac{kv_{Tz}}{\omega_{pe}^2} \right) \left( \frac{T_z}{T_\perp} \right) \left[ c^2 k^2 - \omega_{pe}^2 \left( 1 - \frac{T_\perp}{T_z} \right) \right] \mp \left( 1 - \frac{T_z}{T_\perp} \right) \Omega - \nu_e \quad (62)$$

and

$$\omega = -i(0.79) \left( \frac{kv_{Tz}}{\omega_{pe}^2} \right) \left( \frac{T_z}{T_\perp} \right) \left[ c^2 k^2 - \omega_{pe}^2 \left( 1 - \frac{T_\perp}{T_z} \right) \right] \mp \left( 1 - \frac{T_z}{T_\perp} \right) \Omega - \nu_e \quad (63)$$

where

$$\nu_e = -\frac{1}{p} \frac{dp}{dt} = -\frac{1}{p} \int_0^\infty f_\kappa v_\perp m_e v_z d\mathbf{v} \quad (64)$$

It is obvious from the above relation that collision frequency for particles obeying kappa distribution differs from that of Maxwellian distribution and is dependent on the value of specie of choice  $j = e, i$ . It is seen that collision frequency increases with  $j = e, i$  and is less for kappa distributed particles than that of the Maxwellian particles. It is therefore justified to use appropriate collision frequency for such Kappa distributed particles.

## 6. Conclusion

In this chapter, we have described the electromagnetic waves and instabilities in a generalized Lorentzian plasma including particle streaming and finite and anisotropic thermal spread. It allows to grasp the practical understanding of a complex collisionless system in terms of spectra, bulk relative motion and instabilities. In particular, we have focused on kinetic Alfvén waves and instabilities in a dusty and Lorentzian plasma and several types of modes have been identified under various conditions. We have reviewed the kinetic waves and Weibel instabilities in a non-Maxwellian space and astrophysical plasmas by incorporating some basic concepts of dusty environments. We have found that dispersion characteristics involving kinetic Alfvén waves become significantly modified by superthermality effects and dust plasma parameters. The coupling of magnetized dust to the waves due to cyclotron resonance

is shown to play a vital role on the wave dynamics. Moreover, the dust grain charging yield some additional plasma currents, which depends on the streaming velocity, Lorentzian index and plasma beta. The Lorentzian index is found to either enhance or quench the electromagnetic instabilities. The dust component is found to play an essential role in wave dynamics, i.e., introducing dust lower hybrid frequency when mobile dust particles are included in the plasma. We have seen that the temperature anisotropy in the distribution function has no effect on the wave characteristics, i.e., the employed model inhibits the temperature anisotropy, but supports the velocity anisotropy. Moreover, a brief analysis on Weibel instabilities in a non-Maxwellian plasma is also presented.

Kinetic Alfvén turbulence are always present in the streaming solar wind near 1 AU and in situ measurements have confirmed the presence of non-Maxwellian proton distribution function. The present investigations show that the Lorentzian charged particle distributions in space lead to a essentially new physical situation as compared to the plasma with equilibrium distribution functions. Our results of the present analysis opens a new window of investigation to study various streaming and anisotropic modes in different plasma scenarios when Lorentzian distribution function is employed.

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