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Formation of Nonlinear Solitary Vortical Structures by Coupled Electrostatic Drift and Ion-Acoustic Waves

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Propagation of coupled electrostatic drift and ion-acoustic waves (DIAWs) is presented. It is shown that nonlinear solitary vortical structures can be formed by low-frequency coupled electrostatic DIAWs. Primary waves of distinct (small, intermediate and large) scales are considered. Appropriate set of 3D equations consisting of the generalized Hasegawa–Mima equation for the electrostatic potential (involving both vector and scalar nonlinearities) and the equation of motion of ions parallel to magnetic field are obtained. According to experiments of laboratory plasma mainly focused to large scale DIAWs, the possibility of self-organization of DIAWs into the nonlinear solitary vortical structures is shown analytically. Peculiarities of scalar nonlinearities in the formation of solitary vortical structures are widely discussed.

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Drift waves play a crucial role in the magnetic trapping of plasmas. The drift waves were anticipated by Rudakov and Sagdeev,[1] and later Mikhailovskii[2–4] greatly contributed to this research. Horton[5] elucidated a detailed survey of drift waves, turbulence and associated anomalous transport phenomena. Evolution of this nonlinear theory brings out the concepts of solitons, solitary waves, solitary vortices, jets, filaments, convective cells, double layers, shocks, and zonal flow, which are discussed in an intensive manner for the last years.

In the present work, we discuss nonlinear solitary vortical structures on the low-frequency coupled electrostatic drift and ion-acoustic waves (DIAWs). Drift wave propagation oscillation velocity becomes greater than the phase velocity even at a low energy density, thus the wave front may become curved, incorporating so-called trapped particles, and traveling vortices may form. Description of such an effect requires examination of nonlinear equations involving the dispersion and non-homogeneity of plasmas. Such self-organized structural formations have great importance to understand the macroscopic behavior of plasmas in laboratory and space. Nonlinear dynamics of drift waves in plasmas is primarily identified by the classical Hasegawa–Mima (HM) equation[6,7] giving different solutions which involve turbulent, coherent and wave behaviors. It should be noted that the nonlinear term in the standard HM equation has the structure of type $J(a,b) = [\nabla a, \nabla b]_z$, where a and b are certain functions of wave field. Such nonlinearity is known as vector nonlinearity and it gives the existence of dipolar vortices. In the nonlinear theory of drift waves, Petviashvili[8] indicated the importance of other nonlinearity, called scalar, Korteweg de Vries (KdV)-type nonlinearity $\propto \varphi^2$. This nonlinearity is responsible for the existence of monopolar vortices. When Petviashvili[9] was investigating the problem of Jovian great red spot, he simultaneous firstly took into account both vector and scalar nonlinearities. The detailed analysis of both (monopolar and dipolar) types of drift vortical structures was provided by Mikhailovskii.[10] Later, new localizing role of both (scalar and vector) nonlinearities in the process of formation of nonlinear solitary structures was provided by Nezlin and Chernikov,[11] and it was emphasized that depending on the wavelengths scale drift waves, turbulence should be described by the more complex so-called generalized HM equation.

As in the experiments of tokamak plasmas, large-scale drift waves ($k_\perp \rho_{\rm s} \leqslant 1$, where $\rho_{\rm s}$ is the ion Larmor radius defined at the electron temperature) are mainly observed, here we will keep our attention to the large-scale solitary nonlinear structures and derive the generalized HM equation for the coupled drift-ion-acoustic waves. A system of basic equations, consisting of the general HM equation for electrostatic potential and an equation describing parallel to magnetic field ions motion valid for arbitrary wavelengths of primary waves, is inferred. Linear regime is discussed in detail. Basic nonlinear equations are separately considered in accordance with the wavelengths (small-, intermediate- and large-scales) of primary waves. The possibility of formation of such spatially localized nonlinear vortical structures by coupled electrostatic DIAWs is considered. It is noted that such investigation was started by Meiss and Horton[12] in the case of small-scale structures. Finally, we discuss the obtained results.

We take low-frequency electrostatic waves with the frequency much smaller than the ion cyclotron frequency (i.e., $\omega \ll \omega_{\rm ci}$) in an inhomogeneous (with the density $n_0(x)$ and temperature $T_{\rm e}(x)$) and a magnetized (with the magnetic field) plasma. The linear waves will be presented in such a plasma in form of DIAWs if the phase velocity ω/k_z in the direction of the magnetic field is between the electron and ion thermal velocities, i.e., $v_{\rm Te}$ and $v_{\rm Ti}$.

The complete descriptions of low-frequency electrostatic DIAWs in plasmas are carried out by the equation of motion and the continuity for the ions, and the Boltzmann distribution of electrons,

$$\frac{\partial v}{\partial t} + (v \cdot \nabla)v = -\frac{e}{m}\nabla\varphi + \omega_{\rm ci} v \times e_z, \tag{1}$$

$$\frac{\partial n}{\partial t} + \nabla \cdot (nv) = 0, \tag{2}$$

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$$n = n_0(x) \exp\Big(\frac{e\varphi}{T_{\rm e}(x)}\Big), \quad (3)$$

where n, v, e and m represent the ion density, velocity, charge and mass, respectively, $\varphi(t,x,y,z)$ is the electrostatic potential, and $\omega_{\rm ci} = eB/m$ is the ion cyclotron frequency. The plasma is assumed to be quasineutral, for which $n_{\rm e} = n$. The magnetic field $B = Be_z$ is assumed to be constant, homogeneous, and $T_{\rm e} \gg T_{\rm i}$, which means that the ion pressure in the equation of motion can be ignored. We assume the equilibrium density $n_0(x)$ and electron temperature $T_{\rm e}(x)$ to be inhomogeneous in the x direction.

Electrons attain the thermal equilibrium along the magnetic field lines by Eq. (3), thus we must need that the electrostatic perturbation's phase velocity $\omega/k_z \ll v_{\rm Te}$ along the magnetic field. Also, this must be smaller than the Alfven velocity $c_{\rm A} = B/(\mu_0 n_0 m)^{1/2}$, which is why we can ignore magnetic field perturbations due to the parallel current. Therefore, k_z must be finite.

We take the drift wave's coupling with ion-acoustic ones and assume z-dependence of the fields is weak. Taking the curl of Eq. (1) and using Eq. (2) we obtain the following 'freezing-in field equation'[13]

$$\Big(\frac{\partial}{\partial t} + v\cdot\nabla\Big)\Big(\frac{e_z\omega_{\rm ci} + \varOmega}{n}\Big) = \Big(\frac{e_z\omega_{\rm ci} + \varOmega}{n}\cdot\nabla\Big)v, \quad (4)$$

where $\varOmega = \nabla\times v$ is the vorticity. This equation is valid for the 3D perturbations, and the new term describes vortex stretching on the right-hand side.

Further we will apply the small expansion parameter ε,

$$\varepsilon \sim \frac{1}{\omega_{\rm ci}}\frac{\partial}{\partial t} \sim \frac{1}{k_z v_{\rm Te}}\frac{\partial}{\partial t} \sim \frac{\varOmega}{\omega_{\rm ci}} \sim \frac{e\varphi}{T_{\rm e}} \sim \frac{L}{L_{n,T}} \ll 1, \quad (5)$$

where L is the typical length scale of fluctuation, $L_{n,T}$ is the equilibrium density's inhomogeneity scale and temperature, respectively. The ion Larmor radius, which is the characteristic wave dispersion scale length, is $\rho_{\rm s} = (T_{\rm e}/m\omega_{\rm ci}^2)^{1/2}$ defined at the electron temperature $T_{\rm e}$.

We represent the particle's total velocity as $v = v_\perp + e_z w$ to express Eq. (4) in terms of potential $\varphi(t,x,y,z)$. For low-frequency waves $\omega \ll \omega_{\rm ci}$, Eq. (1) implies[10]

$$v_\perp = v_{\rm E} + v_{\rm I}, \quad (6)$$

where $v_{\rm E}$ is the electric drift velocity (or cross field drift velocity) defined as

$$v_{\rm E} = \frac{1}{B}e_z \times \nabla_\perp\varphi = \frac{1}{B}E\times e_z, \quad (7)$$

and $v_{\rm I}$ is the inertial part of the transverse velocity

$$v_{\rm I} = \frac{1}{\omega_{\rm ci}}e_z \times \frac{d_0}{dt}v_{\rm E}, \quad (8)$$

where $d_0/dt = \partial/\partial t + v_{\rm E}\cdot\nabla + w\partial/\partial z$.

Taking into account the conditions (5) and using Eqs. (6)–(8) in the z-component of Eq. (4) we obtain

$$\begin{aligned} &\frac{\partial\varphi}{\partial t} - \rho_{\rm s}^2\frac{\partial}{\partial t}\Delta_\perp\varphi - \rho_{\rm s}^2\omega_{\rm ci}\frac{1}{n_0}\frac{dn_0}{dx}\frac{\partial\varphi}{\partial y}\\ &+ \rho_{\rm s}^2\omega_{\rm ci}\frac{1}{T_{\rm e}}\frac{dT_{\rm e}}{dx}\varphi\frac{\partial\varphi}{\partial y} - \rho_{\rm s}^4\omega_{\rm ci}J(\varphi,\Delta_\perp\varphi)\\ &+ w\frac{\partial\varphi}{\partial z} + \frac{\partial w}{\partial z} - \rho_{\rm s}^2 w\frac{\partial}{\partial z}\Delta_\perp\varphi\\ &+ \rho_{\rm s}^2\Delta_\perp\varphi\frac{\partial w}{\partial z} - \rho_{\rm s}^2\frac{\partial w}{\partial x}\frac{\partial^2\varphi}{\partial x\partial z}\\ &- \rho_{\rm s}^2\frac{\partial w}{\partial y}\frac{\partial^2\varphi}{\partial y\partial z} = 0. \end{aligned} \quad (9)$$

From the z-component of the equation of motion (1), we obtain the other equation which describes the parallel to the magnetic field ions motion,

$$\frac{\partial w}{\partial t} + \rho_{\rm s}^2\omega_{\rm ci}J(\varphi,w) + w\frac{\partial w}{\partial z} = -v_{\rm s}^2\frac{\partial\varphi}{\partial z}, \quad (10)$$

where $v_{\rm s} = (T_{\rm e}/m)^{1/2}$ is the ion-acoustic speed, $J(a,b) = \partial_x a\partial_y b - \partial_y a\partial_x b$ is the Jacobian, and $\Delta_\perp = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the 2D Laplacian. Equations (9) and (10) describe the initial closed system of equations which are valid for arbitrary $k_\perp\rho_{\rm s}$. In these equations potential φ is normalized by $T_{\rm e}/e$. As to the ion parallel motion, Eq. (10) contains both vector and scalar nonlinearities. Note that in Eq. (9) we keep the fourth small linear term to show the tendency of linear waves to instability.

The generalized Eq. (9) contains an additional three new scalar nonlinearities of KdV type: $\rho_{\rm s}^2\omega_{\rm ci}\frac{1}{T_{\rm e}}\frac{dT_{\rm e}}{dx}\varphi\frac{\partial\varphi}{\partial y}$ compared with the classical HM equation with respect to the drift waves. The standard HM equation contains only the vector nonlinearity $\rho_{\rm s}^4\omega_{\rm ci}J(\varphi,\Delta_\perp\varphi)$, which is valid only for the small-scale structures when the characteristic size $L \leqslant \rho_{\rm s}$ and it predicts the existence of only dipolar vortices (cyclone–anticyclone pairs). Generalizing the HM equation, Eq. (9) containing scalar nonlinearities could describe solitary monopole-type vortices (i.e., either cyclones or anticyclones). Monopolar solitary structures were first observed in laboratory modeling of solitary Rossby vortices.[14] In the numerical work of Kaladze *et al.*,[15] it was shown that the presence of the scalar nonlinearity plays the role of instability forming monopole vortical structures of definite polarity as a result of breaking large-scale dipole ones. The new type mechanism of the formation of solitary drift vortical structures due to the mutual action of scalar and vector nonlinearities was elucidated by Nezlin and Chernikov.[11] Dynamics of large-scale drift vortical structures in electron-positron-ion plasmas was discussed in Kaladze *et al.*[16] It was shown by Kaladze *et al.*[17] that in the Earth's Hall conductive ionospheric E-layer, due to the latitudinal inhomogeneity of both the Coriolis parameter and the geomagnetic field, large-scale ULF coupled Rossby–Khantadze electromagnetic (EM) waves can be self-organized into localized (solitary) dipole nonlinear structures propagating parallel against the background of mean flow.

Recently, the generation of zonal flow by coupled electrostatics drift and ion-acoustic waves has been discussed by Kaladze *et al.*[18]

From Eqs. (9) and (10) for the linear regime, we obtain the following system of equations

$$\frac{\partial\varphi}{\partial t} - \rho_{\rm s}^2\frac{\partial}{\partial t}\Delta_\perp\varphi - \rho_{\rm s}^2\omega_{\rm ci}\frac{1}{n_0}\frac{dn_0}{dx}\frac{\partial\varphi}{\partial y} + \frac{\partial w}{\partial z} = 0,$$
$$\frac{\partial w}{\partial t} = -v_{\rm s}^2\frac{\partial\varphi}{\partial z}. \tag{11}$$

Taking the derivative of the first equation over t and using the second equation we obtain the following coupled drift-ion acoustic waves equation

$$\frac{\partial^2\varphi}{\partial t^2} - \rho_{\rm s}^2\frac{\partial^2}{\partial t^2}\Delta_\perp\varphi - \rho_{\rm s}^2\omega_{\rm ci}\frac{1}{n_0}\frac{dn_0}{dx}\frac{\partial^2\varphi}{\partial t\partial y} - v_{\rm s}^2\frac{\partial^2\varphi}{\partial z^2} = 0. \tag{12}$$

In the (k,ω) space we obtain the following appropriate algebraic equation

$$\omega^2(1+k_\perp^2\rho_{\rm s}^2) - \omega k_y\rho_{\rm s}^2\beta_n\omega_{\rm ci} - k_z^2 v_{\rm s}^2 = 0, \tag{13}$$

where $\beta_n = -\frac{1}{n_0}\frac{dn_0}{dx} > 0$. The roots of this equation are given as follows:

$$\omega_{1,2} = \frac{k_y\rho_{\rm s}^2\omega_{\rm ci}\beta_n \pm \sqrt{k_y^2\rho_{\rm s}^4\omega_{\rm ci}^2\beta_n^2 + 4k_z^2v_{\rm s}^2(1+k_\perp^2\rho_{\rm s}^2)}}{2(1+k_\perp^2\rho_{\rm s}^2)}, \tag{14}$$

where ω_1 corresponds to the fast, and ω_2 to the slow coupled DIAWs.

From Eq. (14) for the generated wave frequencies we obtain the following expressions for linear phase velocities,

$$\Big(\frac{\omega}{k_y}\Big)_{1,2} = \frac{v^*}{2(1+k_\perp^2\rho_{\rm s}^2)}\Big\{1 + \Big[1 + 4\frac{k_z^2}{k_y^2}\frac{1}{\rho_{\rm s}^2\beta_n^2}(1 + k_\perp^2\rho_{\rm s}^2)\Big]^{1/2}\Big\}, \tag{15}$$

where $v^* = \beta\rho_{\rm s}^2\omega_{\rm ci}$ is diamagnetic drift velocity obtained at the electron temperature.

In the case of $k_z = 0$, we have the pure drift waves

$$\omega_1 = \frac{k_y v^*}{1+k_\perp^2\rho_{\rm s}^2}. \tag{16}$$

In the case of $k_y = 0$, we have the pure ion-acoustic waves

$$\omega_{1,2} = \pm\frac{k_z v_{\rm s}}{\sqrt{1+k_\perp^2\rho_{\rm s}^2}}. \tag{17}$$

It is seen that the ion-acoustic waves become dispersive due to the coupling with drift waves.

For the sufficiently small longitudinal wave numbers $k_z \ll k_y$, we will obtain the following mixed frequencies

$$\omega_1 = \frac{k_y v^*}{1+k_\perp^2\rho_{\rm s}^2}\Big(1+\frac{k_z^2v_{\rm s}^2(1+k_\perp^2\rho_{\rm s}^2)}{k_y^2v^{*2}}\Big),\ \ \omega_2 = -\frac{k_z^2v_{\rm s}^2}{k_yv^*}, \tag{18}$$

where ω_1 and ω_2 correspond to upper and bottom signs, respectively, in Eq. (17).

For sufficiently small $k_y \ll k_z$, we obtain

$$\omega_1 = \frac{k_z v_{\rm s}}{\sqrt{1+k_\perp^2\rho_{\rm s}^2}}\Big(1 + \frac{k_y v^*}{2k_z v_{\rm s}\sqrt{1+k_\perp^2\rho_{\rm s}^2}}\Big),$$
$$\omega_2 = -\frac{k_z v_{\rm s}}{\sqrt{1+k_\perp^2\rho_{\rm s}^2}}\Big(1 - \frac{k_y v^*}{2k_z v_{\rm s}\sqrt{1+k_\perp^2\rho_{\rm s}^2}}\Big). \tag{19}$$

For nonlinear regime, here we will take Eqs. (9) and (10) for different wavelength's scales and obtain appropriate nonlinear equations.

For intermediate wavelengths $k_\perp\rho_{\rm s} \geqslant 1$, using estimations

$$\omega \sim k_y\Big|\frac{1}{n_0}\frac{dn_0}{dx}\Big|\rho_{\rm s}^2\omega_{\rm ci} \sim k_\perp^2\rho_{\rm s}^2\omega_{\rm ci}\frac{L}{L_n} \sim \omega_{\rm ci}\frac{L}{L_n} \sim k_z v_{\rm s}, \tag{20}$$

we obtain $k_z \sim 1/L_n$. Further comparing first terms of both sides in Eq. (10) we obtain $w \sim k_z\varphi v_{\rm s}^2/\omega$. Putting here $\omega \approx k_z v_{\rm s}$, we obtain estimation

$$\omega\varphi \sim k_z w \sim \frac{w}{L_n}. \tag{21}$$

Under conditions (20) and (21) we obtain the following system of simplified initial equations of Eqs. (9) and (10) as follows:[14]

$$\frac{\partial\varphi}{\partial t} - \rho_{\rm s}^2\frac{\partial}{\partial t}\Delta_\perp\varphi + \rho_{\rm s}^2\omega_{\rm ci}\beta_n\frac{\partial\varphi}{\partial y} - \rho_{\rm s}^4\omega_{\rm ci}J(\varphi,\Delta_\perp\varphi) + \frac{\partial w}{\partial z} = 0,$$
$$\frac{\partial w}{\partial t} + \rho_{\rm s}^2\omega_{\rm ci}J(\varphi,w) = -v_{\rm s}^2\frac{\partial\varphi}{\partial z}. \tag{22}$$

For large-scale wavelengths $k_\perp\rho_{\rm s} \ll 1$, as it is seen from Eq. (20),

$$\omega\varphi \sim k_z w \sim \omega_{\rm ci}k_\perp^2\rho_{\rm s}^2\frac{L}{L_n}\varphi. \tag{23}$$

Under this condition, Eqs. (9) and (10) can be reduced to

$$\frac{\partial\varphi}{\partial t} - \rho_{\rm s}^2\frac{\partial}{\partial t}\Delta_\perp\varphi + \rho_{\rm s}^2\omega_{\rm ci}\beta_n\frac{\partial\varphi}{\partial y} - \rho_{\rm s}^2\omega_{\rm ci}\beta_{\rm T}\varphi\frac{\partial\varphi}{\partial y} - \rho_{\rm s}^4\omega_{\rm ci}J(\varphi,\Delta_\perp\varphi) + w\frac{\partial\varphi}{\partial z} + \frac{\partial w}{\partial z} = 0,$$
$$\frac{\partial w}{\partial t} + \rho_{\rm s}^2\omega_{\rm ci}J(\varphi,w) + v_{\rm s}^2\frac{\partial\varphi}{\partial z} = 0, \tag{24}$$

where $\beta_{\rm T} = -\frac{1}{T_{\rm e}}\frac{dT_{\rm e}}{dx} > 0$. It is seen that both nonlinearities in Eq. (24) are of the same order when $k_\perp^2\rho_{\rm s}^2 \sim L/L_{\rm T}$.

Vortex structures deserve attention because they carry along trapped medium particles and thus make an essential contribution to global circulation processes. Next, we will construct the appropriate nonlinear solitary vortices on the basis of equations obtained above.[19]

For intermediate-scale structures, $k_\perp\rho_{\rm s} \geqslant 1$, if we normalize time by $\omega_{\rm ci}^{-1}$, lengths by $\rho_{\rm s}$, and velocity w

by $\rho_s\omega_{ci}$ then we can rewrite the system (22) in the following dimensionless form

$$\frac{\partial\varphi}{\partial t}-\frac{\partial}{\partial t}\Delta_\perp\varphi+\beta_n\frac{\partial\varphi}{\partial y}-J(\varphi,\Delta_\perp\varphi)+\frac{\partial w}{\partial z}=0,$$
$$\frac{\partial w}{\partial t}+J(\varphi,w)+\frac{\partial\varphi}{\partial z}=0, \tag{25}$$

where $\beta_n=-\frac{1}{n_0}\frac{dn_0}{dx}>0$ is the dimensionless value. As the system (25) contains only the vector nonlinearity, it is expected that it has a dipolar vortex solution.[20] Such solutions have been constructed by Meiss and Horton[12] and we will represent here only the final results. Let us find the solutions ϕ and w of system (25) in the form of traveling along the y-axis waves, i.e., $X(x,y,z,t)\Rightarrow X(x,\eta)$, where $\eta=y-Ut+\alpha z$. In this transformation, U is the velocity of vortical structure propagating at the right angle to the magnetic field, and α is the inclination angle of vortex front with respect to the plane, which is normal to B_0e_z. Thus small angles $\alpha\ll 1$ correspond to the waves propagating almost perpendicular to the equilibrium magnetic field B_0e_z. Then the system (25) can be transformed in terms of Jacobians as follows:

$$J(\varphi-Ux,\Delta_\perp\varphi-Ux+\beta_n x)=J(\alpha x,w),$$
$$J(\varphi-Ux,w-\alpha x)=0. \tag{26}$$

Choosing zero boundary conditions at infinity ($w,\varphi\to 0$) we determine the particular solution of the second equation of (26) as

$$w=\frac{\alpha}{U}\varphi. \tag{27}$$

Substituting Eq. (27) into the first equation of (26) gives

$$J\Big(\varphi-Ux,\Delta_\perp\varphi-Ux+\beta_n x+\frac{\alpha^2}{U}x\Big)=0. \tag{28}$$

Introducing the circle of $r=a$ radius we divide the integration area into the internal ($r<a$) and the external ($r>a$) regions. Accordingly, we obtain the following equation for external region,[12]

$$\Delta_\perp\varphi_e=\Big(1-\frac{\beta_n}{U}-\frac{\alpha^2}{U^2}\Big)\varphi_e,\ \text{for } r>a. \tag{29}$$

Similarly for the inner region we obtain the following appropriate equation

$$\Delta_\perp\varphi_i=\Big(U-\beta_n-\frac{\alpha^2}{U}\Big)x-C_i(\varphi_i-Ux),\ \text{for } r>a, \tag{30}$$

where C_i is an integration constant. Solutions to Eq. (29), and Eq. (30) in the (x,η) plane can be represented in the polar coordinates r and ϑ, i.e., $x=r\cos\vartheta$ and $y=r\sin\vartheta$. As we are looking for the exponentially vanishing at infinity ($r\to\infty$) solution to Eq. (29) then the following condition should be satisfied,

$$p^2=1-\frac{\alpha^2}{U^2}-\frac{\beta_n}{U}>0. \tag{31}$$

This equation is called the modified dispersion equation (MDE) of vortex. Then particular solution to Eq. (29) in the external region is[12]

$$\varphi_e(r,\vartheta)=BK_1(pr)\cos\vartheta,\ \text{for } r>a, \tag{32}$$

where B and $p>0$ are constants, while K_n is the McDonald function. Using formal substitutions $p^2\to -k_\perp^2$, $U\to\omega/k_y$, $\alpha\to k_z/k_y$ from the MDE equation we obtain the linear dispersion relation (14) in dimensionless variables. The inequality (31) defines the following areas of vortex velocity:

$$U>\frac{1}{2}(\beta_n+\sqrt{\beta_n^2+4\alpha^2}),\ \text{positive velocities}, \tag{33}$$
$$U<\frac{1}{2}(\beta_n-\sqrt{\beta_n^2+4\alpha^2}),\ \text{negative velocities}. \tag{34}$$

It is obvious that these intervals are outside the linear waves phase velocities intervals (15), which is the necessary condition for zero linear wave radiation by vortices.

As to the internal solution to Eq. (30) it can be found in terms of the Bessel functions of first kind J_n as follows:[12]

$$\varphi_i(r,\vartheta)=\Big[AJ_1(kr)+\frac{p^2+k^2}{k^2}Ur\Big]\cos\vartheta,\ \text{for } r<a. \tag{35}$$

where A is a constant, and $k^2=C_i$.

Correspondingly we can find the following expressions for the vorticities,

$$\Delta_\perp\varphi_e=Bp^2K_1(pr)\cos\vartheta,$$
$$\Delta_\perp\varphi_i=-Ak^2J_1(kr)\cos\vartheta. \tag{36}$$

To satisfy the solutions (26) on the circle $r=a$ (i.e., on the whole x,η plane) we require the fulfillment of the following boundary conditions

$$\varphi_i-Ur\cos\vartheta|_{r=a}=\varphi_e-Ur\cos\vartheta|_{r=a}=0. \tag{37}$$

From these conditions we define the integration constants

$$A=-\frac{p^2}{k^2}\frac{Ua}{J_1(ka)},\quad B=\frac{Ua}{K_1(ka)}. \tag{38}$$

Note that with Eq. (38) the solutions (32), (35) and vorticities (36) are also continuous. Condition (37) is equivalent to the demand radial velocity component in the reference frame moving with velocity U to be zero on the circle $r=a$. Owing to Eq. (37) the circle $r=a$ becomes a streamline. Lastly, requiring the continuity of the first derivatives

$$\frac{\partial\varphi_i}{\partial r}\Big|_{r=a}=\frac{\partial\varphi_e}{\partial r}\Big|_{r=a}, \tag{39}$$

we obtain the so-called parameter matching condition (PMC)

$$\frac{J_2(ka)}{kaJ_1(ka)}=-\frac{K_2(pa)}{paJ_1(pa)}, \tag{40}$$

which connects the parameters k, p and a, and two of them may be considered as the independent ones. Physically the condition (39) corresponds to the continuity of tangent velocities, i.e., to the slipless motion of the fluid. Thus only three parameters α, U and a remain undefined in the solution.

For large-scale structures, $k_\perp\rho_s \ll 1$, if we normalize the time by ω_{ci}^{-1}, the lengths by ρ_s, and the velocity w by $\rho_s\omega_{ci}$ then we rewrite the system of Eq. (24) in the following dimensionless system form

$$\frac{\partial\varphi}{\partial t} - \frac{\partial}{\partial t}\Delta_\perp\varphi + \beta_n\frac{\partial\varphi}{\partial y} - \beta_T\varphi\frac{\partial\varphi}{\partial y} - J(\varphi,\Delta_\perp\varphi) + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial z} = 0,$$

$$\frac{\partial w}{\partial t} + J(\varphi,w) + \frac{\partial\varphi}{\partial z} = 0, \quad (41)$$

where $\beta_n = -\frac{1}{n_0}\frac{dn_0}{dx} > 0$ and $\beta_T = -\frac{1}{T_e}\frac{dT_e}{dx} > 0$ are the dimensionless values. Note that in Eq. (41) the scalar and vector nonlinearities are of the same order only in the case of large-scale structures, when $L^3/L_T \sim 1$.

If $L^3/L_T \ll 1$, we can ignore the scalar nonlinearity and the sixth nonlinear term in Eq. (41). Then we obtain the system (25) describing dipole vortical structures which have been considered. Significant difference from the solution (32) is the weak spatial localization of dipole vortices owing to the condition $k_\perp\rho_s \ll 1$, which means the parameter $p \to 0$.

If $L^3/L_T \gg 1$ from the system (41) we obtain the following one

$$\frac{\partial\varphi}{\partial t} - \frac{\partial}{\partial t}\Delta_\perp\varphi + \beta_n\frac{\partial\varphi}{\partial y} - \beta_T\varphi\frac{\partial\varphi}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial z} = 0,$$

$$\frac{\partial w}{\partial t} + J(\varphi,w) + \frac{\partial\varphi}{\partial z} = 0. \quad (42)$$

This system contains the scalar nonlinearities and we expect to find the monopolar vortical solutions. In the case of the traveling waves $\eta = y - Ut + \alpha z$, solution of the second equation of (42) can be represented in the form (33). Substituting this solution w into Eq. (40) gives

$$\Big(1 - \frac{\alpha^2}{U^2} - \frac{\beta_n}{U}\Big)\frac{\partial\varphi}{\partial\eta} - \frac{\partial}{\partial\eta}\Delta_\perp\varphi + \frac{1}{2}\Big(\frac{\beta_T}{U} - \frac{\alpha^2}{U^2}\Big)\frac{\partial\varphi^2}{\partial\eta} = 0. \quad (43)$$

Integration over η yields

$$\Delta_\perp\varphi - \Lambda\varphi + S\varphi^2 = 0, \quad (44)$$

where

$$\Lambda = 1 - \frac{\alpha^2}{U^2} - \frac{\beta_n}{U}, \quad S = -\frac{1}{2U}\Big(\beta_T - \frac{\alpha^2}{U}\Big). \quad (45)$$

Equation (44) describes the scalar structures on the drift-ion-acoustic waves. It is evident that even when the temperature gradient parameter $\beta_T = 0$, the scalar nonlinearity coming from the fifth term of Eq. (43) supports the solitary structure.

Further we use the technique developed by Mikhailovskii[10] to obtain the explicit soliton solution. For sufficiently far distances, Eq. (44) reduces to linear Eq. (29) with $\Lambda > 0$. Thus in the case of the scalar structures (as in the case of vector ones), the condition of spatially isolated structures means $\Lambda > 0$. Multiplying Eq. (44) by φ and integrating over space we obtain the following integral form

$$\int [(\nabla_\perp\varphi)^2 + \Lambda\varphi^2 - S\varphi^3] dr_\perp = 0. \quad (46)$$

As $\Lambda > 0$, the given integral can be zero if $\mathrm{sgn}\varphi = \mathrm{sgn}S$.

To obtain the analytical solution of scalar vortical structures we examine the one-dimensional case when $\partial^2/\partial x^2 \ll \partial^2/\partial\eta^2$, then Eq. (44) reduces to

$$\frac{\partial^2\varphi}{\partial\eta^2} - \Lambda\varphi + S\varphi^2 = 0. \quad (47)$$

This equation has the following soliton solution

$$\varphi(\eta) = \varphi_0 \cosh^{-2}(\Lambda^{1/2}\eta/2), \quad (48)$$

where the amplitude is $\varphi_0 = \frac{3}{2}\frac{\Lambda}{S}$.

In summary, we have shown the possibility of formation of nonlinear solitary vortical structures on the low-frequency coupled electrostatic DIAWs. Taking into account results of laboratory plasma experiments we focus on large-scale ($k_\perp\rho_s \ll 1$) coupled DIAWs.

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